



NEET — JEE MAINS - ADVANCED

Chapter 2 : Motion in a Straight Line (Solved Examples & Exercises)

by - Tomar Sir

Notes

Step-by-step solutions · NCERT Examples 2.1–2.7 and Exercises 2.1–2.18 · Class 11 Physics

Part A — NCERT Solved Examples

Example 2.1

Question: The position of an object moving along the x-axis is given by $x = a + bt^2$, where $a = 8.5$ m, $b = 2.5$ m s⁻² and t is measured in seconds. What is its velocity at $t = 0$ s and $t = 2.0$ s? What is the average velocity between $t = 2.0$ s and $t = 4.0$ s?

Solution: Velocity:

$$v = \frac{dx}{dt} = \frac{d}{dt}(a + bt^2) = 2bt = 5.0 t \text{ m s}^{-1}$$

At $t = 0$ s: $v = 0$ m s⁻¹. At $t = 2.0$ s: $v = 10$ m s⁻¹.

Average velocity:

$$\bar{v} = \frac{x(4.0) - x(2.0)}{4.0 - 2.0} = \frac{a + 16b - a - 4b}{2.0} = 6.0 \times b = 6.0 \times 2.5 = 15 \text{ m s}^{-1}$$

Answer: $v = 0$ at $t = 0$; $v = 10$ m s⁻¹ at $t = 2.0$ s; average velocity = 15 m s⁻¹.

Example 2.2

Question: Obtain equations of motion for constant acceleration using the method of calculus.

Solution: By definition, $a = \frac{dv}{dt}$, so $dv = a dt$. Integrating:

$$\int_{v_0}^v dv = \int_0^t a dt = a \int_0^t dt \quad (a \text{ is constant}) \Rightarrow v - v_0 = at \Rightarrow v = v_0 + at$$

Further, $v = \frac{dx}{dt}$, so $dx = v dt$:

$$\int_{x_0}^x dx = \int_0^t v dt = \int_0^t (v_0 + at) dt \Rightarrow x - x_0 = v_0 t + \frac{1}{2} at^2$$

$$x = x_0 + v_0 t + \frac{1}{2} at^2$$

We can also write $a = \frac{dv}{dt} = \frac{dv}{dx} \frac{dx}{dt} = v \frac{dv}{dx}$, so $v dv = a dx$:

$$\int_{v_0}^v v \, dv = \int_{x_0}^x a \, dx \Rightarrow \frac{v^2 - v_0^2}{2} = a(x - x_0) \Rightarrow v^2 = v_0^2 + 2a(x - x_0)$$

Answer: $v = v_0 + at$; $x = x_0 + v_0 t + \frac{1}{2}at^2$; $v^2 = v_0^2 + 2a(x - x_0)$. The advantage of this method is that it can be used for motion with non-uniform acceleration also.

Example 2.3

Question: A ball is thrown vertically upwards with a velocity of 20 m s^{-1} from the top of a multistorey building. The height of the point from where the ball is thrown is 25.0 m from the ground. (a) How high will the ball rise? (b) How long will it be before the ball hits the ground? Take $g = 10 \text{ m s}^{-2}$.

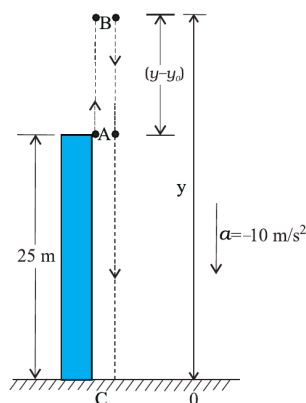


Fig. 2.6

Solution: Take the y -axis vertically upward with zero at the ground. $v_0 = +20 \text{ m s}^{-1}$, $a = -g = -10 \text{ m s}^{-2}$, $y_0 = 25 \text{ m}$.

(a) At the highest point, $v = 0$. Using $v^2 = v_0^2 + 2a(y - y_0)$:

$$0 = (20)^2 + 2(-10)(y - y_0) \Rightarrow y - y_0 = 20 \text{ m}$$

(b) **First method:** Split the motion into upward (A to B) and downward (B to C) parts.

- Upward: $v = v_0 + at \Rightarrow 0 = 20 - 10t_1 \Rightarrow t_1 = 2 \text{ s}$.
- Downward from B (rest, $y_0 = 45 \text{ m}$) to the ground ($y = 0$): $0 = 45 + \frac{1}{2}(-10)t_2^2 \Rightarrow t_2 = 3 \text{ s}$.

Total time $= t_1 + t_2 = 2 + 3 = 5 \text{ s}$.

Second method: Use the initial and final positions directly ($y_0 = 25 \text{ m}$, $y = 0$, $v_0 = 20 \text{ m s}^{-1}$, $a = -10 \text{ m s}^{-2}$):

$$0 = 25 + 20t + \frac{1}{2}(-10)t^2 \Rightarrow 5t^2 - 20t - 25 = 0 \Rightarrow t = 5 \text{ s}$$

Answer: (a) 20 m above the point of launch; (b) 5 s . The second method is better, since we do not have to worry about the path when the acceleration is constant.

Example 2.4

Question: Free-fall: Discuss the motion of an object under free fall. Neglect air resistance.

Solution: An object released near the earth's surface is accelerated downward by gravity, g . With air resistance neglected, it is in free fall. If the height of fall is small compared with the earth's radius, g is constant, so free fall is motion with uniform acceleration.

Take the upward direction as positive: $a = -g = -9.8 \text{ m s}^{-2}$. The object is released from rest at $y = 0$, so $v_0 = 0$:

$$v = 0 - gt = -9.8 t \text{ m s}^{-1}$$

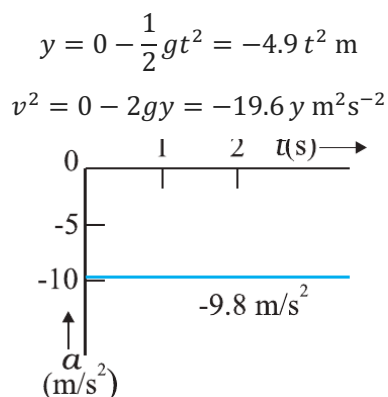


Fig. 2.7 (a): Variation of acceleration with time

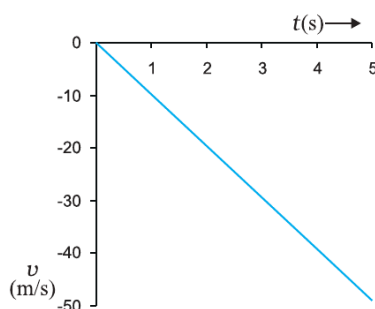


Fig. 2.7 (b): Variation of velocity with time

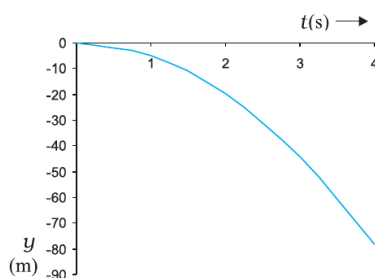


Fig. 2.7 (c): Variation of distance with time

Answer: These equations give velocity and distance as functions of time, and velocity as a function of distance. The a - t graph is a horizontal line, the v - t graph is a straight line, and the y - t graph is a parabola.

Example 2.5

Question: Galileo's law of odd numbers: "The distances traversed, during equal intervals of time, by a body falling from rest, stand to one another in the same ratio as the odd numbers beginning with unity [namely, $1 : 3 : 5 : 7 \dots$]." Prove it.

Solution: Divide the time of fall into equal intervals τ . With zero initial velocity, $y = -\frac{1}{2}gt^2$. Let $y_0 = -\frac{1}{2}g\tau^2$ (position after the first interval).

t	y in terms of y_0	Distance in successive intervals	Ratio
τ	y_0	y_0	1
2τ	$4y_0$	$3y_0$	3
3τ	$9y_0$	$5y_0$	5
4τ	$16y_0$	$7y_0$	7

t	y in terms of y_0	Distance in successive intervals	Ratio
5τ	$25y_0$	$9y_0$	9
6τ	$36y_0$	$11y_0$	11

Answer: The distances in successive equal intervals are in the ratio **1 : 3 : 5 : 7 : 9 : 11 ...** This law was established by Galileo Galilei (1564–1642), the first to make quantitative studies of free fall.

Example 2.6

Question: Stopping distance of vehicles: When brakes are applied to a moving vehicle, the distance it travels before stopping is called stopping distance. It is an important factor for road safety and depends on the initial velocity (v_0) and the braking capacity, or deceleration, $-a$, that is caused by the braking. Derive an expression for the stopping distance of a vehicle in terms of v_0 and a .

Solution: Let the distance travelled before stopping be d_s . Using $v^2 = v_0^2 + 2ax$ with $v = 0$:

$$d_s = \frac{-v_0^2}{2a}$$

(with a negative, d_s is positive).

Answer: The stopping distance is proportional to the **square of the initial velocity**. Doubling the initial velocity increases the stopping distance by a factor of 4 (for the same deceleration). For a car of a particular make, braking distances of 10 m, 20 m, 34 m and 50 m at 11, 15, 20 and 25 m/s are nearly consistent with this formula. Stopping distance is considered in setting speed limits, for example in school zones.

Example 2.7

Question: Reaction time: When a situation demands our immediate action, it takes some time before we really respond. Reaction time is the time a person takes to observe, think and act. You can measure your reaction time by a simple experiment. Take a ruler and ask your friend to drop it vertically through the gap between your thumb and forefinger. After you catch it, find the distance d travelled by the ruler. In a particular case, d was found to be 21.0 cm. Estimate the reaction time.

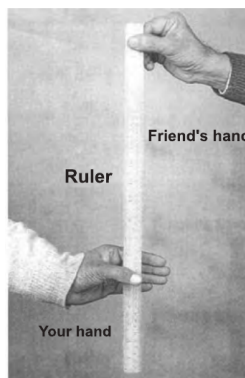


Fig. 2.8

Solution: The ruler falls freely: $v_0 = 0$, $a = -g = -9.8 \text{ m s}^{-2}$. The distance d and the reaction time t_r are related by

$$d = -\frac{1}{2}gt_r^2 \Rightarrow t_r = \sqrt{\frac{2d}{g}} = \sqrt{\frac{2 \times 0.21}{9.8}} \text{ s}$$

Answer: $t_r \approx 0.2 \text{ s}$.

Part B — NCERT Exercises

Exercise 2.1

Question: In which of the following examples of motion can the body be considered approximately a point object: (a) a railway carriage moving without jerks between two stations, (b) a monkey sitting on top of a man cycling smoothly on a circular track, (c) a spinning cricket ball that turns sharply on hitting the ground, (d) a tumbling beaker that has slipped off the edge of a table?

Answer:

- **(a) Yes.** The size of the carriage is very small compared with the distance between two stations.
- **(b) Yes.** The size of the monkey is very small compared with the length of the circular track.
- **(c) No.** The spinning ball turns sharply; the distance it moves after hitting the ground is not large compared with its size, and its rotation matters.
- **(d) No.** The size of the beaker is comparable to the height of the table it falls from.

Exercise 2.2

Question: The position-time (x - t) graphs for two children A and B returning from their school O to their homes P and Q respectively are shown in Fig. 2.9. Choose the correct entries in the brackets below: (a) (A/B) lives closer to the school than (B/A); (b) (A/B) starts from the school earlier than (B/A); (c) (A/B) walks faster than (B/A); (d) A and B reach home at the (same/different) time; (e) (A/B) overtakes (B/A) on the road (once/twice).

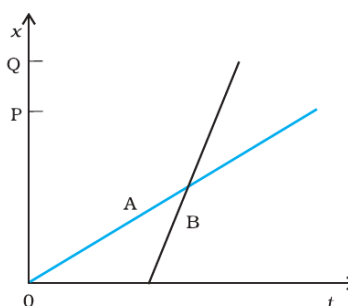


Fig. 2.9

Answer:

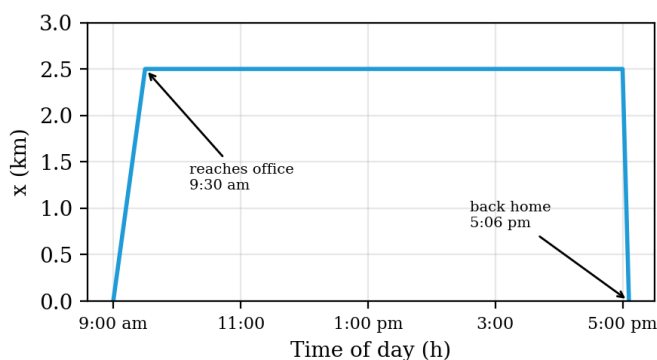
- **(a) A** lives closer to the school than **B** ($OP < OQ$).
- **(b) A** starts from the school earlier than **B** (A's graph starts at $t = 0$; B's starts later).
- **(c) B** walks faster than **A** (B's graph has a greater slope).
- **(d)** A and B reach home at **different** times.
- **(e) B** overtakes **A** on the road **once** (the two graphs cross once).

Exercise 2.3

Question: A woman starts from her home at 9.00 am, walks with a speed of 5 km h^{-1} on a straight road up to her office 2.5 km away, stays at the office up to 5.00 pm, and returns home by an auto with a speed of 25 km h^{-1} . Choose suitable scales and plot the x - t graph of her motion.

Solution:

- Time to walk to the office $= \frac{2.5}{5} \text{ h} = 0.5 \text{ h} = 30 \text{ min}$. She reaches at **9:30 am**.
- She stays at $x = 2.5 \text{ km}$ from 9:30 am to 5:00 pm (horizontal line).
- Time to return by auto $= \frac{2.5}{25} \text{ h} = 0.1 \text{ h} = 6 \text{ min}$. She reaches home at **5:06 pm**.



x-t graph for Exercise 2.3

Answer: The graph rises from 0 to 2.5 km between 9:00 and 9:30 am, stays flat until 5:00 pm, and falls steeply back to 0 at 5:06 pm.

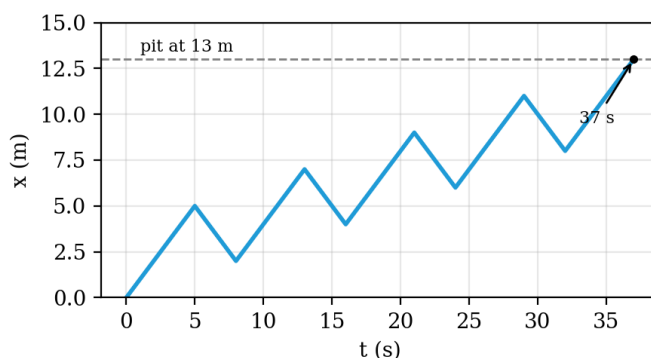
Exercise 2.4

Question: A drunkard walking in a narrow lane takes 5 steps forward and 3 steps backward, followed again by 5 steps forward and 3 steps backward, and so on. Each step is 1 m long and requires 1 s. Plot the x-t graph of his motion. Determine graphically and otherwise how long the drunkard takes to fall in a pit 13 m away from the start.

Solution: In each cycle of 8 steps (8 s), the net displacement is $5 - 3 = 2$ m.

- After 4 cycles (32 s), he is at 8 m.
- He then takes 5 steps forward (5 s) and reaches $8 + 5 = 13$ m — the pit.

$$\text{Time} = 32 + 5 = 37 \text{ s}$$



x-t graph for Exercise 2.4

Answer: The drunkard falls into the pit after **37 s**.

Exercise 2.5

Question: A car moving along a straight highway with speed of 126 km h^{-1} is brought to a stop within a distance of 200 m. What is the retardation of the car (assumed uniform), and how long does it take for the car to stop?

Solution: $v_0 = 126 \times \frac{5}{18} = 35 \text{ m s}^{-1}$, $v = 0$, $x = 200$ m.

$$v^2 = v_0^2 + 2ax \Rightarrow a = -\frac{v_0^2}{2x} = -\frac{(35)^2}{2 \times 200} = -3.06 \text{ m s}^{-2}$$

$$t = \frac{v - v_0}{a} = \frac{0 - 35}{-3.06} = 11.4 \text{ s}$$

Answer: Retardation = 3.06 m s^{-2} ; time to stop = 11.4 s.

Exercise 2.6

Question: A player throws a ball upwards with an initial speed of 29.4 m s^{-1} . (a) What is the direction of acceleration during the upward motion of the ball? (b) What are the velocity and acceleration of the ball at the highest point of its motion? (c) Choose the $x = 0 \text{ m}$ and $t = 0 \text{ s}$ to be the location and time of the ball at its highest point, vertically downward direction to be the positive direction of x -axis, and give the signs of position, velocity and acceleration of the ball during its upward, and downward motion. (d) To what height does the ball rise and after how long does the ball return to the player's hands? (Take $g = 9.8 \text{ m s}^{-2}$ and neglect air resistance.)

Solution:

(a) Vertically **downward** (acceleration due to gravity).

(b) At the highest point, velocity = **0** and acceleration = **9.8 m s^{-2} downward**.

(c) With $x = 0$ at the highest point and downward positive:

Motion	Position	Velocity	Acceleration
Upward	negative	negative	positive
Downward	positive	positive	positive

(d) Height: $h = \frac{u^2}{2g} = \frac{(29.4)^2}{2 \times 9.8} = 44.1 \text{ m}$.

Time to reach the top: $t = \frac{u}{g} = \frac{29.4}{9.8} = 3 \text{ s}$. Time up = time down, so total time = 6 s.

Answer: (d) The ball rises **44.1 m** and returns to the player's hands after **6 s**.

Exercise 2.7

Question: Read each statement below carefully and state with reasons and examples, if it is true or false. A particle in one-dimensional motion (a) with zero speed at an instant may have non-zero acceleration at that instant, (b) with zero speed may have non-zero velocity, (c) with constant speed must have zero acceleration, (d) with positive value of acceleration must be speeding up.

Answer:

- (a) **True.** A ball thrown up has zero speed at its highest point, but its acceleration is g .
- (b) **False.** Speed is the magnitude of velocity. If the speed is zero, the velocity must be zero.
- (c) **True.** In one dimension, constant speed means the velocity stays the same (the direction cannot reverse without the speed passing through zero), so the acceleration is zero.
- (d) **False.** It depends on the choice of positive direction. A particle moving in the negative direction with positive acceleration **slows down**. A particle speeds up only when acceleration and velocity are in the same direction.

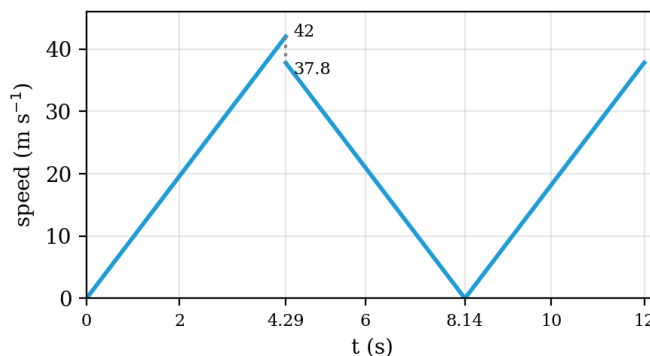
Exercise 2.8

Question: A ball is dropped from a height of 90 m on a floor. At each collision with the floor, the ball loses one tenth of its speed. Plot the speed-time graph of its motion between $t = 0$ to 12 s.

Solution:

- **Fall:** $t_1 = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 90}{9.8}} = 4.29 \text{ s}$. Speed on hitting the floor: $v = gt_1 = 42 \text{ m s}^{-1}$.

- **After the first bounce:** speed = $0.9 \times 42 = 37.8 \text{ m s}^{-1}$ (upward). Time to rise = $\frac{37.8}{9.8} = 3.86 \text{ s}$, so the ball stops at $4.29 + 3.86 = 8.14 \text{ s}$.
- **Falls again** for 3.86 s and hits the floor at $8.14 + 3.86 = 12 \text{ s}$ with speed 37.8 m s^{-1} .



Speed-time graph for Exercise 2.8

Answer: The speed rises linearly from 0 to 42 m s^{-1} (0 to 4.29 s), drops to 37.8 m s^{-1} at the bounce, falls linearly to 0 at 8.14 s , and rises again to 37.8 m s^{-1} at 12 s .

Exercise 2.9

Question: Explain clearly, with examples, the distinction between: (a) magnitude of displacement (sometimes called distance) over an interval of time, and the total length of path covered by a particle over the same interval; (b) magnitude of average velocity over an interval of time, and the average speed over the same interval. [Average speed of a particle over an interval of time is defined as the total path length divided by the time interval.] Show in both (a) and (b) that the second quantity is either greater than or equal to the first. When is the equality sign true? [For simplicity, consider one-dimensional motion only.]

Answer:

(a) Magnitude of displacement is the shortest distance between the initial and final positions. Total path length is the actual distance travelled. Example: a particle goes from O to A (5 m) and back to B (3 m from A). Displacement = 2 m , path length = 8 m . Since the path can never be shorter than the straight-line distance, **path length $\geq$ |displacement|**.

(b) $|\bar{v}| = \frac{|\text{displacement}|}{\Delta t}$ and average speed = $\frac{\text{path length}}{\Delta t}$. Dividing the inequality in (a) by the same Δt gives **average speed $\geq$ |average velocity|**. In the example above, if the time is 4 s : |average velocity| = 0.5 m s^{-1} and average speed = 2 m s^{-1} .

Equality holds in both cases when the particle moves along a straight line **in one direction only** (it does not turn back).

Exercise 2.10

Question: A man walks on a straight road from his home to a market 2.5 km away with a speed of 5 km h^{-1} . Finding the market closed, he instantly turns and walks back home with a speed of 7.5 km h^{-1} . What is the (a) magnitude of average velocity, and (b) average speed of the man over the interval of time (i) 0 to 30 min , (ii) 0 to 50 min , (iii) 0 to 40 min ?

Solution: Time to reach the market = $\frac{2.5}{5} \text{ h} = 30 \text{ min}$. Time to return = $\frac{2.5}{7.5} \text{ h} = 20 \text{ min}$ (home at 50 min).

(i) 0 to 30 min (0.5 h): displacement = path = 2.5 km .

$$|\bar{v}| = \frac{2.5}{0.5} = 5 \text{ km h}^{-1}, \quad \text{average speed} = 5 \text{ km h}^{-1}$$

(ii) **0 to 50 min (5/6 h):** he is back home, so displacement = 0; path = 5 km.

$$|\bar{v}| = 0, \quad \text{average speed} = \frac{5}{5/6} = 6 \text{ km h}^{-1}$$

(iii) **0 to 40 min (2/3 h):** in the last 10 min he walks back $7.5 \times \frac{10}{60} = 1.25$ km. Displacement = $2.5 - 1.25 = 1.25$ km; path = $2.5 + 1.25 = 3.75$ km.

$$|\bar{v}| = \frac{1.25}{2/3} = 1.875 \text{ km h}^{-1}, \quad \text{average speed} = \frac{3.75}{2/3} = 5.625 \text{ km h}^{-1}$$

Answer: (i) 5 km h^{-1} , 5 km h^{-1} ; (ii) 0, 6 km h^{-1} ; (iii) 1.875 km h^{-1} , 5.625 km h^{-1} .

Exercise 2.11

Question: In Exercises 2.9 and 2.10, we have carefully distinguished between average speed and magnitude of average velocity. No such distinction is necessary when we consider instantaneous speed and magnitude of velocity. The instantaneous speed is always equal to the magnitude of instantaneous velocity. Why?

Answer: Instantaneous quantities are defined over an **infinitesimally small** time interval. In such a small interval, the particle cannot change its direction of motion, so the path length equals the magnitude of the displacement. Hence the instantaneous speed always equals the magnitude of the instantaneous velocity.

Exercise 2.12

Question: Look at the graphs (a) to (d) (Fig. 2.10) carefully and state, with reasons, which of these cannot possibly represent one-dimensional motion of a particle.

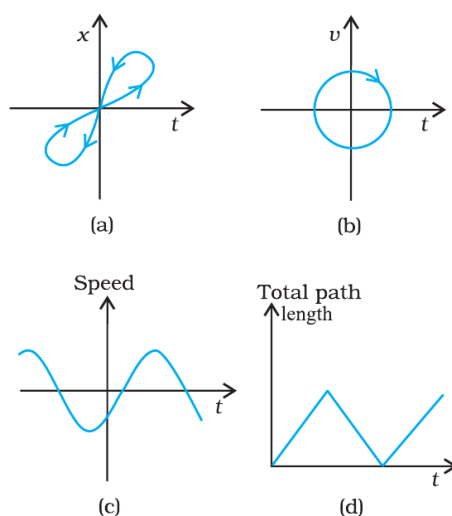


Fig. 2.10

Answer: None of the four graphs can represent one-dimensional motion:

- (a) At one instant of time, the particle would have **two positions**. Impossible.
- (b) At one instant, the particle would have **two velocities** (one positive, one negative). Impossible.
- (c) The graph shows **negative speed** at some times. Speed can never be negative.
- (d) The **total path length decreases** with time in some intervals. Path length can never decrease.

Exercise 2.13

Question: Figure 2.11 shows the x - t plot of one-dimensional motion of a particle. Is it correct to say from the graph that the particle moves in a straight line for $t < 0$ and on a parabolic path for $t > 0$? If not, suggest a suitable physical context for this graph.

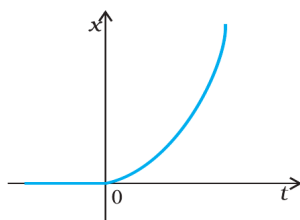


Fig. 2.11

Answer: No. An x - t graph does not show the **path (trajectory)** of a particle; it shows how its position changes with time. The motion here is along a straight line throughout. For $t < 0$, $x = 0$ (the particle is at rest at the origin); for $t > 0$, $x \propto t^2$ (uniformly accelerated motion from rest).

Physical context: A stone held at rest and then **dropped from the top of a tower** at $t = 0$ (with x measured downward).

Exercise 2.14

Question: A police van moving on a highway with a speed of 30 km h^{-1} fires a bullet at a thief's car speeding away in the same direction with a speed of 192 km h^{-1} . If the muzzle speed of the bullet is 150 m s^{-1} , with what speed does the bullet hit the thief's car? (Note: Obtain that speed which is relevant for damaging the thief's car.)

Solution: Speed of the van $= 30 \times \frac{5}{18} = 8.33 \text{ m s}^{-1}$. Speed of the thief's car $= 192 \times \frac{5}{18} = 53.33 \text{ m s}^{-1}$.

The bullet's speed relative to the ground $= 150 + 8.33 = 158.33 \text{ m s}^{-1}$ (muzzle speed is relative to the van).

The speed relevant for damage is the bullet's speed **relative to the car**:

$$v_{bc} = 158.33 - 53.33 = 105 \text{ m s}^{-1}$$

Answer: The bullet hits the thief's car with a speed of **105 m s^{-1}** .

Exercise 2.15

Question: Suggest a suitable physical situation for each of the following graphs (Fig 2.12).

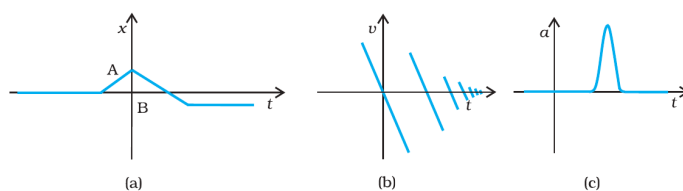


Fig. 2.12

Answer:

- **(a)** A ball lying at rest on a table is kicked (thrown up) slightly, rises, then falls onto the ground below the table level and stays there at rest.
- **(b)** A ball thrown up (or dropped) that **rebounds** from the floor, losing some speed at each collision. The velocity reverses suddenly at each bounce, and the jumps get smaller each time.
- **(c)** A ball moving with uniform velocity is **hit by a bat** for a very short time. The acceleration is non-zero only during the brief impact.

Exercise 2.16

Question: Figure 2.13 gives the x - t plot of a particle executing one-dimensional simple harmonic motion. Give the signs of position, velocity and acceleration variables of the particle at $t = 0.3$ s, 1.2 s, -1.2 s.

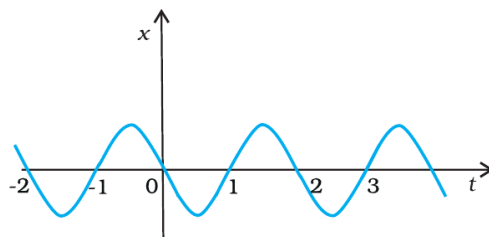


Fig. 2.13

Solution: In SHM, the acceleration is always opposite to the displacement ($a = -\omega^2 x$). The velocity sign is the slope of the x - t graph.

t	Position x	Velocity v	Acceleration a
0.3 s	Negative	negative	positive
1.2 s	Positive	positive	negative
-1.2 s	Negative	positive	positive

Exercise 2.17

Question: Figure 2.14 gives the x - t plot of a particle in one-dimensional motion. Three different equal intervals of time are shown. In which interval is the average speed greatest, and in which is it the least? Give the sign of average velocity for each interval.

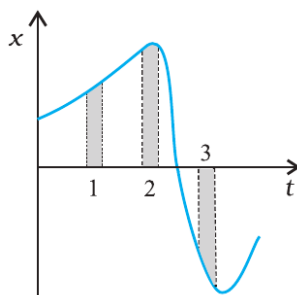


Fig. 2.14

Answer: Average speed is proportional to the steepness of the graph in each interval.

- **Greatest** in interval 3 (steepest).
- **Least** in interval 2 (flattest, near the peak).
- Average velocity is **positive** in intervals 1 and 2 (x increasing) and **negative** in interval 3 (x decreasing).

Exercise 2.18

Question: Figure 2.15 gives a speed-time graph of a particle in motion along a constant direction. Three equal intervals of time are shown. In which interval is the average acceleration greatest in magnitude? In which interval is the average speed greatest? Choosing the positive direction as the constant direction of motion, give the signs of v and a in the three intervals. What are the accelerations at the points A, B, C and D?

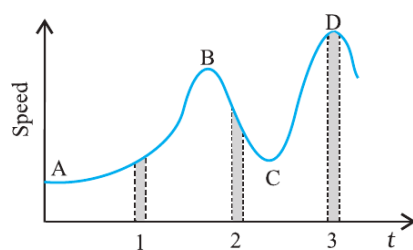


Fig. 2.15

Answer:

- **Average acceleration greatest** in magnitude in interval **2** (the speed changes most steeply).
- **Average speed greatest** in interval **3** (highest speed values).
- **v** is **positive** in all three intervals (constant direction of motion).
- **a** is **positive** in intervals 1 and 3 (speed increasing) and **negative** in interval 2 (speed decreasing).
- At points **A, B, C and D**, the tangents to the graph are horizontal, so the **acceleration is zero** at each.