



Chapter 3 : Motion in a Plane (Solved Examples & Exercises)

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Notes

Step-by-step solutions · NCERT Examples 3.1–3.9 and Exercises 3.1–3.22 · Class 11 Physics

Part A — NCERT Solved Examples

Example 3.1

Question: Rain is falling vertically with a speed of 35 m s^{-1} . Winds start blowing after some time with a speed of 12 m s^{-1} in the east to west direction. In which direction should a boy waiting at a bus stop hold his umbrella?

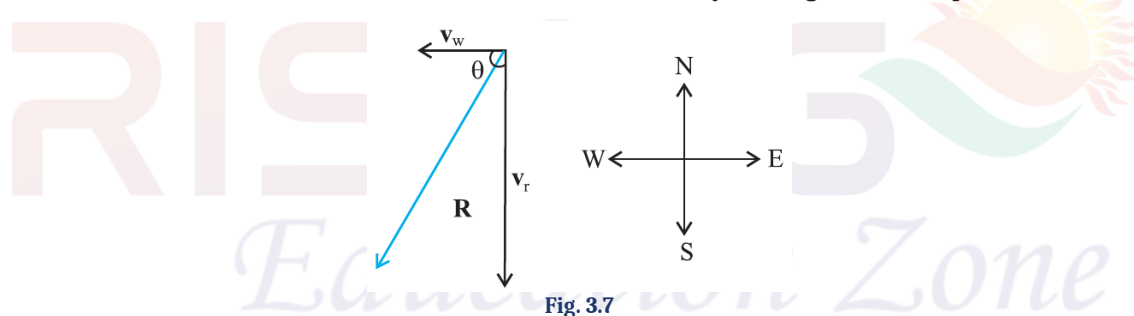


Fig. 3.7

Solution: The resultant of the rain velocity $\vec{v}_r$ (downward) and the wind velocity $\vec{v}_w$ (westward) is $\vec{R}$:

$$R = \sqrt{v_r^2 + v_w^2} = \sqrt{35^2 + 12^2} = 37 \text{ m s}^{-1}$$

The angle θ that $\vec{R}$ makes with the vertical:

$$\tan \theta = \frac{v_w}{v_r} = \frac{12}{35} = 0.343 \quad \Rightarrow \quad \theta = \tan^{-1} (0.343) \approx 19^\circ$$

Answer: The boy should hold his umbrella in the vertical plane at about **19° with the vertical, towards the east.**

Example 3.2

Question: Find the magnitude and direction of the resultant of two vectors $\vec{A}$ and $\vec{B}$ in terms of their magnitudes and the angle θ between them.

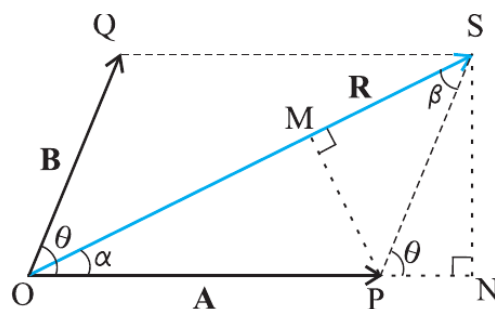


Fig. 3.10

Solution: Let OP and OQ represent **A** and **B** at angle θ . By the parallelogram method, OS represents **R**. SN is normal to OP and PM is normal to OS.

$$OS^2 = ON^2 + SN^2, \quad ON = A + B \cos \theta, \quad SN = B \sin \theta$$

$$R^2 = (A + B \cos \theta)^2 + (B \sin \theta)^2 = A^2 + B^2 + 2AB \cos \theta$$

$$R = \sqrt{A^2 + B^2 + 2AB \cos \theta}$$

In $\triangle OSN$, $SN = R \sin \alpha$; in $\triangle PSN$, $SN = B \sin \theta$. So $R \sin \alpha = B \sin \theta$. Similarly $PM = A \sin \alpha = B \sin \beta$:

$$\frac{R}{\sin \theta} = \frac{A}{\sin \beta} = \frac{B}{\sin \alpha}, \quad \tan \alpha = \frac{B \sin \theta}{A + B \cos \theta}$$

Answer: The magnitude is given by the **law of cosines**, $R = \sqrt{A^2 + B^2 + 2AB \cos \theta}$, and the direction by $\tan \alpha = \frac{B \sin \theta}{A + B \cos \theta}$ or the **law of sines**.

Example 3.3

Question: A motorboat is racing towards north at 25 km/h and the water current in that region is 10 km/h in the direction of 60° east of south. Find the resultant velocity of the boat.

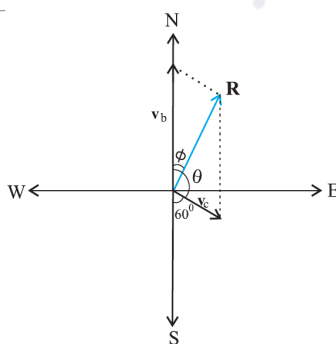


Fig. 3.11

Solution: The angle between $\vec{v}_b$ (north) and $\vec{v}_c$ (60° east of south) is $\theta = 120^\circ$. By the law of cosines:

$$R = \sqrt{v_b^2 + v_c^2 + 2v_b v_c \cos 120^\circ} = \sqrt{25^2 + 10^2 + 2 \times 25 \times 10 \times (-1/2)} \approx 22 \text{ km/h}$$

By the law of sines:

$$\sin \phi = \frac{v_c}{R} \sin \theta = \frac{10 \times \sin 120^\circ}{21.8} = \frac{10\sqrt{3}}{2 \times 21.8} \approx 0.397 \Rightarrow \phi \approx 23.4^\circ$$

Answer: About **22 km/h**, at about 23.4° east of north.

Example 3.4

Question: The position of a particle is given by $\vec{r} = 3.0t\hat{i} + 2.0t^2\hat{j} + 5.0\hat{k}$, where t is in seconds and the coefficients have the proper units for **r** to be in metres. (a) Find **v(t)** and **a(t)** of the particle. (b) Find the

magnitude and direction of $\mathbf{v}(t)$ at $t = 1.0$ s.

Solution:

(a)

$$\vec{v}(t) = \frac{d\vec{r}}{dt} = 3.0\hat{i} + 4.0t\hat{j}, \quad \vec{a}(t) = \frac{d\vec{v}}{dt} = +4.0\hat{j}$$

So $a = 4.0 \text{ m s}^{-2}$ along the y-direction.

(b) At $t = 1.0$ s, $\vec{v} = 3.0\hat{i} + 4.0\hat{j}$:

$$v = \sqrt{3^2 + 4^2} = 5.0 \text{ m s}^{-1}, \quad \theta = \tan^{-1} \left(\frac{v_y}{v_x} \right) = \tan^{-1} \left(\frac{4}{3} \right) \approx 53^\circ$$

Answer: (a) $\vec{v} = 3.0\hat{i} + 4.0t\hat{j}$, $\vec{a} = 4.0\hat{j} \text{ m s}^{-2}$; (b) 5.0 m s^{-1} at 53° with the x-axis.

Example 3.5

Question: A particle starts from the origin at $t = 0$ with a velocity $5.0\hat{i} \text{ m/s}$ and moves in the x-y plane under the action of a force which produces a constant acceleration of $(3.0\hat{i} + 2.0\hat{j}) \text{ m/s}^2$. (a) What is the y-coordinate of the particle at the instant its x-coordinate is 84 m? (b) What is the speed of the particle at this time?

Solution: With $\vec{r}_0 = 0$:

$$\vec{r}(t) = \vec{v}_0 t + \frac{1}{2} \vec{a} t^2 = 5.0\hat{i}t + \frac{1}{2} (3.0\hat{i} + 2.0\hat{j}) t^2 = (5.0t + 1.5t^2)\hat{i} + 1.0t^2\hat{j}$$

(a) $x(t) = 5.0t + 1.5t^2 = 84 \Rightarrow t = 6 \text{ s}$. Then $y = 1.0(6)^2 = 36.0 \text{ m}$.

(b) $\vec{v} = \frac{d\vec{r}}{dt} = (5.0 + 3.0t)\hat{i} + 2.0t\hat{j}$. At $t = 6 \text{ s}$, $\vec{v} = 23.0\hat{i} + 12.0\hat{j}$:

$$v = \sqrt{23^2 + 12^2} \approx 26 \text{ m s}^{-1}$$

Answer: (a) $y = 36.0 \text{ m}$; (b) speed $\approx 26 \text{ m s}^{-1}$.

Example 3.6

Question: Galileo, in his book *Two new sciences*, stated that “for elevations which exceed or fall short of 45° by equal amounts, the ranges are equal”. Prove this statement.

Solution: For a projectile launched with velocity v_0 at angle θ_0 :

$$R = \frac{v_0^2 \sin 2\theta_0}{g}$$

For angles $(45^\circ + \alpha)$ and $(45^\circ - \alpha)$, $2\theta_0$ is $(90^\circ + 2\alpha)$ and $(90^\circ - 2\alpha)$. Now

$$\sin(90^\circ + 2\alpha) = \sin(90^\circ - 2\alpha) = \cos 2\alpha$$

Answer: The ranges are equal for elevations that exceed or fall short of 45° by equal amounts α . Hence proved.

Example 3.7

Question: A hiker stands on the edge of a cliff 490 m above the ground and throws a stone horizontally with an initial speed of 15 m s^{-1} . Neglecting air resistance, find the time taken by the stone to reach the ground, and the speed with which it hits the ground. (Take $g = 9.8 \text{ m s}^{-2}$.)

Solution: Take the origin at the edge of the cliff, x along the initial velocity and y vertically upward.

$$x(t) = x_0 + v_{0x}t, \quad y(t) = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$$

with $x_0 = y_0 = 0$, $v_{0y} = 0$, $a_y = -9.8 \text{ m s}^{-2}$, $v_{0x} = 15 \text{ m s}^{-1}$.

The stone hits the ground when $y = -490$ m:

$$-490 = -\frac{1}{2} (9.8) t^2 \Rightarrow t = 10 \text{ s}$$

Velocity components at that moment: $v_x = 15 \text{ m s}^{-1}$, $v_y = 0 - 9.8 \times 10 = -98 \text{ m s}^{-1}$.

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{15^2 + 98^2} = 99 \text{ m s}^{-1}$$

Answer: Time = 10 s; speed = 99 m s⁻¹.

Example 3.8

Question: A cricket ball is thrown at a speed of 28 m s^{-1} in a direction 30° above the horizontal. Calculate (a) the maximum height, (b) the time taken by the ball to return to the same level, and (c) the distance from the thrower to the point where the ball returns to the same level.

Solution:

(a)

$$h_m = \frac{(v_0 \sin \theta_0)^2}{2g} = \frac{(28 \sin 30^\circ)^2}{2(9.8)} = \frac{14 \times 14}{2 \times 9.8} = 10.0 \text{ m}$$

(b)

$$T_f = \frac{2v_0 \sin \theta_0}{g} = \frac{2 \times 28 \times \sin 30^\circ}{9.8} = \frac{28}{9.8} = 2.9 \text{ s}$$

(c)

$$R = \frac{v_0^2 \sin 2\theta_0}{g} = \frac{28 \times 28 \times \sin 60^\circ}{9.8} = 69 \text{ m}$$

Answer: (a) 10.0 m; (b) 2.9 s; (c) 69 m.

Example 3.9

Question: An insect trapped in a circular groove of radius 12 cm moves along the groove steadily and completes 7 revolutions in 100 s. (a) What is the angular speed, and the linear speed of the motion? (b) Is the acceleration vector a constant vector? What is its magnitude?

Solution: This is uniform circular motion with $R = 12$ cm.

(a)

$$\omega = \frac{2\pi}{T} = 2\pi \times \frac{7}{100} = 0.44 \text{ rad/s}, \quad v = \omega R = 0.44 \times 12 = 5.3 \text{ cm s}^{-1}$$

(b) The velocity is along the tangent and the acceleration is towards the centre. Since this direction changes continuously, the acceleration is **not a constant vector**. Its magnitude is constant:

$$a = \omega^2 R = (0.44)^2 \times 12 = 2.3 \text{ cm s}^{-2}$$

Answer: (a) $\omega = 0.44 \text{ rad/s}$, $v = 5.3 \text{ cm s}^{-1}$; (b) not a constant vector; magnitude 2.3 cm s^{-2} .

Part B — NCERT Exercises

Exercise 3.1

Question: State, for each of the following physical quantities, if it is a scalar or a vector: volume, mass, speed, acceleration, density, number of moles, velocity, angular frequency, displacement, angular velocity.

Answer:

- **Scalars:** volume, mass, speed, density, number of moles, angular frequency.
- **Vectors:** acceleration, velocity, displacement, angular velocity.

Exercise 3.2

Question: Pick out the two scalar quantities in the following list: force, angular momentum, work, current, linear momentum, electric field, average velocity, magnetic moment, relative velocity.

Answer: **Work** and **current**. (Current has a direction but does not obey the law of vector addition, so it is a scalar.)

Exercise 3.3

Question: Pick out the only vector quantity in the following list: temperature, pressure, impulse, time, power, total path length, energy, gravitational potential, coefficient of friction, charge.

Answer: **Impulse** (= force $\times$ time, a vector).

Exercise 3.4

Question: State with reasons, whether the following algebraic operations with scalar and vector physical quantities are meaningful: (a) adding any two scalars, (b) adding a scalar to a vector of the same dimensions, (c) multiplying any vector by any scalar, (d) multiplying any two scalars, (e) adding any two vectors, (f) adding a component of a vector to the same vector.

Answer:

- (a) **Not meaningful** in general. Only scalars of the same dimensions (e.g. two masses) can be added.
- (b) **Not meaningful**. A scalar cannot be added to a vector.
- (c) **Meaningful**. It gives a vector (e.g. mass $\times$ velocity = momentum).
- (d) **Meaningful**. E.g. power $\times$ time = energy.
- (e) **Not meaningful** in general. Only vectors of the same dimensions (e.g. two forces) can be added.
- (f) **Meaningful**. A component of a vector is a vector of the same dimensions, so it can be added to that vector.

Exercise 3.5

Question: Read each statement below carefully and state with reasons, if it is true or false: (a) The magnitude of a vector is always a scalar, (b) each component of a vector is always a scalar, (c) the total path length is always equal to the magnitude of the displacement vector of a particle, (d) the average speed of a particle (defined as total path length divided by the time taken to cover the path) is either greater or equal to the magnitude of average velocity of the particle over the same interval of time, (e) three vectors not lying in a plane can never add up to give a null vector.

Answer:

- (a) **True**. Magnitude is a pure number with a unit; it has no direction.
- (b) **False**. Each component of a vector (e.g. $A_x \hat{i}$) is itself a vector along an axis.
- (c) **False**. Path length $\geq$ |displacement|; they are equal only if the particle moves in one direction along a straight line.
- (d) **True**. Because path length $\geq$ |displacement| over the same time interval.
- (e) **True**. For three vectors to add up to zero, the third must be equal and opposite to the sum of the other two, which lies in the plane of those two. So all three must lie in one plane.

Exercise 3.6

Question: Establish the following vector inequalities geometrically or otherwise: (a) $|\vec{a} + \vec{b}| \leq |\vec{a}| + |\vec{b}|$, (b) $|\vec{a} + \vec{b}| \geq ||\vec{a}| - |\vec{b}||$, (c) $|\vec{a} - \vec{b}| \leq |\vec{a}| + |\vec{b}|$, (d) $|\vec{a} - \vec{b}| \geq ||\vec{a}| - |\vec{b}||$. When does the equality sign above apply?

Solution: Using the law of cosines with θ the angle between $\mathbf{a}$ and $\mathbf{b}$:

$$|\vec{a} + \vec{b}|^2 = a^2 + b^2 + 2ab \cos \theta, \quad |\vec{a} - \vec{b}|^2 = a^2 + b^2 - 2ab \cos \theta$$

Since $-1 \leq \cos \theta \leq 1$:

$$(a - b)^2 \leq |\vec{a} \pm \vec{b}|^2 \leq (a + b)^2$$

Taking square roots gives all four inequalities. Geometrically, in the triangle formed by $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{a} \pm \mathbf{b}$, one side is never greater than the sum of the other two and never less than their difference.

Equality:

- a. and (d): when $\mathbf{a}$ and $\mathbf{b}$ are in the **same direction** ($\theta = 0^\circ$).
- b. and (c): when $\mathbf{a}$ and $\mathbf{b}$ are in **opposite directions** ($\theta = 180^\circ$).

Exercise 3.7

Question: Given $\vec{a} + \vec{b} + \vec{c} + \vec{d} = \vec{0}$, which of the following statements are correct: (a) $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$ and $\mathbf{d}$ must each be a null vector, (b) the magnitude of $(\mathbf{a} + \mathbf{c})$ equals the magnitude of $(\mathbf{b} + \mathbf{d})$, (c) the magnitude of $\mathbf{a}$ can never be greater than the sum of the magnitudes of $\mathbf{b}$, $\mathbf{c}$ and $\mathbf{d}$, (d) $\mathbf{b} + \mathbf{c}$ must lie in the plane of $\mathbf{a}$ and $\mathbf{d}$ if $\mathbf{a}$ and $\mathbf{d}$ are not collinear, and in the line of $\mathbf{a}$ and $\mathbf{d}$, if they are collinear?

Answer:

- (a) **Incorrect.** The sum of non-zero vectors can be zero (e.g. four sides of a closed quadrilateral).
- (b) **Correct.** $\vec{a} + \vec{c} = -(\vec{b} + \vec{d})$, so their magnitudes are equal.
- (c) **Correct.** $\vec{a} = -(\vec{b} + \vec{c} + \vec{d})$, so $|\vec{a}| = |\vec{b} + \vec{c} + \vec{d}| \leq |\vec{b}| + |\vec{c}| + |\vec{d}|$.
- (d) **Correct.** $\vec{b} + \vec{c} = -(\vec{a} + \vec{d})$, which lies in the plane of $\mathbf{a}$ and $\mathbf{d}$ (or along their line if they are collinear).

Exercise 3.8

Question: Three girls skating on a circular ice ground of radius 200 m start from a point P on the edge of the ground and reach a point Q diametrically opposite to P following different paths as shown in Fig. 3.19. What is the magnitude of the displacement vector for each? For which girl is this equal to the actual length of path skated?

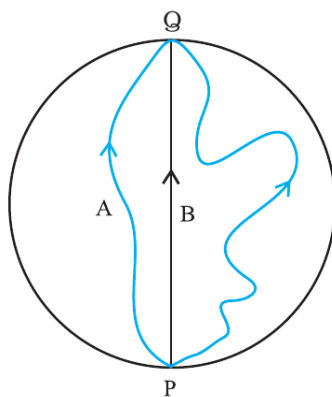


Fig. 3.19

Answer: Displacement depends only on the end points. For each girl, the displacement is $PQ = \text{diameter} = 400 \text{ m}$. It equals the actual path length only for **girl B**, who skates along the straight diameter.

Exercise 3.9

Question: A cyclist starts from the centre O of a circular park of radius 1 km , reaches the edge P of the park, then cycles along the circumference, and returns to the centre along QO as shown in Fig. 3.20. If the round trip takes 10 min , what is the (a) net displacement, (b) average velocity, and (c) average speed of the cyclist?

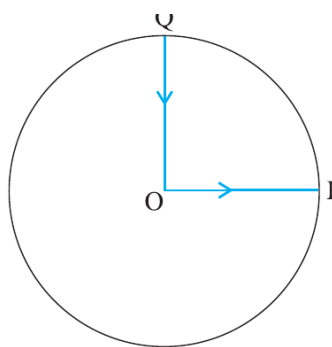


Fig. 3.20

Solution:

(a) He returns to the starting point O , so the **net displacement is zero**.

(b) Average velocity = displacement $\div$ time = **zero**.

(c) Path length = $OP + \text{arc } PQ \text{ (a quarter circle)} + QO$:

$$\text{Path} = 1 + \frac{2\pi \times 1}{4} + 1 = 2 + 1.571 = 3.571 \text{ km}$$

$$\text{Time} = 10 \text{ min} = \frac{1}{6} \text{ h.}$$

$$\text{Average speed} = \frac{3.571}{1/6} = 21.4 \text{ km h}^{-1}$$

Answer: (a) 0 ; (b) 0 ; (c) 21.4 km h^{-1} .

Exercise 3.10

Question: On an open ground, a motorist follows a track that turns to his left by an angle of 60° after every 500 m . Starting from a given turn, specify the displacement of the motorist at the third, sixth and eighth turn. Compare the magnitude of the displacement with the total path length covered by the motorist in each case.

Solution: Turning 60° after every 500 m traces a **regular hexagon** of side 500 m .

- **Third turn:** he is at the opposite vertex. Displacement = $2 \times 500 = 1000$ m; path = 1500 m. Ratio = $1000/1500 = 2/3$.
- **Sixth turn:** he is back at the start. Displacement = **0**; path = 3000 m. Ratio = **0**.
- **Eighth turn:** he is two sides past the start. Displacement = $2 \times 500 \times \cos 30^\circ = 500\sqrt{3} = 866$ m, at 60° to the initial direction; path = 4000 m. Ratio = $866/4000 = 0.22$.

Exercise 3.11

Question: A passenger arriving in a new town wishes to go from the station to a hotel located 10 km away on a straight road from the station. A dishonest cabman takes him along a circuitous path 23 km long and reaches the hotel in 28 min. What is (a) the average speed of the taxi, (b) the magnitude of average velocity? Are the two equal?

Solution: Time = 28 min = $\frac{28}{60}$ h.

(a) Average speed = $\frac{23}{28/60} = 49.3 \text{ km h}^{-1}$.

(b) |Average velocity| = $\frac{10}{28/60} = 21.4 \text{ km h}^{-1}$.

Answer: No, they are **not equal**, because the path length (23 km) is greater than the displacement (10 km). They would be equal only for a straight path.

Exercise 3.12

Question: The ceiling of a long hall is 25 m high. What is the maximum horizontal distance that a ball thrown with a speed of 40 m s^{-1} can go without hitting the ceiling of the hall?

Solution: The maximum height must not exceed 25 m:

$$h_m = \frac{v_0^2 \sin^2 \theta}{2g} = 25 \Rightarrow \sin^2 \theta = \frac{2 \times 9.8 \times 25}{(40)^2} = 0.306 \Rightarrow \sin \theta = 0.553, \theta = 33.6^\circ$$

$$R = \frac{v_0^2 \sin 2\theta}{g} = \frac{1600 \times \sin 67.2^\circ}{9.8} = \frac{1600 \times 0.922}{9.8} = 150.5 \text{ m}$$

Answer: About **150.5 m**.

Exercise 3.13

Question: A cricketer can throw a ball to a maximum horizontal distance of 100 m. How much high above the ground can the cricketer throw the same ball?

Solution: $R_{max} = \frac{u^2}{g} = 100 \text{ m}$, so $u^2 = 100g$.

Thrown vertically upward with the same speed:

$$H = \frac{u^2}{2g} = \frac{100g}{2g} = 50 \text{ m}$$

Answer: **50 m**.

Exercise 3.14

Question: A stone tied to the end of a string 80 cm long is whirled in a horizontal circle with a constant speed. If the stone makes 14 revolutions in 25 s, what is the magnitude and direction of acceleration of the stone?

Solution: $R = 0.80 \text{ m}$, $\nu = 14/25 \text{ s}^{-1}$.

$$\omega = 2\pi\nu = 2\pi \times \frac{14}{25} = 3.52 \text{ rad s}^{-1}$$

$$a = \omega^2 R = (3.52)^2 \times 0.80 = 9.91 \text{ m s}^{-2}$$

Answer: 9.91 m s^{-2} , directed along the string **towards the centre** of the circle.

Exercise 3.15

Question: An aircraft executes a horizontal loop of radius 1.00 km with a steady speed of 900 km/h. Compare its centripetal acceleration with the acceleration due to gravity.

Solution: $v = 900 \times \frac{5}{18} = 250 \text{ m s}^{-1}$, $R = 1000 \text{ m}$.

$$a_c = \frac{v^2}{R} = \frac{(250)^2}{1000} = 62.5 \text{ m s}^{-2}, \quad \frac{a_c}{g} = \frac{62.5}{9.8} = 6.4$$

Answer: The centripetal acceleration is about **6.4 times** g .

Exercise 3.16

Question: Read each statement below carefully and state, with reasons, if it is true or false: (a) The net acceleration of a particle in circular motion is always along the radius of the circle towards the centre, (b) the velocity vector of a particle at a point is always along the tangent to the path of the particle at that point, (c) the acceleration vector of a particle in uniform circular motion averaged over one cycle is a null vector.

Answer:

- **(a) False.** It is along the radius only in **uniform** circular motion. If the speed changes, there is also a tangential acceleration.
- **(b) True.** By definition, velocity is along the tangent to the path, in the direction of motion.
- **(c) True.** Over one cycle the velocity returns to its initial value, so $\Delta \mathbf{v} = 0$ and the average acceleration is zero.

Exercise 3.17

Question: The position of a particle is given by $\vec{r} = 3.0t\hat{i} - 2.0t^2\hat{j} + 4.0\hat{k} \text{ m}$, where t is in seconds and the coefficients have the proper units for $\mathbf{r}$ to be in metres. (a) Find the $\mathbf{v}$ and $\mathbf{a}$ of the particle. (b) What is the magnitude and direction of velocity of the particle at $t = 2.0 \text{ s}$?

Solution:

(a)

$$\vec{v} = \frac{d\vec{r}}{dt} = (3.0\hat{i} - 4.0t\hat{j}) \text{ m s}^{-1}, \quad \vec{a} = \frac{d\vec{v}}{dt} = -4.0\hat{j} \text{ m s}^{-2}$$

(b) At $t = 2.0 \text{ s}$, $\vec{v} = 3.0\hat{i} - 8.0\hat{j}$:

$$v = \sqrt{3^2 + 8^2} = \sqrt{73} = 8.54 \text{ m s}^{-1}, \quad \theta = \tan^{-1} \left(\frac{8}{3} \right) \approx 69.4^\circ$$

Answer: (b) 8.54 m s^{-1} , at about 69.4° **below** the x-axis (with the negative y-direction component).

Exercise 3.18

Question: A particle starts from the origin at $t = 0 \text{ s}$ with a velocity of $10.0\hat{j} \text{ m/s}$ and moves in the x-y plane with a constant acceleration of $(8.0\hat{i} + 2.0\hat{j}) \text{ m s}^{-2}$. (a) At what time is the x-coordinate of the particle 16 m? What is the y-coordinate of the particle at that time? (b) What is the speed of the particle at the time?

Solution:

$$x = \frac{1}{2} (8.0) t^2 = 4.0t^2, \quad y = 10.0t + \frac{1}{2} (2.0) t^2 = 10.0t + t^2$$

(a) $4.0t^2 = 16 \Rightarrow t = 2 \text{ s}$. Then $y = 10.0 (2) + (2)^2 = 24 \text{ m}$.

(b) $v_x = 8.0t = 16 \text{ m/s}$, $v_y = 10.0 + 2.0t = 14 \text{ m/s}$.

$$v = \sqrt{16^2 + 14^2} = 21.26 \text{ m s}^{-1}$$

Answer: (a) $t = 2 \text{ s}$, $y = 24 \text{ m}$; (b) speed $\approx 21.3 \text{ m s}^{-1}$.

Exercise 3.19

Question: $\hat{i}$ and $\hat{j}$ are unit vectors along x- and y-axis respectively. What is the magnitude and direction of the vectors $\hat{i} + \hat{j}$ and $\hat{i} - \hat{j}$? What are the components of a vector $\vec{A} = 2\hat{i} + 3\hat{j}$ along the directions of $\hat{i} + \hat{j}$ and $\hat{i} - \hat{j}$? [You may use graphical method]

Solution:

- $|\hat{i} + \hat{j}| = \sqrt{1^2 + 1^2} = \sqrt{2}$, at 45° with the x-axis.
- $|\hat{i} - \hat{j}| = \sqrt{2}$, at -45° (45° below the x-axis).

Unit vectors: $\hat{n}_1 = \frac{\hat{i} + \hat{j}}{\sqrt{2}}$, $\hat{n}_2 = \frac{\hat{i} - \hat{j}}{\sqrt{2}}$.

Component of $\vec{A}$ along $\hat{n}_1$: $\vec{A} \cdot \hat{n}_1 = \frac{2+3}{\sqrt{2}} = \frac{5}{\sqrt{2}}$, i.e. the vector $\frac{5}{2}(\hat{i} + \hat{j})$.

Component of $\vec{A}$ along $\hat{n}_2$: $\vec{A} \cdot \hat{n}_2 = \frac{2-3}{\sqrt{2}} = -\frac{1}{\sqrt{2}}$, i.e. the vector $-\frac{1}{2}(\hat{i} - \hat{j})$.

Check: $\frac{5}{2}(\hat{i} + \hat{j}) - \frac{1}{2}(\hat{i} - \hat{j}) = 2\hat{i} + 3\hat{j} \checkmark$

Exercise 3.20

Question: For any arbitrary motion in space, which of the following relations are true: (a) $\vec{v}_{\text{average}} = \frac{1}{2} [\vec{v}(t_1) + \vec{v}(t_2)]$, (b) $\vec{v}_{\text{average}} = [\vec{r}(t_2) - \vec{r}(t_1)] / (t_2 - t_1)$, (c) $\vec{v}(t) = \vec{v}(0) + \vec{a}t$, (d) $\vec{r}(t) = \vec{r}(0) + \vec{v}(0)t + \frac{1}{2}\vec{a}t^2$, (e) $\vec{a}_{\text{average}} = [\vec{v}(t_2) - \vec{v}(t_1)] / (t_2 - t_1)$? (The 'average' stands for average of the quantity over the time interval t_1 to t_2 .)

Answer: (b) and (e) are true for any motion — they are the definitions of average velocity and average acceleration. (a), (c) and (d) hold only for motion with **constant acceleration**, so they are not true for arbitrary motion.

Exercise 3.21

Question: Read each statement below carefully and state, with reasons and examples, if it is true or false. A scalar quantity is one that (a) is conserved in a process, (b) can never take negative values, (c) must be dimensionless, (d) does not vary from one point to another in space, (e) has the same value for observers with different orientations of axes.

Answer:

- (a) **False.** Kinetic energy is a scalar but is not conserved in an inelastic collision.
- (b) **False.** Temperature (in $^\circ\text{C}$) and potential energy can be negative.
- (c) **False.** Mass, length and speed are scalars with dimensions.
- (d) **False.** Temperature and gravitational potential vary from point to point.

- (e) **True.** A scalar has no direction, so its value does not change when the axes are rotated.

Exercise 3.22

Question: An aircraft is flying at a height of 3400 m above the ground. If the angle subtended at a ground observation point by the aircraft positions 10.0 s apart is 30° , what is the speed of the aircraft?

Solution: The observation point is directly below the midpoint of the two positions (symmetric case). Each half-angle is 15° . The distance travelled in 10 s:

$$d = 2 \times 3400 \times \tan 15^\circ = 2 \times 3400 \times 0.268 = 1822 \text{ m}$$
$$v = \frac{1822}{10} = 182 \text{ m s}^{-1}$$

Answer: About 182 m s^{-1} .

