



Chapter 3 : Motion in a Plane (Notes)

by - Tomar Sir

Notes

GSEB and CBSE Board Exam preparation subjective notes · Class 11 Physics · NCERT Chapter 3

Q.1 Distinguish between scalar and vector quantities. Define position and displacement vectors. (3 Marks)

Scalar quantity: A quantity with **magnitude only**. It is completely specified by a single number with a proper unit. Scalars are combined by the rules of ordinary algebra. Examples: distance, mass, temperature, time, volume, density, speed, work, energy.

Vector quantity: A quantity that has **both magnitude and direction** and obeys the **triangle law** (or parallelogram law) of addition. Examples: displacement, velocity, acceleration, force, momentum.

Scalar	Vector
Has magnitude only.	Has magnitude and direction.
Added by ordinary algebra.	Added by the triangle / parallelogram law.
Examples: mass, speed, time, work.	Examples: displacement, velocity, force.
Represented by a number and unit.	Represented by a bold letter $\mathbf{v}$ or $\vec{v}$.

Note: Having magnitude and direction is not enough to be a vector; the quantity must also obey the law of vector addition. Electric current has magnitude and direction but is a **scalar**.

Position vector: Choose an origin O. If an object is at P at time t , the vector $\vec{OP} = \vec{r}$ is its **position vector**. Its length is the magnitude of $\vec{r}$ and its direction is the direction in which P lies as seen from O.

Displacement vector: If the object moves from P (time t) to P' (time t'), the vector $\vec{PP'}$ is the **displacement vector**.

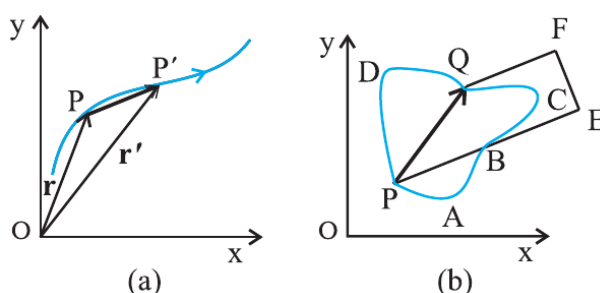


Fig. 3.1: (a) Position and displacement vectors. (b) Displacement vector PQ and different courses of motion

Key Examination Points for Full Marks:

- The displacement vector is the straight line joining the initial and final positions; it **does not depend on the actual path**. In Fig. 3.1(b), PQ is the same for paths PABCQ, PDQ and PBEFQ.
- Hence, **|displacement| ≤ path length**.

Q.2 Explain equality of vectors and multiplication of a vector by a real number (2 Marks)

Equality of vectors: Two vectors **A** and **B** are equal **if and only if** they have the **same magnitude and the same direction**. Shift **B** parallel to itself until its tail coincides with the tail of **A**; if their tips also coincide, **A = B**.

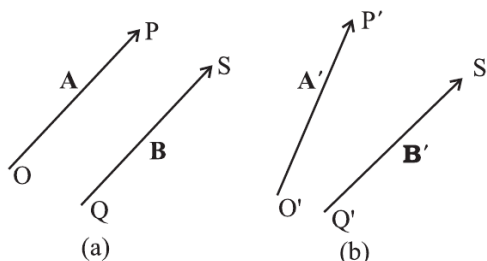


Fig. 3.2: (a) Two equal vectors **A** and **B**. (b) Two vectors **A'** and **B'** are unequal though they are of the same length

In Fig. 3.2(b), **A'** and **B'** have the same magnitude but different directions, so they are **not equal**.

Free vectors: Displacing a vector parallel to itself leaves it unchanged. Such vectors are called **free vectors**. When the location or line of action matters (e.g. force), they are called **localised vectors**.

Multiplication by a real number λ :

- If $\lambda > 0$: $\lambda \vec{A}$ has magnitude $\lambda |\vec{A}|$ and the **same** direction as $\vec{A}$.
- If $\lambda < 0$: $\lambda \vec{A}$ has magnitude $|\lambda| |\vec{A}|$ and the **opposite** direction.

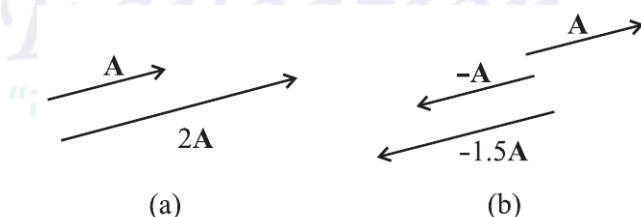


Fig. 3.3: (a) Vector **A** and the resultant vector after multiplying **A** by a positive number 2. (b) Vector **A** and resultant vectors after multiplying it by a negative number -1 and -1.5

Key Examination Points for Full Marks:

- λ may be a scalar with its own dimension. Then the dimension of $\lambda \mathbf{A}$ is the product of the dimensions of λ and **A**. Example: velocity $\times$ time = displacement.

Q.3 Explain the triangle method of vector addition. Show that vector addition is commutative and associative. (3 Marks)

Triangle (head-to-tail) method: To find $\vec{A} + \vec{B}$, place the **tail of B at the head of A**. The vector from the tail of **A** to the head of **B** is the resultant $\vec{R} = \vec{A} + \vec{B}$. The two vectors and their resultant form three sides of a triangle.

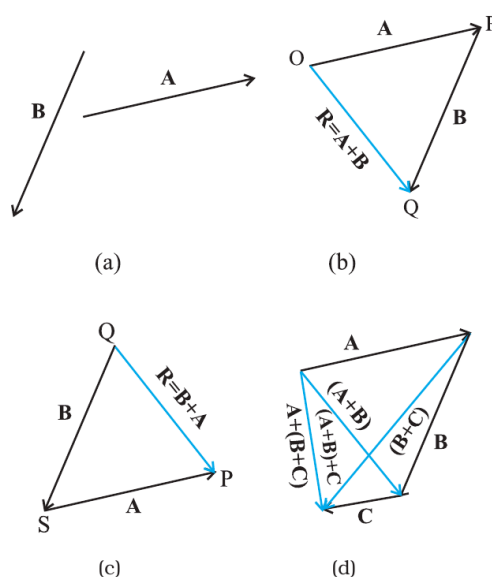


Fig. 3.4: (a) Vectors A and B . (b) Vectors A and B added graphically. (c) Vectors B and A added graphically. (d) Illustrating the associative law of vector addition

Commutative law: Finding $\vec{B} + \vec{A}$ (Fig. 3.4(c)) gives the same vector $\vec{R}$:

$$\vec{A} + \vec{B} = \vec{B} + \vec{A}$$

Associative law: Adding A and B first and then C gives the same result as adding B and C first and then A (Fig. 3.4(d)):

$$(\vec{A} + \vec{B}) + \vec{C} = \vec{A} + (\vec{B} + \vec{C})$$

Null (zero) vector: The sum of two equal and opposite vectors: $\vec{A} + (-\vec{A}) = \vec{0}$, $|\vec{0}| = 0$. Its direction cannot be specified. Properties:

$$\vec{A} + \vec{0} = \vec{A}, \quad \lambda \vec{0} = \vec{0}, \quad 0 \vec{A} = \vec{0}$$

Physical meaning: If an object moves from P to P' and comes back to P , its displacement is a null vector.

Q.4 Explain subtraction of vectors and the parallelogram law of vector addition. (2 Marks)

Subtraction: The difference of A and B is the sum of A and $-B$:

$$\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$$

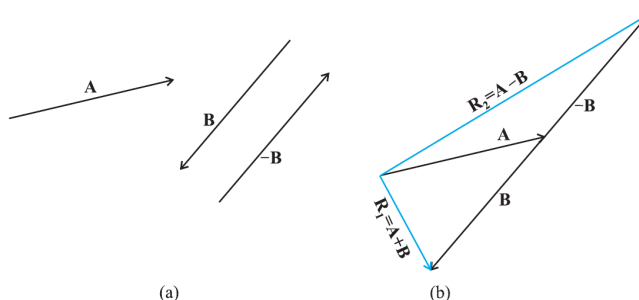


Fig. 3.5: (a) Two vectors A and B , $-B$ is also shown. (b) Subtracting vector B from vector A — the result is R_2 . For comparison, addition of vectors A and B , i.e. R_1 is also shown

Parallelogram law: If two vectors acting at a point are represented in magnitude and direction by the two adjacent sides of a parallelogram drawn from that point, their resultant is represented by the diagonal of the parallelogram passing through that point.

To add **A** and **B**: bring their tails to a common origin **O**. Complete the parallelogram **OQSP**. The diagonal **OS** is the resultant **R** (Fig. 3.6).

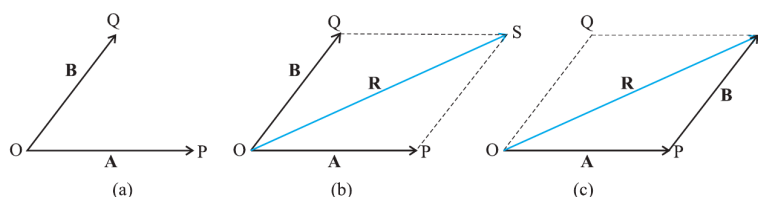


Fig. 3.6: (a) Two vectors **A** and **B** with their tails brought to a common origin. (b) The sum $\mathbf{A} + \mathbf{B}$ obtained using the parallelogram method. (c) The parallelogram method of vector addition is equivalent to the triangle method

Key Examination Points for Full Marks:

- The parallelogram and triangle methods give the **same** result; they are equivalent.

Illustration (NCERT Example 3.1): Rain falls vertically at 35 m s^{-1} and wind blows east to west at 12 m s^{-1} .

$R = \sqrt{35^2 + 12^2} = 37 \text{ m s}^{-1}$; $\tan \theta = 12/35 = 0.343$, $\theta \approx 19^\circ$. The boy should hold his umbrella at about **19° with the vertical towards the east**.

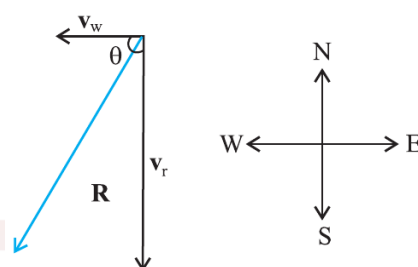


Fig. 3.7: Rain and wind velocities

Q.5 What is a unit vector? Resolve a vector into rectangular components. (3 Marks)

Resolution of a vector: Splitting a vector into two or more vectors (components) whose sum is the original vector. Any vector **A** in a plane can be written along two non-collinear vectors **a** and **b**: $\vec{A} = \lambda \vec{a} + \mu \vec{b}$.

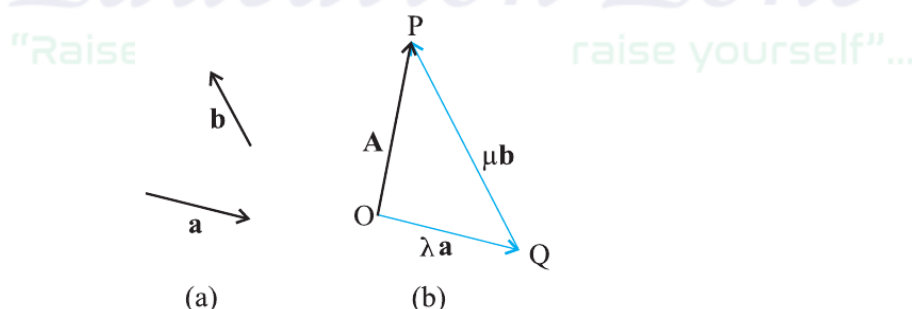


Fig. 3.8: (a) Two non-collinear vectors **a** and **b**. (b) Resolving a vector **A** in terms of vectors **a** and **b**

Unit vector: A vector of **unit magnitude** pointing in a particular direction. It has **no dimension and no unit**; it only specifies a direction. Unit vectors along the **x**-, **y**- and **z**-axes are $\hat{i}$, $\hat{j}$, $\hat{k}$:

$$|\hat{i}| = |\hat{j}| = |\hat{k}| = 1$$

A unit vector along **A**: $\hat{n} = \frac{\vec{A}}{|\vec{A}|}$, so $\vec{A} = |\vec{A}| \hat{n}$.

Rectangular components (two dimensions): Draw perpendiculars from the head of **A** to the axes (Fig. 3.9):

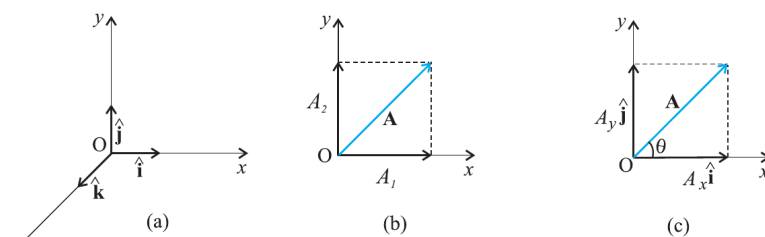


Fig. 3.9: (a) Unit vectors $\hat{i}$, $\hat{j}$ and $\hat{k}$ lie along the x-, y-, and z-axes. (b) A vector A is resolved into its components A_x and A_y along x-, and y-axes. (c) A_x and A_y expressed in terms of $\hat{i}$ and $\hat{j}$

$$\vec{A} = A_x \hat{i} + A_y \hat{j}, \quad A_x = A \cos \theta, \quad A_y = A \sin \theta$$

Conversely,

$$A = \sqrt{A_x^2 + A_y^2}, \quad \tan \theta = \frac{A_y}{A_x}$$

Three dimensions: If α , β , γ are the angles of A with the x-, y-, z-axes,

$$A_x = A \cos \alpha, \quad A_y = A \cos \beta, \quad A_z = A \cos \gamma, \quad \vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$$

$$A = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

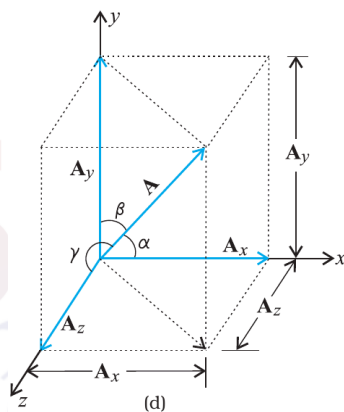


Fig. 3.9 (d): A vector A resolved into components along x-, y-, and z-axes

Position vector: $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$.

Key Examination Points for Full Marks:

- A_x is a **number** (not a vector); $A_x \hat{i}$ is a vector.
- A component can be positive, negative or zero depending on θ .

Q.6 Explain the analytical method of vector addition.

(2 Marks)

The graphical method has limited accuracy. It is easier to add vectors by combining their **components**.

Let $\vec{A} = A_x \hat{i} + A_y \hat{j}$ and $\vec{B} = B_x \hat{i} + B_y \hat{j}$. Then

$$\vec{R} = \vec{A} + \vec{B} = (A_x + B_x) \hat{i} + (A_y + B_y) \hat{j}$$

$$R_x = A_x + B_x, \quad R_y = A_y + B_y$$

Each component of the resultant is the sum of the corresponding components of the vectors.

Three dimensions:

$$R_x = A_x + B_x, \quad R_y = A_y + B_y, \quad R_z = A_z + B_z$$

Any number of vectors: For $\vec{T} = \vec{a} + \vec{b} - \vec{c}$:

$$T_x = a_x + b_x - c_x, \quad T_y = a_y + b_y - c_y, \quad T_z = a_z + b_z - c_z$$

Q.7 Derive the magnitude and direction of the resultant of two vectors (3 Marks) (laws of cosines and sines).

Let OP and OQ represent $\mathbf{A}$ and $\mathbf{B}$ with angle θ between them. By the parallelogram method, OS is the resultant $\mathbf{R}$ (Fig. 3.10). Draw SN normal to OP (extended) and PM normal to OS .

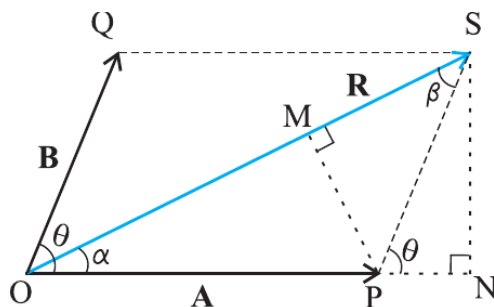


Fig. 3.10: Resultant of two vectors by the parallelogram method

Magnitude: In right triangle OSN ,

$$\begin{aligned} OS^2 &= ON^2 + SN^2 \\ ON &= OP + PN = A + B \cos \theta, \quad SN = B \sin \theta \\ R^2 &= (A + B \cos \theta)^2 + (B \sin \theta)^2 = A^2 + B^2 + 2AB \cos \theta \\ R &= \sqrt{A^2 + B^2 + 2AB \cos \theta} \quad (\text{Law of cosines}) \end{aligned}$$

Direction: If α is the angle of $\mathbf{R}$ with $\mathbf{A}$:

$$\tan \alpha = \frac{SN}{OP + PN} = \frac{B \sin \theta}{A + B \cos \theta}$$

Law of sines: In $\triangle OSN$, $SN = R \sin \alpha$; in $\triangle PSN$, $SN = B \sin \theta$. So $R \sin \alpha = B \sin \theta$. Similarly $PM = A \sin \alpha = B \sin \beta$. Hence

$$\frac{R}{\sin \theta} = \frac{A}{\sin \beta} = \frac{B}{\sin \alpha} \quad (\text{Law of sines})$$

Special cases:

- $\theta = 0^\circ$: $R = A + B$ (maximum).
- $\theta = 180^\circ$: $R = |A - B|$ (minimum).
- $\theta = 90^\circ$: $R = \sqrt{A^2 + B^2}$.

Illustration (NCERT Example 3.3): A motorboat at 25 km/h north, current 10 km/h at 60° east of south ($\theta = 120^\circ$): $R = \sqrt{25^2 + 10^2 + 2 \times 25 \times 10 \times (-1/2)} \approx 22$ km/h; $\sin \phi = \frac{10 \sin 120^\circ}{21.8} = 0.397$, $\phi \approx 23.4^\circ$ (east of north).

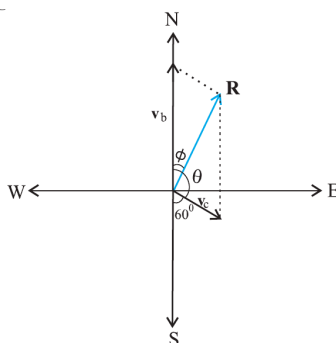


Fig. 3.11: Resultant velocity of the boat

Q.8 Define average velocity, instantaneous velocity and acceleration for (3 Marks) motion in a plane.

Position and displacement: $\vec{r} = x\hat{i} + y\hat{j}$. If the particle moves from P (position $\vec{r}$) to P' (position $\vec{r}'$), the displacement is

$$\Delta\vec{r} = \vec{r}' - \vec{r} = \Delta x\hat{i} + \Delta y\hat{j}$$

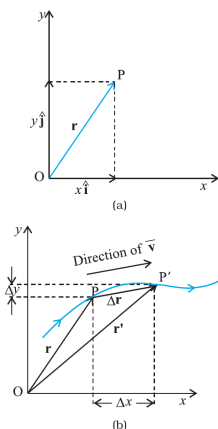


Fig. 3.12: (a) Position vector $\vec{r}$. (b) Displacement $\Delta\vec{r}$ and average velocity of a particle

Average velocity:

$$\vec{v} = \frac{\Delta\vec{r}}{\Delta t} = \frac{\Delta x}{\Delta t}\hat{i} + \frac{\Delta y}{\Delta t}\hat{j}$$

Its direction is the same as that of $\Delta\vec{r}$.

Instantaneous velocity:

$$\vec{v} = \lim_{\Delta t \rightarrow 0} \frac{\Delta\vec{r}}{\Delta t} = \frac{d\vec{r}}{dt} = v_x\hat{i} + v_y\hat{j}, \quad v_x = \frac{dx}{dt}, \quad v_y = \frac{dy}{dt}$$

$$v = \sqrt{v_x^2 + v_y^2}, \quad \tan \theta = \frac{v_y}{v_x}$$

As $\Delta t \rightarrow 0$, $\Delta\vec{r}$ lies along the **tangent** to the path (Fig. 3.13). So the **direction of velocity at any point on the path is tangential to the path, in the direction of motion**.

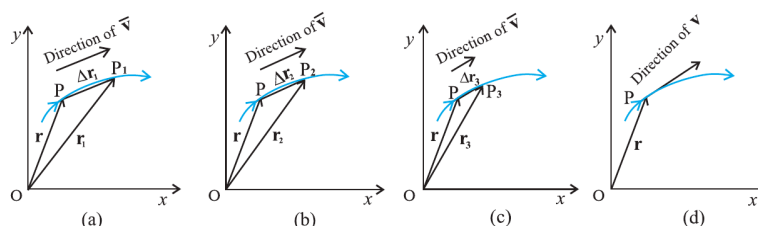


Fig. 3.13: As the time interval Δt approaches zero, the average velocity approaches the velocity $\vec{v}$. The direction of $\vec{v}$ is parallel to the line tangent to the path

Average acceleration:

$$\vec{a} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\Delta v_x}{\Delta t}\hat{i} + \frac{\Delta v_y}{\Delta t}\hat{j}$$

Instantaneous acceleration:

$$\vec{a} = \lim_{\Delta t \rightarrow 0} \frac{\Delta\vec{v}}{\Delta t} = a_x\hat{i} + a_y\hat{j}, \quad a_x = \frac{dv_x}{dt} = \frac{d^2x}{dt^2}, \quad a_y = \frac{dv_y}{dt} = \frac{d^2y}{dt^2}$$

Key Examination Points for Full Marks:

- In **one dimension**, velocity and acceleration are always along the same line. In **two or three dimensions**, they may have **any angle between 0° and 180°**.

Illustration (NCERT Example 3.4): $\vec{r} = 3.0t\hat{i} + 2.0t^2\hat{j} + 5.0\hat{k}$; $\vec{v} = 3.0\hat{i} + 4.0t\hat{j}$, $\vec{a} = 4.0\hat{j} \text{ m s}^{-2}$. At $t = 1.0 \text{ s}$, $v = \sqrt{3^2 + 4^2} = 5.0 \text{ m s}^{-1}$ at $\theta = \tan^{-1}(4/3) \approx 53^\circ$ with the x-axis.

Q.9 Derive the equations of motion in a plane with constant acceleration. (2 Marks)

Let an object move in the x-y plane with constant acceleration $\mathbf{a}$. Its velocity is $\vec{v}_0$ at $t = 0$ and $\mathbf{v}$ at time t .

Velocity:

$$\vec{a} = \frac{\vec{v} - \vec{v}_0}{t - 0} \Rightarrow \vec{v} = \vec{v}_0 + \vec{a}t$$

In components: $v_x = v_{0x} + a_x t$, $v_y = v_{0y} + a_y t$.

Position: The average velocity over time t is $(\vec{v}_0 + \vec{v})/2$, so the displacement is

$$\begin{aligned} \vec{r} - \vec{r}_0 &= \left(\frac{\vec{v}_0 + \vec{v}}{2} \right) t = \left(\frac{(\vec{v}_0 + \vec{a}t) + \vec{v}_0}{2} \right) t = \vec{v}_0 t + \frac{1}{2} \vec{a} t^2 \\ \vec{r} &= \vec{r}_0 + \vec{v}_0 t + \frac{1}{2} \vec{a} t^2 \end{aligned}$$

In components:

$$x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2, \quad y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$$

Key conclusion: Motion in a plane can be treated as two separate, simultaneous one-dimensional motions with constant acceleration along two perpendicular directions. The x- and y-motions are independent of each other.

Illustration (NCERT Example 3.5): Starting at the origin with $5.0\hat{i} \text{ m/s}$ and $\vec{a} = (3.0\hat{i} + 2.0\hat{j}) \text{ m/s}^2$: $x = 5.0t + 1.5t^2 = 84 \text{ m}$ gives $t = 6 \text{ s}$; then $y = 36.0 \text{ m}$ and $v = \sqrt{23^2 + 12^2} \approx 26 \text{ m s}^{-1}$.

Q.10 What is projectile motion? Derive the equation of the trajectory and show that it is a parabola. (3 Marks)

Projectile: An object that is in flight after being thrown or projected. Examples: a football, a cricket ball, a baseball.

Projectile motion is the result of **two independent, simultaneous motions**: a horizontal motion with **no acceleration** and a vertical motion with **constant acceleration** g downward. (This independence was first stated by **Galileo** in his *Dialogue on the great world systems*, 1632.) Air resistance is neglected.

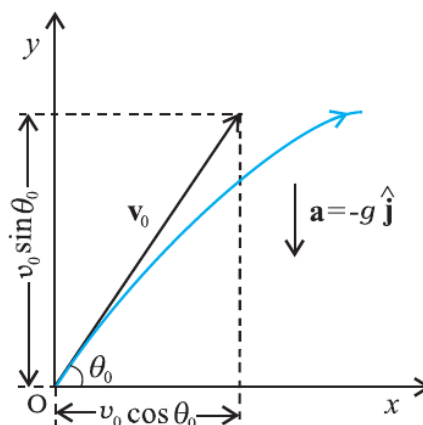


Fig. 3.16: Motion of an object projected with velocity v_0 at angle θ_0

Equations: The object is launched from the origin with velocity v_0 at angle θ_0 with the x-axis. Acceleration: $a_x = 0$, $a_y = -g$. Initial components: $v_{0x} = v_0 \cos \theta_0$, $v_{0y} = v_0 \sin \theta_0$.

$$x = (v_0 \cos \theta_0) t, \quad y = (v_0 \sin \theta_0) t - \frac{1}{2}gt^2$$

$$v_x = v_0 \cos \theta_0 \text{ (constant)}, \quad v_y = v_0 \sin \theta_0 - gt$$

Equation of path: From the first equation, $t = \frac{x}{v_0 \cos \theta_0}$. Substituting in y :

$$y = (\tan \theta_0) x - \frac{g}{2(v_0 \cos \theta_0)^2} x^2$$

This is of the form $y = ax + bx^2$ (a, b constants), which is the equation of a **parabola**. So the **path of a projectile is a parabola**.

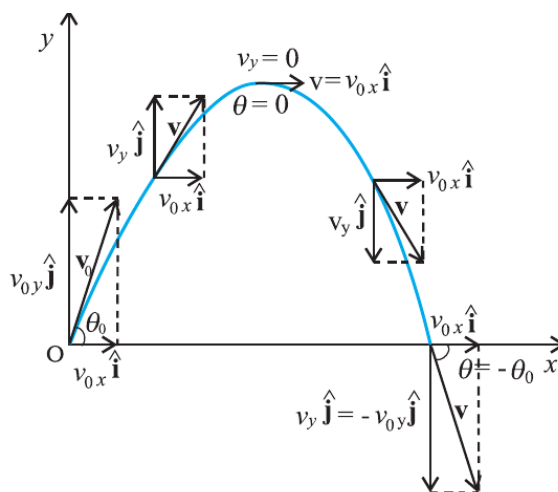


Fig. 3.17: The path of a projectile is a parabola

Key Examination Points for Full Marks:

- The horizontal component of velocity **remains constant** throughout; only the vertical component changes.
- At the highest point, $v_y = 0$, so the velocity is horizontal ($v = v_0 \cos \theta_0$) and the angle is 0; the acceleration is still g downward.

Q.11 Derive the time of flight, maximum height and horizontal range of a projectile. (3 Marks)

Time to reach maximum height (t_m): At the highest point, $v_y = 0$:

$$v_0 \sin \theta_0 - gt_m = 0 \Rightarrow t_m = \frac{v_0 \sin \theta_0}{g}$$

Time of flight (T_f): Put $y = 0$ in $y = (v_0 \sin \theta_0) t - \frac{1}{2}gt^2$:

$$T_f = \frac{2v_0 \sin \theta_0}{g} = 2t_m$$

($T_f = 2t_m$, as expected from the symmetry of the parabolic path.)

Maximum height (h_m): Put $t = t_m$ in the equation for y :

$$h_m = (v_0 \sin \theta_0) \left(\frac{v_0 \sin \theta_0}{g} \right) - \frac{g}{2} \left(\frac{v_0 \sin \theta_0}{g} \right)^2$$

$$h_m = \frac{(v_0 \sin \theta_0)^2}{2g}$$

Horizontal range (R): The horizontal distance travelled during the time of flight:

$$R = (v_0 \cos \theta_0) (T_f) = (v_0 \cos \theta_0) \frac{2v_0 \sin \theta_0}{g}$$

$$R = \frac{v_0^2 \sin 2\theta_0}{g}$$

Maximum range: R is maximum when $\sin 2\theta_0 = 1$, i.e. $\theta_0 = 45^\circ$:

$$R_{\max} = \frac{v_0^2}{g}$$

Illustration (NCERT Example 3.6): For angles $(45^\circ + \alpha)$ and $(45^\circ - \alpha)$, $2\theta_0 = 90^\circ \pm 2\alpha$, and $\sin(90^\circ + 2\alpha) = \sin(90^\circ - 2\alpha) = \cos 2\alpha$. So the ranges are equal for complementary angles (elevations exceeding or falling short of 45° by equal amounts).

Illustration (NCERT Example 3.8): A ball thrown at 28 m s^{-1} at 30° : $h_m = \frac{(28 \sin 30^\circ)^2}{2 \times 9.8} = 10.0 \text{ m}$;
 $T_f = \frac{2 \times 28 \times \sin 30^\circ}{9.8} = 2.9 \text{ s}$; $R = \frac{28^2 \sin 60^\circ}{9.8} = 69 \text{ m}$.

Q.12 What is uniform circular motion? Derive the expression for centripetal acceleration. (3 Marks)

Uniform circular motion: When an object follows a circular path at a **constant speed**, the motion is called uniform circular motion. The speed is constant, but the **direction of velocity changes continuously**, so the object is **accelerated**.

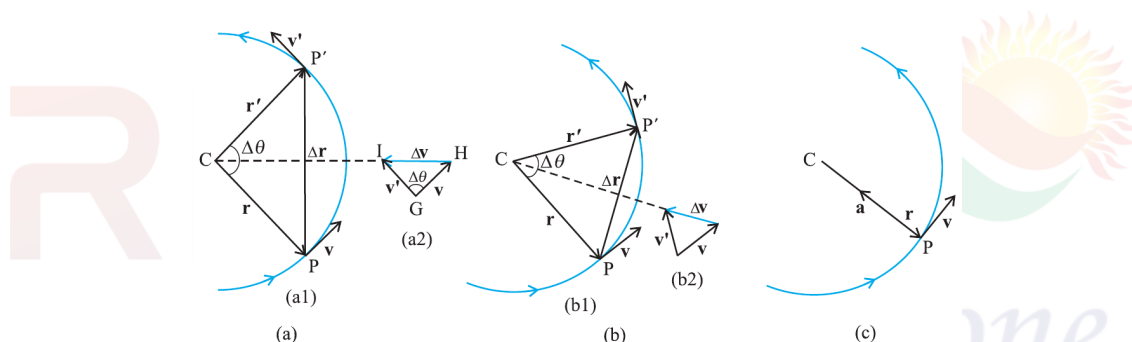


Fig. 3.18: Velocity and acceleration of an object in uniform circular motion. The time interval Δt decreases from (a) to (c) where it is zero. The acceleration is directed, at each point of the path, towards the centre of the circle

Direction: Let $\mathbf{r}$ and $\mathbf{r}'$ be the position vectors, and $\mathbf{v}$ and $\mathbf{v}'$ the velocities, at P and P' (Fig. 3.18(a)). Velocity is along the tangent, so $\mathbf{v} \perp \mathbf{r}$ and $\mathbf{v}' \perp \mathbf{r}'$. Hence $\Delta \mathbf{v} \perp \Delta \mathbf{r}$. The average acceleration $\vec{a} = \Delta \vec{v} / \Delta t$ is along $\Delta \mathbf{v}$, which lies on the bisector of $\mathbf{r}$ and $\mathbf{r}'$, pointing towards the centre. As $\Delta t \rightarrow 0$, the acceleration is **directed towards the centre** at every point.

Magnitude: The angle between $\mathbf{r}$ and $\mathbf{r}'$ is $\Delta \theta$. Since $\mathbf{v}$ and $\mathbf{v}'$ are perpendicular to $\mathbf{r}$ and $\mathbf{r}'$, the angle between them is also $\Delta \theta$. So triangle CPP' (position vectors) and triangle GHI (velocity vectors) are **similar**:

$$\frac{|\Delta \vec{v}|}{v} = \frac{|\Delta \vec{r}|}{R} \Rightarrow |\Delta \vec{v}| = v \frac{|\Delta \vec{r}|}{R}$$

$$|\vec{a}| = \lim_{\Delta t \rightarrow 0} \frac{|\Delta \vec{v}|}{\Delta t} = \frac{v}{R} \lim_{\Delta t \rightarrow 0} \frac{|\Delta \vec{r}|}{\Delta t}$$

For small Δt , the arc $PP' \approx |\Delta \mathbf{r}| = v\Delta t$, so $\lim_{\Delta t \rightarrow 0} \frac{|\Delta \vec{r}|}{\Delta t} = v$:

$$a_c = \frac{v^2}{R}$$

Centripetal acceleration ("centre-seeking", a term proposed by Newton) has magnitude v^2/R and is **always directed towards the centre**. Its magnitude is constant but its **direction changes**, so it is **not a constant vector**. (Its first thorough analysis was published by Christiaan Huygens in 1673.)

Q.13 Define angular speed, period and frequency. Derive $v = R\omega$ and $a = \omega^2 R$. (2 Marks)

Angular displacement ($\Delta\theta$): The angle turned through by the line from the centre to the object in time Δt .

Angular speed (ω): The time rate of change of angular displacement:

$$\omega = \frac{\Delta\theta}{\Delta t}$$

SI unit: rad s^{-1} . **Dimensions:** $[T^{-1}]$.

Relation $v = R\omega$: In time Δt the object moves an arc $\Delta s = R \Delta\theta$, so

$$v = \frac{\Delta s}{\Delta t} = R \frac{\Delta\theta}{\Delta t} \Rightarrow v = R\omega$$

Centripetal acceleration in terms of ω :

$$a_c = \frac{v^2}{R} = \frac{\omega^2 R^2}{R} \Rightarrow a_c = \omega^2 R$$

Time period (T): Time for one revolution. **Frequency (ν):** Number of revolutions per second, $\nu = 1/T$. In one revolution, the distance is $2\pi R$:

$$v = \frac{2\pi R}{T} = 2\pi R\nu, \quad \omega = 2\pi\nu, \quad a_c = 4\pi^2\nu^2 R$$

Illustration (NCERT Example 3.9): An insect in a circular groove of radius 12 cm makes 7 revolutions in 100 s. $\omega = 2\pi \times 7/100 = 0.44 \text{ rad/s}$; $v = \omega R = 5.3 \text{ cm s}^{-1}$; $a = \omega^2 R = 2.3 \text{ cm s}^{-2}$, directed towards the centre. The acceleration is **not a constant vector** (its direction changes), but its magnitude is constant.

Q.14 Write the important points to ponder about motion in a plane. (2 Marks)

1. **Path length $\geq$ |displacement|.** They are equal only if the object does not change direction.
2. Hence **average speed $\geq$ |average velocity|**, with equality only when path length = |displacement|.
3. The vector equations $\vec{v} = \vec{v}_0 + \vec{a}t$ and $\vec{r} = \vec{r}_0 + \vec{v}_0 t + \frac{1}{2}\vec{a}t^2$ do not involve any choice of axes; they can be resolved along any two independent axes.
4. The kinematic equations for **uniform acceleration do not apply to uniform circular motion**, because the magnitude of acceleration is constant but its **direction changes**.
5. An object subjected to two velocities $\vec{v}_1$ and $\vec{v}_2$ has a resultant velocity $\vec{v} = \vec{v}_1 + \vec{v}_2$. Distinguish this from the **relative velocity** $\vec{v}_{12} = \vec{v}_1 - \vec{v}_2$.
6. The acceleration of an object in circular motion is towards the centre **only if the speed is constant**.
7. The shape of the trajectory depends not only on the acceleration but also on the **initial conditions**. Under the same gravity, the path can be a straight line or a parabola.

Formula Sheet — Motion in a Plane

Quantity	Formula	SI Unit
Vector in components	$\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$	—
Magnitude	$A = \sqrt{A_x^2 + A_y^2 + A_z^2}$	—
Components (2D)	$A_x = A \cos \theta, A_y = A \sin \theta,$ $\tan \theta = A_y / A_x$	—
Unit vector	$\hat{n} = \vec{A} / \vec{A} $	—
Resultant (law of cosines)	$R = \sqrt{A^2 + B^2 + 2AB \cos \theta}$	—
Direction of resultant	$\tan \alpha = \frac{B \sin \theta}{A + B \cos \theta}$	—
Law of sines	$\frac{R}{\sin \theta} = \frac{A}{\sin \beta} = \frac{B}{\sin \alpha}$	—
Velocity	$\vec{v} = \frac{d\vec{r}}{dt} = v_x \hat{i} + v_y \hat{j}$	m s^{-1}
Acceleration	$\vec{a} = \frac{d\vec{v}}{dt} = a_x \hat{i} + a_y \hat{j}$	m s^{-2}
Constant acceleration	$\vec{v} = \vec{v}_0 + \vec{a}t; \vec{r} = \vec{r}_0 + \vec{v}_0 t + \frac{1}{2} \vec{a}t^2$	—
Projectile path	$y = x \tan \theta_0 - \frac{gx^2}{2v_0^2 \cos^2 \theta_0}$	m
Time of flight	$T_f = \frac{2v_0 \sin \theta_0}{g}$	s
Maximum height	$h_m = \frac{v_0^2 \sin^2 \theta_0}{2g}$	m
Horizontal range	$R = \frac{v_0^2 \sin 2\theta_0}{g}; R_{\max} = \frac{v_0^2}{g} \text{ at } 45^\circ$	m
Horizontal projection (height h)	$t = \sqrt{\frac{2h}{g}}, R = u \sqrt{\frac{2h}{g}}$	s, m
Angular speed	$\omega = \frac{\Delta \theta}{\Delta t} = 2\pi \nu = \frac{2\pi}{T}$	rad s^{-1}
Linear speed	$v = R\omega = 2\pi R\nu$	m s^{-1}
Centripetal acceleration	$a_c = \frac{v^2}{R} = \omega^2 R = 4\pi^2 \nu^2 R$	m s^{-2}
Relative velocity	$\vec{v}_{AB} = \vec{v}_A - \vec{v}_B$	m s^{-1}

NEET / JEE Quick Points and Tricks

1. **Range of resultant:** $|A - B| \leq R \leq A + B$. Two vectors of equal magnitude A at angle θ :

$$R = 2A \cos (\theta/2); R = A \text{ when } \theta = 120^\circ.$$

2. **Complementary angles** (θ and $90^\circ - \theta$) give the **same range**. Then $h_1 + h_2 = \frac{v_0^2}{2g}, h_1 h_2 = R^2/16$, and

$$T_1 T_2 = 2R/g.$$

3. **Range = maximum height** when $\tan \theta = 4$ ($\theta \approx 76^\circ$). Generally, $R = 4h_m \cot \theta$.

4. **At the highest point:** speed = $v_0 \cos \theta$; KE = $K_0 \cos^2 \theta$; velocity $\perp$ acceleration.

5. **Velocity $\perp$ initial velocity:** only possible if $\theta > 45^\circ$; occurs at $t = \frac{v_0}{g \sin \theta}$.

6. **Horizontal projection from height h:** time depends only on h (same as free fall). A ball dropped and a ball thrown horizontally from the same height hit the ground **together**.
7. **Projectile equation shortcut:** If $y = ax - bx^2$, then $\tan \theta = a$, $R = a/b$, $h_m = a^2/4b$.
8. **Rain-umbrella:** Tilt the umbrella in the direction of the rain's velocity **relative to the person**:
 $\vec{v}_{rm} = \vec{v}_r - \vec{v}_m$; $\tan \theta = v_m/v_r$.
9. **River crossing (width d, boat speed v, river speed u):** Shortest time $t = d/v$ (head straight across), drift $= ud/v$. Shortest path (zero drift): head upstream at $\sin \theta = u/v$; $t = \frac{d}{\sqrt{v^2 - u^2}}$.
10. **Change in velocity in UCM:** after angle θ , $|\Delta \vec{v}| = 2v \sin (\theta/2)$. Half revolution $\rightarrow 2v$; full revolution $\rightarrow 0$.
 Average acceleration over a full cycle = 0.
11. **Non-uniform circular motion:** total acceleration $a = \sqrt{a_c^2 + a_t^2}$, where $a_t = dv/dt$ is tangential.
12. **Dot / cross products (for vector questions):** $\vec{A} \cdot \vec{B} = AB \cos \theta$; perpendicular vectors have $\vec{A} \cdot \vec{B} = 0$.
 $|\vec{A} + \vec{B}| = |\vec{A} - \vec{B}|$ means $\vec{A} \perp \vec{B}$.
13. **Angle with axes:** $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$ (direction cosines).
14. **Trajectory depends on frame:** a ball dropped in a moving train falls straight down for a passenger but follows a parabola for a person on the ground.

