



NEET — JEE MAINS - ADVANCED

Chapter 1 : Units and Measurement (Solved Examples & Exercises)

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Notes

Step-by-step solutions • NCERT Examples 1.1–1.5 and Exercises 1.1–1.17 • Class 11 Physics

Part A — NCERT Solved Examples

Example 1.1

Question: Each side of a cube is measured to be 7.203 m. What are the total surface area and the volume of the cube to appropriate significant figures?

Solution: The measured length has 4 significant figures, so the area and volume must also be rounded off to 4 significant figures.

$$\text{Surface area} = 6a^2 = 6(7.203)^2 \text{ m}^2 = 311.299254 \text{ m}^2$$

$$\text{Volume} = a^3 = (7.203)^3 \text{ m}^3 = 373.714754 \text{ m}^3$$

Answer: Surface area = 311.3 m²; Volume = 373.7 m³.

Example 1.2

Question: 5.74 g of a substance occupies 1.2 cm³. Express its density by keeping the significant figures in view.

Solution: The mass has 3 significant figures, but the volume has only 2. In division, the result keeps the least number of significant figures, i.e. 2.

$$\text{Density} = \frac{\text{mass}}{\text{volume}} = \frac{5.74 \text{ g}}{1.2 \text{ cm}^3} = 4.783... \text{ g cm}^{-3}$$

Answer: Density = 4.8 g cm⁻³.

Example 1.3

Question: Consider the equation $\frac{1}{2}mv^2 = mgh$, where m is the mass of the body, v its velocity, g the acceleration due to gravity and h the height. Check whether this equation is dimensionally correct.

Solution:

$$\text{LHS: } [M][L T^{-1}]^2 = [M][L^2 T^{-2}] = [M L^2 T^{-2}]$$

$$\text{RHS: } [M][L T^{-2}][L] = [M L^2 T^{-2}]$$

$\left(\frac{1}{2}\right)$ is a pure number and has no dimensions.)

Answer: The dimensions of LHS and RHS are the same, so the equation is **dimensionally correct**.

Example 1.4

Question: The SI unit of energy is $J = kg\ m^2\ s^{-2}$; that of speed v is $m\ s^{-1}$ and of acceleration a is $m\ s^{-2}$. Which of the following formulae for kinetic energy K can you rule out on the basis of dimensional arguments (m is the mass of the body)? (a) $K = m^2v^3$ (b) $K = \frac{1}{2}mv^2$ (c) $K = ma$ (d) $K = \frac{3}{16}mv^2$ (e) $K = \frac{1}{2}mv^2 + ma$

Solution: Kinetic energy has dimensions $[K] = [M\ L^2\ T^{-2}]$. Every correct formula must have the same dimensions on both sides, and only quantities with the same dimensions can be added.

Formula	Dimensions of RHS	Result
(a) m^2v^3	$[M^2L^3T^{-3}]$	Ruled out
(b) $\frac{1}{2}mv^2$	$[M\ L^2\ T^{-2}]$	Possible
(c) ma	$[M\ L\ T^{-2}]$	Ruled out
(d) $\frac{3}{16}mv^2$	$[M\ L^2\ T^{-2}]$	Possible
(e) $\frac{1}{2}mv^2 + ma$	Adds two different dimensions	Ruled out

Answer: Formulae (a), (c) and (e) are ruled out. Dimensional analysis cannot tell which of (b) and (d) is correct, because the constants $\frac{1}{2}$ and $\frac{3}{16}$ have no dimensions. From the definition of kinetic energy (Chapter 5), the correct formula is (b).

Example 1.5

Question: A simple pendulum has a bob attached to a string, oscillating under gravity. Suppose the period of oscillation depends on its length (l), mass of the bob (m) and acceleration due to gravity (g). Derive the expression for its time period using the method of dimensions.

Solution: Let

$$T = k\ l^x\ g^y\ m^z$$

where k is a dimensionless constant. Writing dimensions on both sides:

$$[L^0M^0T^1] = [L]^x[L\ T^{-2}]^y[M]^z = [L^{x+y}T^{-2y}M^z]$$

Equating the powers of L, T and M:

$$\begin{aligned} x + y &= 0, & -2y &= 1, & z &= 0 \\ \Rightarrow x &= \frac{1}{2}, & y &= -\frac{1}{2}, & z &= 0 \end{aligned}$$

$$\text{So } T = k\ l^{1/2}g^{-1/2} = k\ \sqrt{\frac{l}{g}}.$$

Answer: $T = k\ \sqrt{\frac{l}{g}}$. The constant k cannot be found by dimensions; actually $k = 2\pi$, so $T = 2\pi\ \sqrt{\frac{l}{g}}$. The period does not depend on the mass of the bob.

Part B — NCERT Exercises

Note: In stating numerical answers, take care of significant figures.

Exercise 1.1

Question: Fill in the blanks.

(a) The volume of a cube of side 1 cm is equal to m^3 .

$$V = (1 \text{ cm})^3 = (10^{-2} \text{ m})^3 = 10^{-6} \text{ m}^3$$

(b) The surface area of a solid cylinder of radius 2.0 cm and height 10.0 cm is equal to $(\text{mm})^2$.

$$r = 2.0 \text{ cm} = 20 \text{ mm}, h = 10.0 \text{ cm} = 100 \text{ mm}.$$

$$A = 2\pi r(r + h) = 2 \times 3.14 \times 20 \times (20 + 100) = 1.5079 \times 10^4 \text{ mm}^2$$

The radius has only 2 significant figures, so $A = 1.5 \times 10^4 \text{ mm}^2$.

(c) A vehicle moving with a speed of 18 km h^{-1} covers m in 1 s.

$$18 \text{ km h}^{-1} = 18 \times \frac{1000}{3600} \text{ m s}^{-1} = 5 \text{ m s}^{-1}$$

So it covers **5 m** in 1 s.

(d) The relative density of lead is 11.3. Its density is g cm^{-3} or kg m^{-3} .

Density of water = $1 \text{ g cm}^{-3} = 10^3 \text{ kg m}^{-3}$. So density of lead = $11.3 \text{ g cm}^{-3} = 1.13 \times 10^4 \text{ kg m}^{-3}$.

Answer: (a) 10^{-6} (b) 1.5×10^4 (c) 5 (d) 11.3; 1.13×10^4

Exercise 1.2

Question: Fill in the blanks by suitable conversion of units.

(a) $1 \text{ kg m}^2 \text{ s}^{-2} = \dots \text{g cm}^2 \text{ s}^{-2}$

$$1 \text{ kg m}^2 \text{ s}^{-2} = (10^3 \text{ g})(10^2 \text{ cm})^2 \text{ s}^{-2} = 10^3 \times 10^4 \text{ g cm}^2 \text{ s}^{-2} = 10^7 \text{ g cm}^2 \text{ s}^{-2}$$

(b) $1 \text{ m} = \dots \text{ly}$

1 light year = $9.46 \times 10^{15} \text{ m}$, so

$$1 \text{ m} = \frac{1}{9.46 \times 10^{15}} \text{ ly} = 1.06 \times 10^{-16} \text{ ly}$$

(c) $3.0 \text{ m s}^{-2} = \dots \text{km h}^{-2}$

$$3.0 \text{ m s}^{-2} = 3.0 \times \frac{10^{-3} \text{ km}}{\left(\frac{1}{3600} \text{ h}\right)^2} = 3.0 \times 10^{-3} \times (3600)^2 \text{ km h}^{-2} = 3.9 \times 10^4 \text{ km h}^{-2}$$

(d) $G = 6.67 \times 10^{-11} \text{ N m}^2 (\text{kg})^{-2} = \dots (\text{cm})^3 \text{ s}^{-2} \text{ g}^{-1}$

$$\begin{aligned} 1 \text{ N m}^2 \text{ kg}^{-2} &= 1 \text{ kg m s}^{-2} \cdot \text{m}^2 \cdot \text{kg}^{-2} = 1 \text{ m}^3 \text{ s}^{-2} \text{ kg}^{-1} \\ &= (10^2 \text{ cm})^3 \text{ s}^{-2} (10^3 \text{ g})^{-1} = 10^6 \times 10^{-3} \text{ cm}^3 \text{ s}^{-2} \text{ g}^{-1} = 10^3 \text{ cm}^3 \text{ s}^{-2} \text{ g}^{-1} \end{aligned}$$

So $G = 6.67 \times 10^{-11} \times 10^3 = 6.67 \times 10^{-8} \text{ cm}^3 \text{ s}^{-2} \text{ g}^{-1}$.

Answer: (a) 10^7 (b) 1.06×10^{-16} (c) 3.9×10^4 (d) 6.67×10^{-8}

Exercise 1.3

Question: A calorie is a unit of heat (energy in transit) and it equals about 4.2 J where $1 \text{ J} = 1 \text{ kg m}^2 \text{ s}^{-2}$.

Suppose we employ a system of units in which the unit of mass equals $\alpha \text{ kg}$, the unit of length equals $\beta \text{ m}$ and the unit of time is $\gamma \text{ s}$. Show that a calorie has a magnitude $4.2 \alpha^{-1} \beta^{-2} \gamma^2$ in terms of the new units.

Solution: Energy has dimensions $[M L^2 T^{-2}]$. Using $n_1 u_1 = n_2 u_2$:

$$n_2 = n_1 \left[\frac{M_1}{M_2} \right]^1 \left[\frac{L_1}{L_2} \right]^2 \left[\frac{T_1}{T_2} \right]^{-2}$$

Here $n_1 = 4.2$, $M_1 = 1 \text{ kg}$, $L_1 = 1 \text{ m}$, $T_1 = 1 \text{ s}$, $M_2 = \alpha \text{ kg}$, $L_2 = \beta \text{ m}$, $T_2 = \gamma \text{ s}$.

$$n_2 = 4.2 \left[\frac{1}{\alpha} \right] \left[\frac{1}{\beta} \right]^2 \left[\frac{1}{\gamma} \right]^{-2}$$

Answer: 1 calorie = $4.2 \alpha^{-1} \beta^{-2} \gamma^2$ new units of energy. Hence shown.

Exercise 1.4

Question: Explain this statement clearly: “To call a dimensional quantity ‘large’ or ‘small’ is meaningless without specifying a standard for comparison.” In view of this, reframe the following statements wherever necessary.

Explanation: A quantity is large or small only when compared with a standard of the same kind. For example, 1 cm is large compared with the size of an atom, but small compared with the size of a room. So every such statement must name the standard.

Statement	Reframed statement
(a) atoms are very small objects	Atoms are very small compared with a pin head or a grain of sand.
(b) a jet plane moves with great speed	A jet plane moves with great speed compared with a car or a train.
(c) the mass of Jupiter is very large	The mass of Jupiter is very large compared with the mass of the earth.
(d) the air inside this room contains a large number of molecules	The air inside this room contains a large number of molecules compared with a geometry box (or a small cup).
(e) a proton is much more massive than an electron	Already correct — the electron is the standard. (About 1836 times.)
(f) the speed of sound is much smaller than the speed of light	Already correct — the speed of light is the standard.

Exercise 1.5

Question: A new unit of length is chosen such that the speed of light in vacuum is unity. What is the distance between the Sun and the Earth in terms of the new unit if light takes 8 min and 20 s to cover this distance?

Solution: Speed of light = 1 new unit per second, i.e. the new unit of length is the distance light travels in 1 s.

Time = 8 min 20 s = $8 \times 60 + 20 = 500 \text{ s}$.

Distance = speed $\times$ time = $1 \times 500 = 500$ new units

Answer: 500 new units (each equal to $3 \times 10^8 \text{ m}$).

Exercise 1.6

Question: Which of the following is the most precise device for measuring length: (a) a vernier callipers with 20 divisions on the sliding scale (b) a screw gauge of pitch 1 mm and 100 divisions on the circular scale (c) an optical instrument that can measure length to within a wavelength of light?

Solution: The most precise device has the smallest least count.

- **(a)** Vernier callipers: $LC = \frac{1 \text{ mm}}{20} = 0.05 \text{ mm} = 5 \times 10^{-5} \text{ m}$.

- **(b)** Screw gauge: $LC = \frac{\text{pitch}}{\text{no. of divisions}} = \frac{1 \text{ mm}}{100} = 0.01 \text{ mm} = 10^{-5} \text{ m}$.
- **(c)** Optical instrument: $LC \approx \text{wavelength of light} \approx 6 \times 10^{-7} \text{ m}$.

Answer: **(c) the optical instrument** is the most precise.

Exercise 1.7

Question: A student measures the thickness of a human hair by looking at it through a microscope of magnification 100. He makes 20 observations and finds that the average width of the hair in the field of view of the microscope is 3.5 mm. What is the estimate on the thickness of hair?

Solution:

$$\text{Actual thickness} = \frac{\text{observed width}}{\text{magnification}} = \frac{3.5 \text{ mm}}{100}$$

Answer: Thickness of hair = 0.035 mm.

Exercise 1.8

Question: Answer the following.

(a) You are given a thread and a metre scale. How will you estimate the diameter of the thread?

Wind the thread closely on a uniform cylindrical rod (e.g. a pencil) so that the turns touch each other. Count the number of turns n and measure the length L of the winding with the metre scale. Then

$$\text{Diameter of the thread} = \frac{L}{n}$$

Taking a large number of turns improves the accuracy.

(b) A screw gauge has a pitch of 1.0 mm and 200 divisions on the circular scale. Do you think it is possible to increase the accuracy of the screw gauge arbitrarily by increasing the number of divisions on the circular scale?

$LC = \frac{\text{pitch}}{N}$, so in theory a larger N gives a smaller least count. **But in practice, accuracy cannot be increased arbitrarily.** With very many divisions, the divisions become too close to be read by the eye, and errors due to the screw (backlash, uneven threads) become larger than the least count.

(c) Why is a set of 100 measurements of the diameter of a brass rod expected to yield a more reliable estimate than a set of 5 measurements only?

Each measurement has **random errors**, which may be positive or negative. When a large number of readings is taken, these errors tend to cancel out, so the mean of 100 readings is closer to the true value than the mean of 5 readings.

Exercise 1.9

Question: The photograph of a house occupies an area of 1.75 cm^2 on a 35 mm slide. The slide is projected on to a screen, and the area of the house on the screen is 1.55 m^2 . What is the linear magnification of the projector-screen arrangement?

Solution:

$$\text{Areal magnification} = \frac{\text{area on screen}}{\text{area on slide}} = \frac{1.55 \text{ m}^2}{1.75 \times 10^{-4} \text{ m}^2} = 8857$$

Area is proportional to the square of length, so

$$\text{Linear magnification} = \sqrt{\text{areal magnification}} = \sqrt{8857}$$

Answer: Linear magnification ≈ 94.1 .

Exercise 1.10

Question: State the number of significant figures in the following: (a) 0.007 m^2 (b) $2.64 \times 10^{24} \text{ kg}$ (c) 0.2370 g cm^{-3} (d) 6.320 J (e) 6.032 N m^{-2} (f) 0.0006032 m^2

Solution:

Quantity	Significant figures	Reason
(a) 0.007 m^2	1	Zeros before the first non-zero digit are not significant
(b) $2.64 \times 10^{24} \text{ kg}$	3	The power of 10 is not counted
(c) 0.2370 g cm^{-3}	4	Trailing zero after the decimal point is significant
(d) 6.320 J	4	Trailing zero after the decimal point is significant
(e) 6.032 N m^{-2}	4	Zero between non-zero digits is significant
(f) 0.0006032 m^2	4	Leading zeros not significant; zero between 6 and 3 is

Exercise 1.11

Question: The length, breadth and thickness of a rectangular sheet of metal are 4.234 m , 1.005 m and 2.01 cm respectively. Give the area and volume of the sheet to correct significant figures.

Solution: $l = 4.234 \text{ m}$ (4 s.f.), $b = 1.005 \text{ m}$ (4 s.f.), $t = 2.01 \text{ cm} = 0.0201 \text{ m}$ (3 s.f.). The result must have 3 significant figures.

Total surface area of the sheet:

$$A = 2(lb + bt + tl) = 2(4.234 \times 1.005 + 1.005 \times 0.0201 + 0.0201 \times 4.234) \\ = 2(4.25517 + 0.02020 + 0.08510) = 8.7209 \text{ m}^2$$

Volume:

$$V = l \times b \times t = 4.234 \times 1.005 \times 0.0201 = 0.085529 \text{ m}^3$$

Answer: Area = 8.72 m^2 ; Volume = 0.0855 m^3 .

Exercise 1.12

Question: The mass of a box measured by a grocer's balance is 2.30 kg . Two gold pieces of masses 20.15 g and 20.17 g are added to the box. What is (a) the total mass of the box, (b) the difference in the masses of the pieces to correct significant figures?

Solution:

(a) Total mass = $2.30 \text{ kg} + 0.02015 \text{ kg} + 0.02017 \text{ kg} = 2.34032 \text{ kg}$.

In addition, the result keeps the least number of decimal places. 2.30 kg has 2 decimal places, so total mass = 2.34 kg .

(b) Difference = $20.17 \text{ g} - 20.15 \text{ g} = 0.02 \text{ g}$ (2 decimal places).

Answer: (a) 2.34 kg (b) 0.02 g

Exercise 1.13

Question: A famous relation in physics relates ‘moving mass’ m to the ‘rest mass’ m_0 of a particle in terms of its speed v and the speed of light c . A boy recalls the relation almost correctly but forgets where to put the constant c . He writes $m = \frac{m_0}{(1-v^2)^{1/2}}$. Guess where to put the missing c .

Solution: m and m_0 have the same dimensions, so $(1 - v^2)^{1/2}$ must be **dimensionless**. Since 1 is a pure number, v^2 must also be dimensionless. This is possible only if v^2 is divided by c^2 :

$$\frac{v^2}{c^2} = \frac{[L T^{-1}]^2}{[L T^{-1}]^2} = \text{dimensionless}$$

Answer: $m = \frac{m_0}{(1 - \frac{v^2}{c^2})^{1/2}}$

Exercise 1.14

Question: The unit of length convenient on the atomic scale is known as an angstrom and is denoted by Å: $1 \text{ Å} = 10^{-10} \text{ m}$. The size of a hydrogen atom is about 0.5 Å . What is the total atomic volume in m^3 of a mole of hydrogen atoms?

Solution: Taking the size as the radius, $r = 0.5 \text{ Å} = 0.5 \times 10^{-10} \text{ m}$.

$$\text{Volume of one atom} = \frac{4}{3}\pi r^3 = \frac{4}{3} \times 3.14 \times (0.5 \times 10^{-10})^3 = 5.23 \times 10^{-31} \text{ m}^3$$

One mole contains $N_A = 6.023 \times 10^{23}$ atoms.

$$\text{Total atomic volume} = 6.023 \times 10^{23} \times 5.23 \times 10^{-31} = 3.15 \times 10^{-7} \text{ m}^3$$

Answer: About $3 \times 10^{-7} \text{ m}^3$.

Exercise 1.15

Question: One mole of an ideal gas at standard temperature and pressure occupies 22.4 L (molar volume). What is the ratio of molar volume to the atomic volume of a mole of hydrogen? (Take the size of hydrogen molecule to be about 1 Å .) Why is this ratio so large?

Solution: Molar volume $V_m = 22.4 \text{ L} = 22.4 \times 10^{-3} \text{ m}^3$.

Size (diameter) $= 1 \text{ Å}$, so radius $r = 0.5 \times 10^{-10} \text{ m}$. From Exercise 1.14, the atomic volume of one mole $= 3.15 \times 10^{-7} \text{ m}^3$.

$$\frac{\text{Molar volume}}{\text{Atomic volume}} = \frac{22.4 \times 10^{-3}}{3.15 \times 10^{-7}} = 7.1 \times 10^4$$

Answer: The ratio is about 7×10^4 . It is so large because in a gas the **separation between molecules is very large** compared with the size of a molecule. Most of the volume of a gas is empty space.

Exercise 1.16

Question: Explain this common observation clearly: If you look out of the window of a fast moving train, the nearby trees, houses etc. seem to move rapidly in a direction opposite to the train’s motion, but the distant objects (hill tops, the Moon, the stars etc.) seem to be stationary.

Answer: The apparent motion depends on how fast the **line of sight** (angle) to the object changes.

- For a **nearby** object, a small movement of the train changes the direction of the line of sight by a **large angle**. So the object appears to move rapidly backward (opposite to the train).



- For a **distant** object, the same movement of the train changes the line of sight by a **negligibly small angle**, because the distance is very large. So distant objects appear stationary. (Since we know we are moving, they even seem to move along with us.)

This is the same idea as **parallax**: the parallax angle is inversely proportional to the distance of the object.

Exercise 1.17

Question: The Sun is a hot plasma (ionized matter) with its inner core at a temperature exceeding 10^7 K, and its outer surface at a temperature of about 6000 K. At these high temperatures, no substance remains in a solid or liquid phase. In what range do you expect the mass density of the Sun to be, in the range of densities of solids and liquids or gases? Check if your guess is correct from the following data: mass of the Sun = 2.0×10^{30} kg, radius of the Sun = 7.0×10^8 m.

Solution:

$$\begin{aligned}\rho &= \frac{M}{V} = \frac{M}{\frac{4}{3}\pi R^3} = \frac{2.0 \times 10^{30}}{\frac{4}{3} \times 3.14 \times (7.0 \times 10^8)^3} \\ &= \frac{2.0 \times 10^{30}}{1.44 \times 10^{27}} = 1.4 \times 10^3 \text{ kg m}^{-3}\end{aligned}$$

Answer: $\rho \approx 1.4 \times 10^3 \text{ kg m}^{-3}$. This is in the range of densities of **solids and liquids**, not gases. Although the Sun is a hot gas (plasma), its density is high because of the huge **gravitational compression** of its outer layers on the inner layers.