



NEET — JEE MAINS - ADVANCED

Chapter 4 : Laws of Motion (Notes)

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Notes

GSEB and CBSE Board Exam preparation subjective notes · Class 11 Physics · NCERT Chapter 4

Q.1 What was Aristotle's view on motion? Why is it wrong?

(2 Marks)

Aristotle's law of motion: The Greek thinker Aristotle (384–322 B.C.) held that **an external force is required to keep a body in motion**. For example, he thought an arrow keeps flying because the air behind it keeps pushing it.

Common experience seems to support it: A toy car pulled by a string comes to rest when the string is released. Left to themselves, moving bodies eventually stop.

The flaw (Aristotle's fallacy): The toy car stops because of the external force of **friction** from the floor, which opposes its motion. When the car moves uniformly, the child's force only cancels friction, so the **net external force is zero**. If there were no friction, no force would be needed to keep the car moving.

Key Examination Points for Full Marks:

- Opposing forces such as **friction** (solids) and **viscous forces** (fluids) are always present in nature. That is why a force seems necessary to maintain motion.
- The correct idea, given by **Galileo**, is that **no net force is needed for uniform motion**. This became the foundation of Newtonian mechanics.

Q.2 Describe Galileo's experiments that led to the law of inertia. (3 Marks)

Experiment 1 — Single inclined plane: Galileo observed that objects

- moving **down** an inclined plane **accelerate**;
- moving **up** an inclined plane **retard** (slow down);
- moving on a **horizontal** plane is the intermediate case.

He concluded that an object moving on a **frictionless horizontal plane** must have neither acceleration nor retardation, i.e. it should move with **constant velocity**.

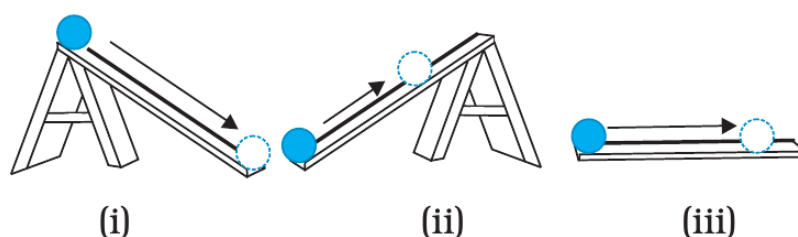


Fig. 4.1 (a): (i) motion down an incline — acceleration; (ii) motion up an incline — retardation; (iii) motion on a horizontal plane — constant velocity

Experiment 2 — Double inclined plane: A ball released from rest on one smooth plane rolls down and climbs up the other plane to **nearly the same height** (never greater). In the ideal case (no friction) it reaches exactly the initial height.

- If the slope of the second plane is decreased, the ball still reaches the same height but travels a **longer distance**.
- If the second plane is made **horizontal** (slope zero), the ball travels an **infinite distance**: its motion never ceases.

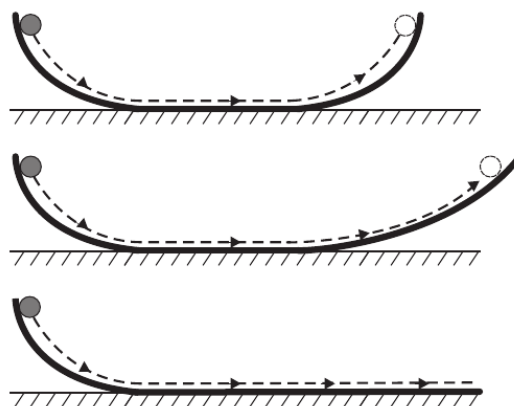


Fig. 4.1 (b): The law of inertia was inferred by Galileo from the motion of a ball on a double inclined plane

Conclusion: The state of rest and the state of uniform linear motion are **equivalent**. In both, the net force on the body is zero. In practice the ball stops because of friction, which can never be totally eliminated.

Inertia: The property of a body by which it cannot change its state of rest or of uniform motion by itself is called **inertia**. Inertia means “**resistance to change**”. Mass is a measure of inertia.

Q.3 State Newton’s first law of motion. Explain it with examples. (2 Marks)

Statement: Every body continues to be in its state of rest or of uniform motion in a straight line unless compelled by some external force to act otherwise.

In simple terms: If the net external force on a body is zero, its acceleration is zero. Acceleration can be non-zero only if there is a net external force on the body.

$$\vec{F}_{\text{net}} = 0 \Rightarrow \vec{a} = 0$$

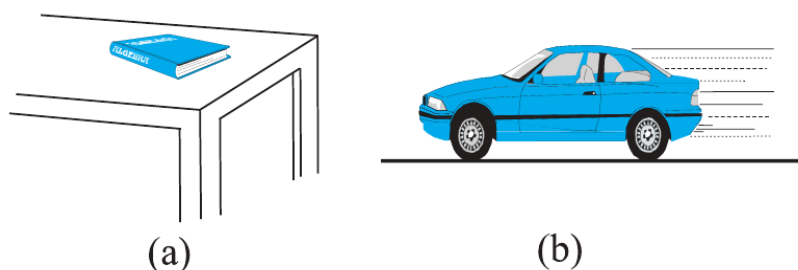


Fig. 4.2: (a) A book at rest on the table and (b) a car moving with uniform velocity. The net force is zero in each case

Examples:

- **Book on a table:** Its weight W acts downward and the normal force R acts upward. The book is at rest, so by the first law the net force is zero, hence $R = W$. (The correct reasoning is: “the book is at rest, *therefore* the net force is zero” — not “the forces cancel, therefore it is at rest”.)
- **Car on a road:** While picking up speed, the car accelerates due to an external force — the **friction** of the road on the tyres. When it moves with constant velocity, the net external force is zero.

- **Jerk in a bus:** When a bus starts suddenly, our feet move with the bus (due to friction), but the rest of the body stays behind due to **inertia**, so we are thrown **backward**. When the bus stops suddenly, we are thrown **forward**.

Illustration (NCERT Example 4.1): An astronaut separates from a spaceship accelerating at 100 m s^{-2} in interstellar space. Once outside, no force acts on him (no nearby stars, negligible pull of the small ship), so by the first law his **acceleration is zero**.

Q.4 Define linear momentum. Explain its importance with examples. (2 Marks)

Definition: The momentum of a body is the product of its mass m and velocity $\vec{v}$:

$$\vec{p} = m\vec{v}$$

It is a **vector** quantity, in the direction of velocity. **SI unit:** kg m s^{-1} (or N s). **Dimensions:** $[\text{M L T}^{-1}]$.

Why momentum matters (common experiences):

- **Mass matters:** A greater force is needed to push a loaded truck than a small car to the same speed in the same time.
- **Speed matters:** A bullet fired from a gun can pierce human tissue, but the same bullet at moderate speed causes little damage.
- **Time matters:** A seasoned cricketer draws his hands backward while catching a ball. This gives a **longer time** to stop the ball, so a **smaller force** is needed and the hands are not hurt.

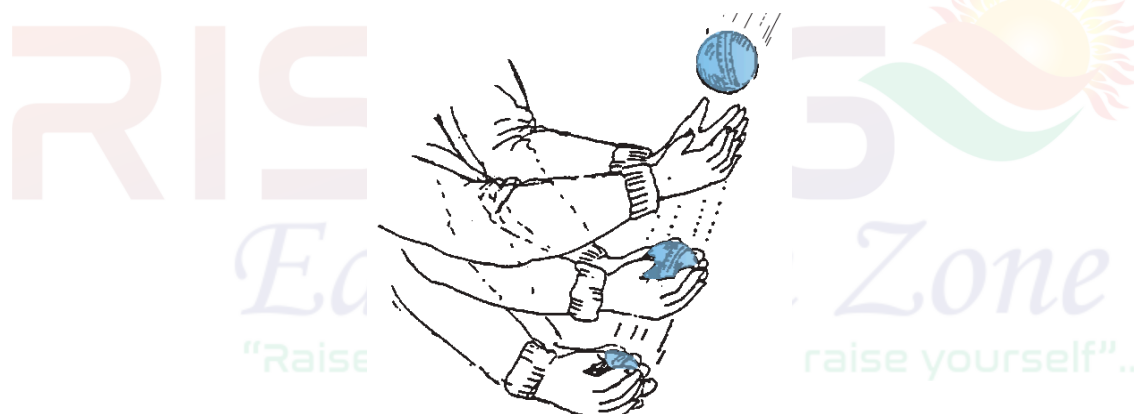


Fig. 4.3: A seasoned cricketer draws in his hands during a catch, allowing greater time for the ball to stop and hence requires a smaller force

- **Direction matters:** A stone whirled in a horizontal circle with uniform speed has constant magnitude of momentum, but its **direction changes**. A force (from our hand, through the string) is needed to change the direction of momentum.

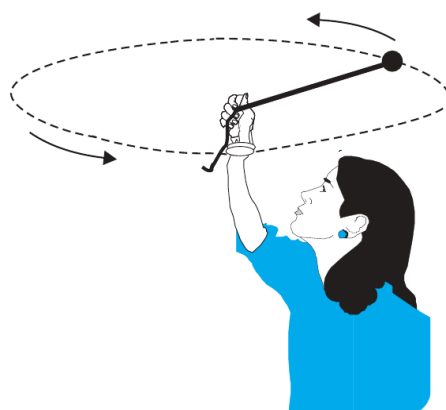


Fig. 4.4: Force is necessary for changing the direction of momentum, even if its magnitude is constant

Conclusion: The same force acting for the same time produces the **same change in momentum** in different bodies. The greater the rate of change of momentum, the greater the force.

Q.5 State Newton's second law of motion. Derive $F = ma$ and define the unit of force. (3 Marks)

Statement: The rate of change of momentum of a body is directly proportional to the applied force and takes place in the direction in which the force acts.

Derivation: Let a force $\vec{F}$ act for a time Δt on a body of mass m , changing its velocity from $\vec{v}$ to $\vec{v} + \Delta\vec{v}$. The change in momentum is $\Delta\vec{p} = m \Delta\vec{v}$. By the second law,

$$\vec{F} \propto \frac{\Delta\vec{p}}{\Delta t} \quad \text{or} \quad \vec{F} = k \frac{\Delta\vec{p}}{\Delta t}$$

Taking the limit $\Delta t \rightarrow 0$,

$$\vec{F} = k \frac{d\vec{p}}{dt}$$

For a body of fixed mass m ,

$$\frac{d\vec{p}}{dt} = \frac{d}{dt}(m\vec{v}) = m \frac{d\vec{v}}{dt} = m\vec{a}$$

So $\vec{F} = k m \vec{a}$. The unit of force is chosen so that $k = 1$. Hence

$$\vec{F} = \frac{d\vec{p}}{dt} = m\vec{a}$$

Unit of force: One **newton** is the force that produces an acceleration of 1 m s^{-2} in a body of mass 1 kg .

$$1 \text{ N} = 1 \text{ kg m s}^{-2}$$

Dimensions: $[M L T^{-2}]$. **CGS unit:** dyne; $1 \text{ N} = 10^5 \text{ dyne}$.

Illustration (NCERT Example 4.2): A bullet of mass 0.04 kg moving at 90 m s^{-1} is stopped by a wooden block in 60 cm .

$$a = \frac{-u^2}{2s} = \frac{-(90)^2}{2 \times 0.6} = -6750 \text{ m s}^{-2}, \quad F = ma = 0.04 \times 6750 = 270 \text{ N}$$

This is the **average** resistive force, since the actual retardation may not be uniform.

Q.6 Write the important points about Newton's second law of motion. (2 Marks)

- Consistent with the first law:** $\vec{F} = 0$ implies $\vec{a} = 0$.
- It is a vector law.** It is equivalent to three equations, one for each component:

$$F_x = \frac{dp_x}{dt} = ma_x, \quad F_y = \frac{dp_y}{dt} = ma_y, \quad F_z = \frac{dp_z}{dt} = ma_z$$

If a force is not parallel to the velocity, it changes only the component of velocity **along the force**. The component normal to the force stays unchanged (e.g. the horizontal velocity of a projectile stays constant under vertical gravity).

- Applies to a particle, a rigid body or a system of particles.** $\vec{F}$ is the total **external** force and $\vec{a}$ is the acceleration of the centre of mass. **Internal forces are not included in $\vec{F}$.**
- It is a local relation.** The force at a point at a certain instant determines the acceleration at that point **at that instant**. Acceleration does not depend on the history of motion.

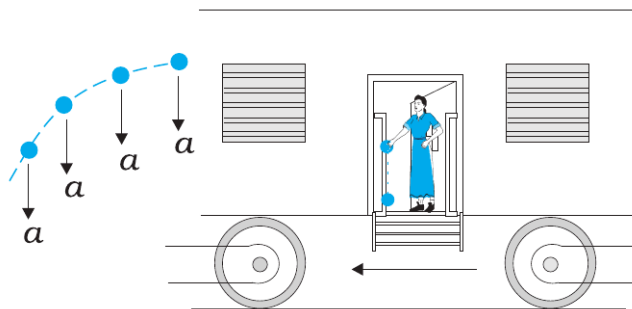


Fig. 4.5: The moment a stone is dropped out of an accelerated train, it has no horizontal acceleration if air resistance is neglected — it carries no memory of its acceleration with the train

Illustration (NCERT Example 4.3): If $y = ut + \frac{1}{2}gt^2$, then $v = \frac{dy}{dt} = u + gt$ and $a = \frac{dv}{dt} = g$. So the force is $F = ma = mg$; the particle moves under gravity.

Q.7 Define impulse. Show that impulse equals the change in momentum. (2 Marks)

Impulsive force: A large force acting for a very short time that produces a finite change in momentum is called an **impulsive force**. Examples: a ball hitting a wall and bouncing back, a bat hitting a ball, a hammer striking a nail.

Impulse: The product of force and the time for which it acts.

Derivation: From the second law, $\vec{F} = \frac{d\vec{p}}{dt}$, so $\vec{F} dt = d\vec{p}$. For a constant force acting for time Δt ,

$$\vec{J} = \vec{F} \Delta t = \Delta \vec{p} = m\vec{v} - m\vec{u}$$

For a variable force, $\vec{J} = \int \vec{F} dt$ = area under the F - t graph.

Key Examination Points for Full Marks:

- Impulse is a **vector**, in the direction of the change in momentum (and of the average force).
- **SI unit:** N s (same as momentum, kg m s^{-1}). **Dimensions:** $[M L T^{-1}]$.
- The force and the contact time are hard to measure separately, but their product (the change in momentum) is easy to find.
- The position of the body hardly changes during the short time of action.

Illustration (NCERT Example 4.4): A batsman hits a 0.15 kg ball straight back at the same speed of 12 m s^{-1} .

$$J = \Delta p = 0.15 \times 12 - (-0.15 \times 12) = 3.6 \text{ N s}$$

directed from the batsman towards the bowler.

Illustration (NCERT Example 4.5): Two identical balls strike a rigid wall with the same speed u — (a) normally, (b) at 30° to the normal — and rebound without change in speed.

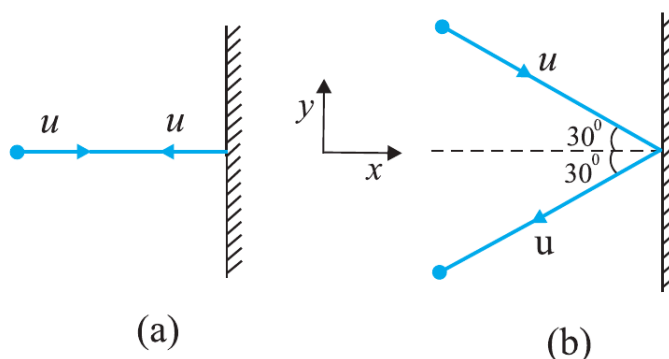


Fig. 4.6: Ball striking a rigid wall (a) normally, (b) at an angle of 30° to the normal

- Case (a): $\Delta p_x = -mu - mu = -2mu$, $\Delta p_y = 0$.

- Case (b): p_x changes sign but p_y does not, so $\Delta p_x = -2mu \cos 30^\circ$ and $\Delta p_y = 0$.

In both cases the force on the ball is normal to the wall (along $-x$), so by the third law the force on the wall is **normal to the wall** (along $+x$). Ratio of impulses: $\frac{2mu}{2mu \cos 30^\circ} = \frac{2}{\sqrt{3}} \approx 1.2$.

Q.8 State Newton's third law of motion. Write its important points. (3 Marks)

Statement: To every action, there is always an equal and opposite reaction.

Simple form: Forces always occur in pairs. The force on a body A by B is equal and opposite to the force on the body B by A.

$$\vec{F}_{AB} = -\vec{F}_{BA}$$

(force on A by B) = – (force on B by A)

Important points:

- **Action and reaction are simply forces.** The terms mean nothing else but “force”.
- **No cause-effect relation.** Action does not come before reaction. The force on A by B and the force on B by A act **at the same instant**. Either one may be called action and the other reaction.
- **They act on different bodies**, not on the same body. So they **never cancel** each other. When studying the motion of one body, only one of the two forces is relevant.
- For a system (A + B) as a whole, $\vec{F}_{AB}$ and $\vec{F}_{BA}$ are **internal forces** and add up to zero. This is why the second law can be applied to a system of particles.
- The law holds whether or not the bodies are in contact. Example: the earth pulls a stone, and the stone pulls the earth with an equal and opposite force. We do not notice the earth's motion because its mass is very large.

Examples: Recoil of a gun; a swimmer pushing water backward; walking (we push the ground backward, the ground pushes us forward); a rocket pushing hot gases backward.

Q.9 State and prove the law of conservation of linear momentum. (3 Marks)

Statement: The total momentum of an isolated system of interacting particles is conserved (an isolated system has no net external force).

Proof (collision of two bodies): Let two bodies A and B have initial momenta $\vec{p}_A$ and $\vec{p}_B$. They collide and get apart with final momenta $\vec{p}_A'$ and $\vec{p}_B'$. The forces act for the same time Δt (the time of contact). By the second law,

$$\vec{F}_{AB} \Delta t = \vec{p}_A' - \vec{p}_A \quad \text{and} \quad \vec{F}_{BA} \Delta t = \vec{p}_B' - \vec{p}_B$$

By the third law, $\vec{F}_{AB} = -\vec{F}_{BA}$. So

$$\begin{aligned} \vec{p}_A' - \vec{p}_A &= -(\vec{p}_B' - \vec{p}_B) \\ \vec{p}_A' + \vec{p}_B' &= \vec{p}_A + \vec{p}_B \end{aligned}$$

Total final momentum = total initial momentum. This holds whether the collision is **elastic or inelastic**.

Application — Recoil of a gun: Before firing, the bullet and gun are at rest, so the total momentum is zero. The force on the bullet by the gun is $\vec{F}$ and on the gun by the bullet is $-\vec{F}$, for the same time. So

$$\begin{aligned} \vec{p}_b + \vec{p}_g &= 0 \Rightarrow \vec{p}_g = -\vec{p}_b \\ m_g v_g &= -m_b v_b \Rightarrow v_g = -\frac{m_b}{m_g} v_b \end{aligned}$$

The gun recoils backward. As $m_g \gg m_b$, the recoil speed is small.

Key Examination Points for Full Marks:

- Internal (mutual) forces change the momenta of individual particles, but the changes cancel in pairs.
- The law follows from the **second and third laws**.

Q.10 What is meant by equilibrium of a particle? Write the conditions for equilibrium under concurrent forces. (3 Marks)

Definition: A particle is in equilibrium when the **net external force** on it is **zero**. By the first law, the particle is then either at rest or in uniform motion.

Two forces: Equilibrium requires the forces to be equal and opposite:

$$\vec{F}_1 = -\vec{F}_2$$

Three concurrent forces: The vector sum must be zero:

$$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 0$$

The resultant of any two forces (by the parallelogram law) must be equal and opposite to the third force. The three forces can be represented by the **sides of a triangle** taken in the same sense.

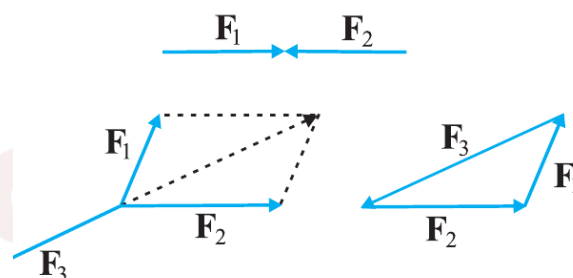


Fig. 4.7: Equilibrium under concurrent forces

n forces: A particle is in equilibrium under forces $\vec{F}_1, \vec{F}_2, \dots, \vec{F}_n$ if they can be represented by the sides of a **closed n-sided polygon** with arrows in the same sense.

Component form:

$$F_{1x} + F_{2x} + F_{3x} = 0, \quad F_{1y} + F_{2y} + F_{3y} = 0, \quad F_{1z} + F_{2z} + F_{3z} = 0$$

Note: Equilibrium of an extended body also needs rotational equilibrium (zero net torque), studied in Chapter 6.

Illustration (NCERT Example 4.6): A 6 kg mass hangs by a 2 m rope. A horizontal force of 50 N is applied at the mid-point P. Find the angle of the rope with the vertical ($g = 10 \text{ m s}^{-2}$).

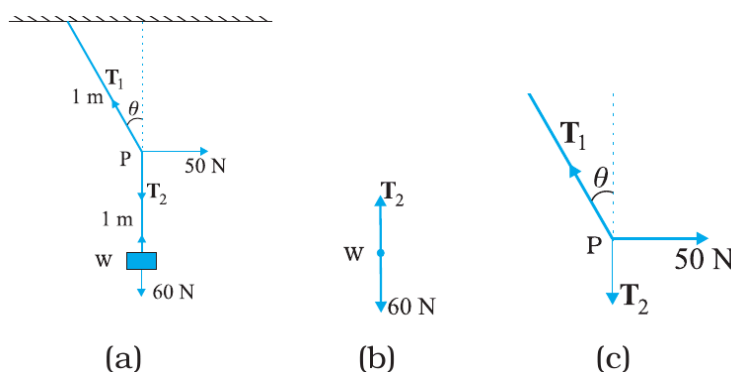


Fig. 4.8: (a) the arrangement; (b) free-body diagram of the weight W ; (c) free-body diagram of the point P

For W : $T_2 = 6 \times 10 = 60 \text{ N}$. For P :

$$T_1 \cos \theta = T_2 = 60 \text{ N}, \quad T_1 \sin \theta = 50 \text{ N}$$

$$\tan \theta = \frac{5}{6} \Rightarrow \theta = \tan^{-1} \left(\frac{5}{6} \right) \approx 40^\circ$$

The answer does not depend on the length of the (massless) rope or the point where the force is applied.

Q.11 Explain the common forces in mechanics.

(2 Marks)

Gravitational force: It acts on every object on the earth and governs the motion of celestial bodies. It **acts at a distance** without any intervening medium.

Contact forces: All other common forces in mechanics arise from contact with another object, solid or fluid.

- **Normal reaction:** The component of the contact force **normal** to the surfaces in contact.
- **Friction:** The component of the contact force **parallel** to the surfaces in contact.
- **Fluid forces:** buoyant force (equal to the weight of fluid displaced), viscous force, air resistance.
- **Spring force:** When a spring is compressed or extended, a restoring force $F = -kx$ acts, where k is the force constant. The negative sign shows the force is opposite to the displacement.
- **Tension:** The restoring force in a string. For an inextensible string the force constant is very high. The tension T is taken as the same throughout a string of negligible mass.

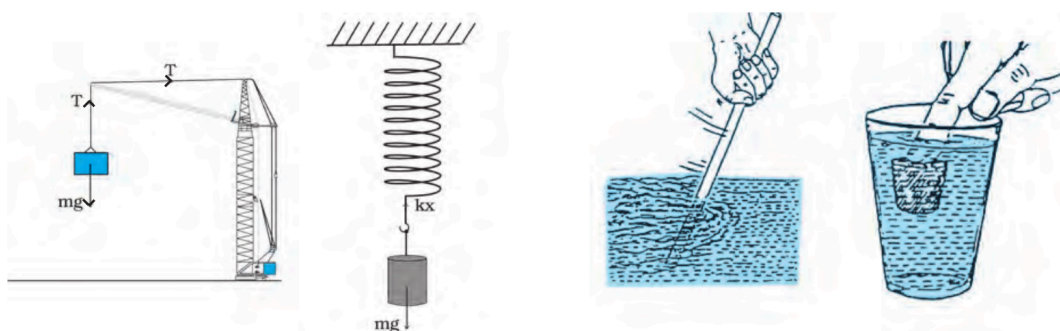


Fig. 4.9: Some examples of contact forces in mechanics

Origin: Of the four fundamental forces, only gravitational and electrical forces matter in mechanics. All contact forces **fundamentally arise from electrical forces** between the charged constituents (nuclei and electrons) of the bodies.

Q.12 What is friction? Explain static and kinetic friction and state the laws of friction.

(3 Marks)

Friction: The component of the contact force parallel to the surfaces in contact, which **opposes impending or actual relative motion** between the two surfaces. Note that it opposes **relative motion**, not motion itself.

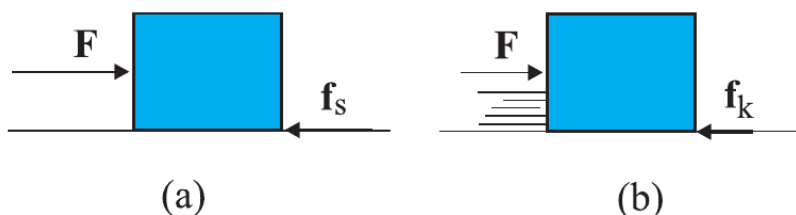


Fig. 4.10: (a) Impending motion is opposed by static friction; (b) once the body moves, it is subject to kinetic friction, which is usually less than the maximum static friction

Static friction (f_s): The friction that acts when there is **no relative motion** and opposes **impending motion**.

- It does not exist by itself. When there is no applied force, there is no static friction.
- It is **self-adjusting**: as the applied force F increases, f_s increases, staying equal and opposite to F , up to a limit.
- **Limiting friction** — the maximum value of static friction:

$$(f_s)_{\max} = \mu_s N \quad \text{so} \quad f_s \leq \mu_s N$$

Kinetic (sliding) friction (f_k): When the applied force exceeds $(f_s)_{max}$, the body starts sliding. The friction opposing actual relative motion is

$$f_k = \mu_k N$$

- $\mu_k < \mu_s$: friction decreases once motion starts.
- Acceleration of the body $= \frac{F - f_k}{m}$. For constant velocity, $F = f_k$. If F is removed, the retardation is $\frac{f_k}{m}$.

Laws of friction (empirical):

1. Limiting friction is **independent of the area of contact**.
2. It is **proportional to the normal force**: $(f_s)_{max} = \mu_s N$.
3. μ_s and μ_k depend only on the **nature of the surfaces** in contact.
4. Kinetic friction is nearly **independent of velocity**.

Illustration (NCERT Example 4.7): A box on the floor of a train stays at rest relative to the train only through static friction. $ma = f_s \leq \mu_s mg$, so $a_{max} = \mu_s g = 0.15 \times 10 = 1.5 \text{ m s}^{-2}$.

Illustration (NCERT Example 4.9): A 20 kg trolley ($\mu_k = 0.04$) is pulled by a 3 kg block hanging over a smooth pulley ($g = 10 \text{ m s}^{-2}$).

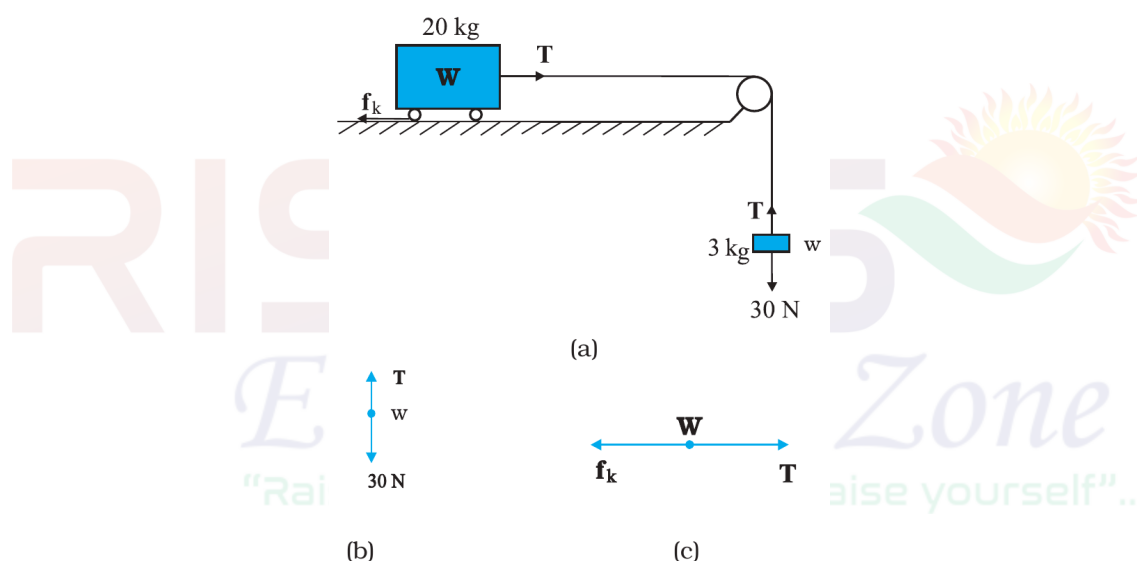


Fig. 4.12: (a) block-trolley system; (b) free-body diagram of the block; (c) free-body diagram of the trolley

$$\text{Block: } 30 - T = 3a \quad \text{Trolley: } T - 0.04 \times 200 = 20a \Rightarrow T - 8 = 20a$$

Adding: $22 = 23a$, so $a = 0.96 \text{ m s}^{-2}$ and $T = 27.1 \text{ N}$.

Q.13 Define angle of repose. Show that the coefficient of static friction equals the tangent of the angle of repose. (2 Marks)

Angle of repose (θ_{max}): The maximum angle of an inclined plane at which a body placed on it just begins to slide.

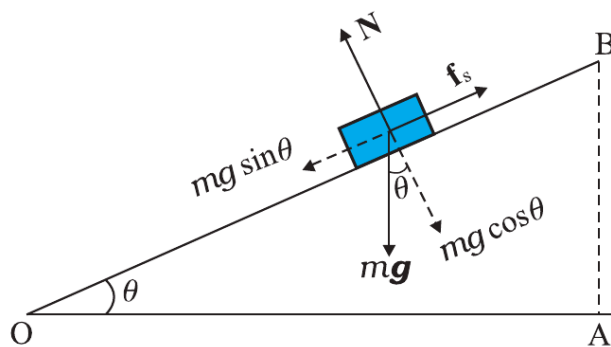


Fig. 4.11: Forces on a block resting on an inclined plane

Proof: Forces on a block of mass m at rest on the incline: (i) weight mg downward, (ii) normal force N , (iii) static friction f_s up the plane, opposing impending motion. Resolving mg along and perpendicular to the plane, for equilibrium

$$mg \sin \theta = f_s, \quad mg \cos \theta = N$$

As θ increases, f_s increases. At $\theta = \theta_{max}$, f_s becomes maximum: $(f_s)_{max} = \mu_s N$. So

$$mg \sin \theta_{max} = \mu_s mg \cos \theta_{max}$$

$$\mu_s = \tan \theta_{max} \quad \text{or} \quad \theta_{max} = \tan^{-1} \mu_s$$

Result: θ_{max} depends only on μ_s and is **independent of the mass** of the block.

Illustration (NCERT Example 4.8): A 4 kg block just begins to slide at $\theta = 15^\circ$. So $\mu_s = \tan 15^\circ = 0.27$.

Q.14 What is rolling friction? Is friction a necessary evil? How can friction be reduced? (2 Marks)

Rolling friction: The resistance to motion when a body (a ring, sphere or wheel) rolls over a surface. Ideally a body rolling without slipping has only one point of contact with no relative motion, so there should be no friction. In practice, the surfaces get slightly **deformed**, so there is a finite area of contact, and a component of the contact force opposes motion.

- For the same weight, rolling friction is **much smaller** (by 2 or 3 orders of magnitude) than static or sliding friction. This is why the wheel was a major milestone in human history.

Friction as an evil (disadvantages): It opposes relative motion and dissipates power as heat in machines, causing wear and tear.

Methods of reducing friction:

- Using **lubricants**.
- Using **ball bearings** between moving parts (converts sliding into rolling friction).
- Maintaining a thin **cushion of air** between surfaces in relative motion.

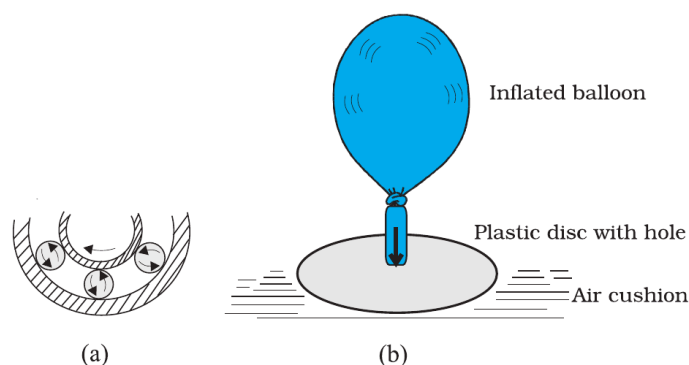


Fig. 4.13: Some ways of reducing friction: (a) ball bearings between moving parts of a machine; (b) compressed cushion of air between surfaces in relative motion

Friction as a necessity (advantages):

- Kinetic friction is used in **brakes** of machines and automobiles.
- We are able to **walk** because of static friction.
- A car cannot move on a very slippery road; friction between tyres and road provides the force to accelerate it.

Hence friction is called a **necessary evil**.

Q.15 What is centripetal force? Obtain the maximum speed of a car on a level circular road. (3 Marks)

Centripetal force: A body moving in a circle of radius R with uniform speed v has an acceleration v^2/R towards the centre. The force providing it is

$$f_c = \frac{mv^2}{R}$$

directed towards the centre. Centripetal force is **not a new kind of force** — it is a name for whichever real force (tension, gravity, friction, normal force) provides the inward acceleration. Examples: tension in the string for a whirled stone; the sun's gravitational pull on a planet.

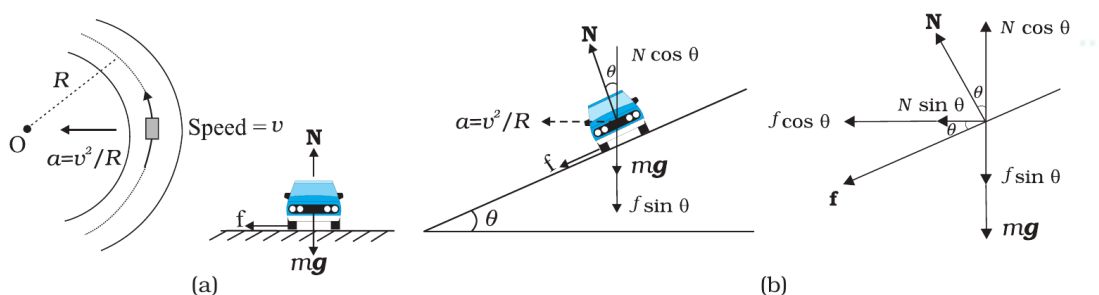


Fig. 4.14: Circular motion of a car on (a) a level road, (b) a banked road

Car on a level road — Fig. 4.14 (a): Three forces act on the car: (i) weight mg , (ii) normal reaction N , (iii) friction f .

No vertical acceleration, so

$$N - mg = 0 \Rightarrow N = mg$$

The centripetal force is provided by **static friction** along the road surface (it opposes the impending motion of the car away from the circle):

$$f = \frac{mv^2}{R} \leq \mu_s N = \mu_s mg$$

$$v^2 \leq \mu_s Rg \Rightarrow v_{max} = \sqrt{\mu_s Rg}$$

Result: v_{max} is independent of the mass of the car. For given μ_s and R there is a maximum speed for safe turning.

Illustration (NCERT Example 4.10): A cyclist at 18 km/h ($= 5 \text{ m s}^{-1}$) takes a turn of radius 3 m, $\mu_s = 0.1$. $\mu_s Rg = 0.1 \times 3 \times 9.8 = 2.94 \text{ m}^2\text{s}^{-2}$, but $v^2 = 25 \text{ m}^2\text{s}^{-2}$. Since $v^2 > \mu_s Rg$, the cyclist will slip.

Q.16 Obtain the maximum permissible speed of a car on a banked road. (3 Marks) Find the optimum speed.

Banking: Raising the outer edge of a curved road above the inner edge by an angle θ . This reduces the contribution of friction needed for circular motion.

Forces — Fig. 4.14 (b): weight mg , normal force N (perpendicular to the road), friction f (down the slope, at maximum speed).

Vertical direction (no acceleration):

$$N \cos \theta = mg + f \sin \theta \quad \dots(i)$$

Horizontal direction (centripetal force):

$$N \sin \theta + f \cos \theta = \frac{mv^2}{R} \quad \dots(ii)$$

For v_{max} , put $f = \mu_s N$:

$$N \cos \theta = mg + \mu_s N \sin \theta \Rightarrow N = \frac{mg}{\cos \theta - \mu_s \sin \theta}$$

$$N (\sin \theta + \mu_s \cos \theta) = \frac{mv_{max}^2}{R}$$

Substituting N :

$$\frac{mg (\sin \theta + \mu_s \cos \theta)}{\cos \theta - \mu_s \sin \theta} = \frac{mv_{max}^2}{R}$$

Dividing numerator and denominator by $\cos \theta$,

$$v_{max} = \sqrt{Rg \frac{\mu_s + \tan \theta}{1 - \mu_s \tan \theta}}$$

This is **greater** than the maximum speed on a flat road.

Optimum speed: Put $\mu_s = 0$:

$$v_o = \sqrt{Rg \tan \theta}$$

At this speed, **no friction is needed**: the horizontal component of the normal force alone provides the centripetal force. This causes the least wear and tear of tyres.

Key Examination Points for Full Marks:

- If $v < v_o$, friction acts **up** the slope.
- A car can be parked on a banked road only if $\tan \theta \leq \mu_s$.

Illustration (NCERT Example 4.11): $R = 300 \text{ m}$, $\theta = 15^\circ$, $\mu_s = 0.2$, $g = 9.8 \text{ m s}^{-2}$.

$$v_o = \sqrt{300 \times 9.8 \times \tan 15^\circ} = 28.1 \text{ m s}^{-1}, \quad v_{max} = \sqrt{300 \times 9.8 \times \frac{0.2 + \tan 15^\circ}{1 - 0.2 \tan 15^\circ}} = 38.1 \text{ m s}^{-1}$$

Q.17 What is a free-body diagram? Write the steps for solving problems (3 Marks) in mechanics.

System and environment: In a problem with an assembly of bodies, we choose any part as the **system**. The remaining part (plus any other agencies of force) is the **environment**.

Free-body diagram: A separate diagram that shows the chosen system and **all the forces acting on it** by the remaining parts and other agencies. It does **not** show the forces exerted by the system on the environment.

Steps:

1. Draw a schematic diagram showing the parts of the assembly, links, supports, etc.
2. Choose a convenient part of the assembly as one system.
3. Draw a free-body diagram of the system with all forces on it. **Do not include forces on the environment by the system.**
4. Include the known magnitudes and directions (e.g. tension along the string). Treat the rest as unknowns found using the laws of motion.
5. If needed, repeat for another system, using the **third law**: if the force on A due to B is $\vec{F}$, the force on B due to A is $-\vec{F}$.

Illustration (NCERT Example 4.12): A 2 kg block rests on a soft floor. A 25 kg iron cylinder is placed on it, and both go down with acceleration 0.1 m s^{-2} ($g = 10 \text{ m s}^{-2}$).

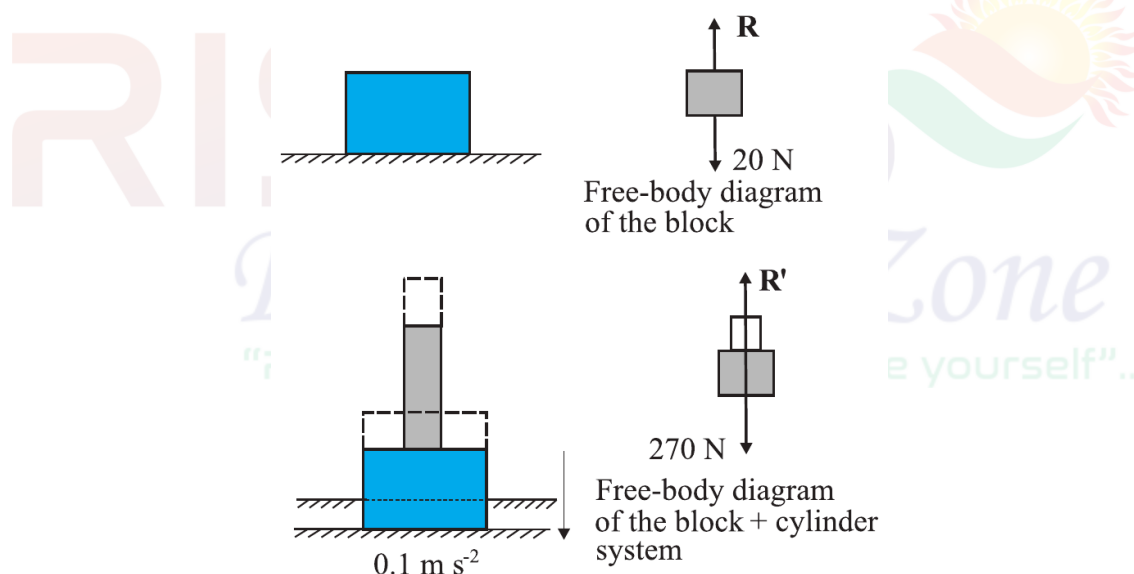


Fig. 4.15: Free-body diagrams of the block, and of the block + cylinder system

- Before the floor yields:** The block is at rest, so $R = 2 \times 10 = 20 \text{ N}$. By the third law, the action of the block on the floor is **20 N downward**.
- After the floor yields:** For the system (block + cylinder), weight = 270 N downward and normal force R' upward:

$$270 - R' = 27 \times 0.1 \quad \Rightarrow \quad R' = 267.3 \text{ N}$$

So the action on the floor is **267.3 N downward**.

Action-reaction pairs: (i) gravity on the block by the earth, and gravity on the earth by the block; (ii) force on the floor by the block (system) and force on the block (system) by the floor; (iii) in (b), force on the block by the cylinder and on the cylinder by the block.

Note: Weight mg and the normal force R on the same body are **not** an action-reaction pair. They are equal only when the body is in equilibrium (case (a)), not in case (b).

Formula Sheet — Laws of Motion

Quantity	Formula	SI Unit
Momentum	$\vec{p} = m\vec{v}$	kg m s^{-1}
Newton's second law	$\vec{F} = \frac{d\vec{p}}{dt} = m\vec{a}$	N
Components	$F_x = ma_x, F_y = ma_y, F_z = ma_z$	N
Impulse	$\vec{J} = \vec{F}\Delta t = \Delta\vec{p} = \int \vec{F} dt$	N s
Third law	$\vec{F}_{AB} = -\vec{F}_{BA}$	N
Conservation of momentum	$\vec{p}_A + \vec{p}_B = \vec{p}_A' + \vec{p}_B'$	kg m s^{-1}
Recoil velocity of gun	$v_g = -\frac{m_b}{m_g}v_b$	m s^{-1}
Equilibrium (concurrent)	$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 0$	N
Spring force	$F = -kx$	N
Static friction	$f_s \leq (f_s)_{\max} = \mu_s N$	N
Kinetic friction	$f_k = \mu_k N$	N
Angle of repose	$\mu_s = \tan \theta_{\max}$	—
Centripetal force	$f_c = \frac{mv^2}{R}$	N
Level road	$v_{\max} = \sqrt{\mu_s Rg}$	m s^{-1}
Banked road (max)	$v_{\max} = \sqrt{Rg \frac{\mu_s + \tan \theta}{1 - \mu_s \tan \theta}}$	m s^{-1}
Banked road (optimum)	$v_o = \sqrt{Rg \tan \theta}$	m s^{-1}

Dimensions: Force $[M L T^{-2}]$; momentum and impulse $[M L T^{-1}]$. **Conversions:** $1 \text{ N} = 10^5 \text{ dyne}$;
 $1 \text{ kg wt} = 9.8 \text{ N}$; $36 \text{ km h}^{-1} = 10 \text{ m s}^{-1}$

NEET / JEE Quick Points and Tricks

- Force is parallel to acceleration, not to velocity.** At the top of a vertical throw, $v = 0$ but the force is still mg and $a = g$.
- Action-reaction never cancel.** They act on different bodies. Weight and normal force on the same body are *not* an action-reaction pair.
- Apparent weight in a lift:** Scale reading $R = m(g + a)$ going up with acceleration a ; $R = m(g - a)$ going down with acceleration a ; $R = mg$ at uniform velocity; $R = 0$ in free fall.
- Two blocks in contact on a smooth surface:** If F pushes m_1 , which pushes m_2 : $a = \frac{F}{m_1 + m_2}$ and the contact force between them $= \frac{m_2}{m_1 + m_2} F$.
- Atwood machine** (masses over a smooth pulley, $m_1 > m_2$): $a = \frac{(m_1 - m_2)g}{m_1 + m_2}$, $T = \frac{2m_1 m_2 g}{m_1 + m_2}$.
- Tension in a connecting string** = (mass being dragged behind) $\times$ (common acceleration). So pulling the **lighter** body gives the **larger** tension. NCERT Ex. 4.15: pulling 10 kg gives 400 N; pulling 20 kg gives 200 N.
- Rocket thrust:** $F = m(g + a)$ at lift-off; thrust $= u \frac{dm}{dt}$ (exhaust speed $\times$ rate of mass ejection).

8. **Explosion / split at rest:** Total momentum stays zero, so fragments move in opposite directions; the lighter fragment moves faster ($v \propto 1/m$).
9. **Friction is self-adjusting.** Use $f_s = \mu_s N$ only when the body is on the verge of sliding. Otherwise f_s = the applied force.
10. **Block on a rough incline:** Sliding down, $a = g (\sin \theta - \mu_k \cos \theta)$; going up, retardation $= g (\sin \theta + \mu_k \cos \theta)$. If sliding down takes n times the time on a smooth incline, $\mu_k = \tan \theta \left(1 - \frac{1}{n^2}\right)$.
11. **Pulling is easier than pushing** (lawn mower). Pulling at angle θ reduces N to $mg - F \sin \theta$; pushing increases it to $mg + F \sin \theta$. Least force to drag a block: $\frac{\mu mg}{\sqrt{1 + \mu^2}}$ at $\tan \theta = \mu$.
12. **Impulse = area under the F - t graph.** A ball rebounding elastically from a wall: $\Delta p = 2mv \cos \theta$ (normal to the wall).
13. **String cut in circular motion:** The body flies off **along the tangent** (NCERT Ex. 4.22). Horizontal circle: $T = m\omega^2 r$.
14. **Banking:** Without friction, $\tan \theta = \frac{v^2}{Rg}$. Railway tracks are banked by raising the outer rail.

