



Chapter 5 : Work, Energy and Power (Solved Examples & Exercises)

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Notes

Step-by-step solutions · NCERT Examples 5.1–5.12 and Exercises 5.1–5.23 · Class 11 Physics

Part A — NCERT Solved Examples

Example 5.1

Question: Find the angle between force $\vec{F} = (3\hat{i} + 4\hat{j} - 5\hat{k})$ unit and displacement $\vec{d} = (5\hat{i} + 4\hat{j} + 3\hat{k})$ unit. Also find the projection of F on d .

Solution:

$$\vec{F} \cdot \vec{d} = F_x d_x + F_y d_y + F_z d_z = 3(5) + 4(4) + (-5)(3) = 16 \text{ unit}$$

$$F^2 = 9 + 16 + 25 = 50, \quad d^2 = 25 + 16 + 9 = 50$$

$$\cos \theta = \frac{\vec{F} \cdot \vec{d}}{Fd} = \frac{16}{\sqrt{50}\sqrt{50}} = \frac{16}{50} = 0.32 \Rightarrow \theta = \cos^{-1} 0.32 \approx 71.3^\circ$$

Projection of $\vec{F}$ on $\vec{d}$:

$$F \cos \theta = \frac{\vec{F} \cdot \vec{d}}{d} = \frac{16}{\sqrt{50}} = 2.26 \text{ unit}$$

Answer: $\theta = \cos^{-1}(0.32)$; projection = 2.26 unit

Example 5.2

Question: It is well known that a raindrop falls under the influence of the downward gravitational force and the opposing resistive force. The latter is known to be proportional to the speed of the drop but is otherwise undetermined. Consider a drop of mass 1.00 g falling from a height 1.00 km. It hits the ground with a speed of 50.0 m s^{-1} . (a) What is the work done by the gravitational force? (b) What is the work done by the unknown resistive force?

Solution: The change in kinetic energy of the drop (initially at rest) is

$$\Delta K = \frac{1}{2}mv^2 - 0 = \frac{1}{2} \times 10^{-3} \times 50 \times 50 = 1.25 \text{ J}$$

(a) Taking $g = 10 \text{ m/s}^2$:

$$W_g = mgh = 10^{-3} \times 10 \times 10^3 = 10.0 \text{ J}$$

(b) From the work–energy theorem, $\Delta K = W_g + W_r$:

$$W_r = \Delta K - W_g = 1.25 - 10 = -8.75 \text{ J}$$

Answer: (a) 10.0 J (b) -8.75 J (negative)

Example 5.3

Question: A cyclist comes to a skidding stop in 10 m. During this process, the force on the cycle due to the road is 200 N and is directly opposed to the motion. (a) How much work does the road do on the cycle? (b) How much work does the cycle do on the road?

Solution:

(a) The stopping (frictional) force and the displacement make an angle of 180° :

$$W_r = Fd \cos \theta = 200 \times 10 \times \cos \pi = -2000 \text{ J}$$

This negative work brings the cycle to a halt, in accordance with the WE theorem.

(b) By Newton's third law, an equal and opposite force of 200 N acts on the road. But the road undergoes no displacement, so the work done by the cycle on the road is **zero**.

Lesson: Though the force on A by B is equal and opposite to that on B by A, the work done on A by B is not necessarily equal and opposite to the work done on B by A.

Example 5.4

Question: In a ballistics demonstration a police officer fires a bullet of mass 50.0 g with speed 200 m s^{-1} on soft plywood of thickness 2.00 cm. The bullet emerges with only 10% of its initial kinetic energy. What is the emergent speed of the bullet?

Solution: Initial KE $= \frac{1}{2}mv^2 = \frac{1}{2} \times 0.05 \times 200^2 = 1000 \text{ J}$. Final KE $= 0.1 \times 1000 = 100 \text{ J}$.

$$\frac{1}{2}mv_f^2 = 100 \Rightarrow v_f = \sqrt{\frac{2 \times 100}{0.05}} = 63.2 \text{ m s}^{-1}$$

Answer: 63.2 m s^{-1} . The speed is reduced by about 68% (not 90%).

Example 5.5

Question: A woman pushes a trunk on a railway platform which has a rough surface. She applies a force of 100 N over a distance of 10 m. Thereafter, she gets progressively tired and her applied force reduces linearly with distance to 50 N. The total distance through which the trunk has been moved is 20 m. Plot the force applied by the woman and the frictional force, which is 50 N, versus displacement. Calculate the work done by the two forces over 20 m.

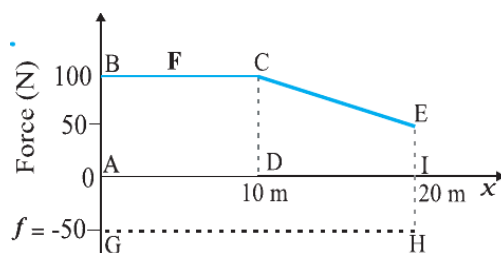


Fig. 5.4: Force F applied by the woman and the opposing frictional force f against displacement

Solution: The friction $\left| \vec{f} \right| = 50 \text{ N}$ opposes the motion, so it is plotted on the negative side of the force axis.

Work done by the woman = area of rectangle ABCD + area of trapezium CEID:

$$W_F = 100 \times 10 + \frac{1}{2} (100 + 50) \times 10 = 1000 + 750 = 1750 \text{ J}$$

Work done by friction = area of rectangle AGHI:

$$W_f = (-50) \times 20 = -1000 \text{ J}$$

Answer: $W_F = 1750 \text{ J}$; $W_f = -1000 \text{ J}$

Example 5.6

Question: A block of mass $m = 1 \text{ kg}$, moving on a horizontal surface with speed $v_i = 2 \text{ m s}^{-1}$, enters a rough patch ranging from $x = 0.10 \text{ m}$ to $x = 2.01 \text{ m}$. The retarding force F_r on the block in this range is inversely proportional to x : $F_r = \frac{-k}{x}$ for $0.1 < x < 2.01 \text{ m}$, and 0 for $x < 0.1 \text{ m}$ and $x > 2.01 \text{ m}$, where $k = 0.5 \text{ J}$. What is the final kinetic energy and speed v_f of the block as it crosses this patch?

Solution: From the WE theorem:

$$\begin{aligned} K_f &= K_i + \int_{0.1}^{2.01} \left(\frac{-k}{x} \right) dx = \frac{1}{2}mv_i^2 - k \ln x \Big|_{0.1}^{2.01} \\ &= \frac{1}{2}mv_i^2 - k \ln \left(\frac{2.01}{0.1} \right) = 2 - 0.5 \ln (20.1) = 2 - 1.5 = 0.5 \text{ J} \\ v_f &= \sqrt{\frac{2K_f}{m}} = 1 \text{ m s}^{-1} \end{aligned}$$

($\ln$ is the natural logarithm: $\ln X = 2.303 \log_{10} X$.)

Answer: $K_f = 0.5 \text{ J}$; $v_f = 1 \text{ m s}^{-1}$

Example 5.7

Question: A bob of mass m is suspended by a light string of length L . It is imparted a horizontal velocity v_0 at the lowest point A such that it completes a semi-circular trajectory in the vertical plane with the string becoming slack only on reaching the topmost point, C. Obtain an expression for (i) v_0 ; (ii) the speeds at points B and C; (iii) the ratio of the kinetic energies (K_B/K_C) at B and C. Comment on the nature of the trajectory of the bob after it reaches the point C.

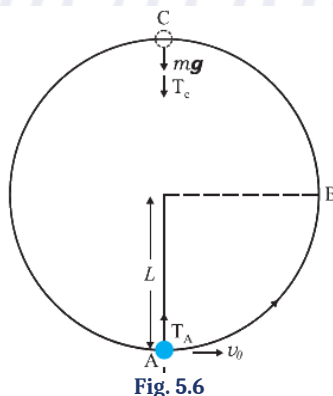


Fig. 5.6

Solution:

(i) Two forces act on the bob: gravity and tension. Tension does no work (it is always normal to the displacement), so mechanical energy is conserved. Taking PE = 0 at A:

$$\text{At A: } E = \frac{1}{2}mv_0^2, \text{ and } T_A - mg = \frac{mv_0^2}{L}.$$

At C the string slackens, so $T_C = 0$: $E = \frac{1}{2}mv_C^2 + 2mgL$ and $mg = \frac{mv_C^2}{L}$. Hence $E = \frac{5}{2}mgL$.

$$\frac{5}{2}mgL = \frac{1}{2}mv_0^2 \Rightarrow v_0 = \sqrt{5gL}$$

(ii) From the condition at C: $v_C = \sqrt{gL}$. At B (height L):

$$\frac{1}{2}mv_B^2 + mgL = \frac{5}{2}mgL \Rightarrow v_B = \sqrt{3gL}$$

(iii)

$$\frac{K_B}{K_C} = \frac{\frac{1}{2}mv_B^2}{\frac{1}{2}mv_C^2} = \frac{3}{1}$$

Comment: At C the string becomes slack. If the string were cut at this instant, the bob would execute projectile motion with horizontal projection (like a rock kicked horizontally from a cliff). Otherwise it continues on its circular path and completes the revolution.

Example 5.8

Question: To simulate car accidents, auto manufacturers study the collisions of moving cars with mounted springs of different spring constants. Consider a typical simulation with a car of mass 1000 kg moving with a speed 18.0 km/h on a smooth road and colliding with a horizontally mounted spring of spring constant $5.25 \times 10^3 \text{ N m}^{-1}$. What is the maximum compression of the spring?

Solution: At maximum compression, all the KE of the car is converted into PE of the spring. $18 \text{ km/h} = 5 \text{ m/s}$ (useful: $36 \text{ km/h} = 10 \text{ m/s}$).

$$K = \frac{1}{2}mv^2 = \frac{1}{2} \times 10^3 \times 5 \times 5 = 1.25 \times 10^4 \text{ J}$$

$$\frac{1}{2}kx_m^2 = K \Rightarrow x_m = \sqrt{\frac{2 \times 1.25 \times 10^4}{5.25 \times 10^3}} = 2.18 \text{ m}$$

Answer: $x_m \approx 2.18 \text{ m}$. (The textbook prints 2.00 m; that value corresponds to $k = 6.25 \times 10^3 \text{ N m}^{-1}$.) The spring is assumed massless and the surface frictionless.

Example 5.9

Question: Consider Example 5.8 taking the coefficient of friction, μ , to be 0.5 and calculate the maximum compression of the spring.

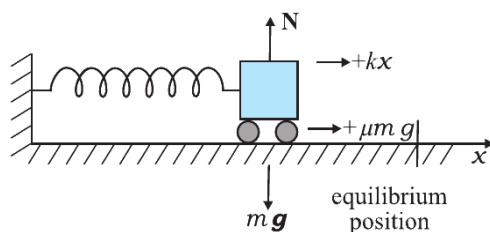


Fig. 5.9: The forces acting on the car

Solution: Both the spring force and friction oppose the compression. Use the WE theorem:

$$\Delta K = K_f - K_i = 0 - \frac{1}{2}mv^2, \quad W = -\frac{1}{2}kx_m^2 - \mu mg x_m$$

$$\frac{1}{2}mv^2 = \frac{1}{2}kx_m^2 + \mu mg x_m$$

With $\mu mg = 0.5 \times 10^3 \times 10 = 5 \times 10^3 \text{ N}$ ($g = 10 \text{ m s}^{-2}$):

$$kx_m^2 + 2\mu mg x_m - mv^2 = 0 \Rightarrow x_m = \frac{-\mu mg + [\mu^2 m^2 g^2 + mkv^2]^{1/2}}{k}$$

$$x_m = \frac{-5000 + \sqrt{(5000)^2 + 1000 \times 5250 \times 25}}{5250} = \frac{-5000 + 12500}{5250} = 1.43 \text{ m}$$

Answer: $x_m \approx 1.43 \text{ m}$ (the textbook prints 1.35 m), less than in Example 5.8, as expected.

In general, with a conservative force F_c and a non-conservative force F_{nc} : $E_f - E_i = W_{nc}$, where W_{nc} depends on the path.

Example 5.10

Question: An elevator can carry a maximum load of 1800 kg (elevator + passengers) is moving up with a constant speed of 2 m s^{-1} . The frictional force opposing the motion is 4000 N. Determine the minimum power delivered by the motor to the elevator in watts as well as in horse power.

Solution: The downward force on the elevator is

$$F = mg + F_f = 1800 \times 10 + 4000 = 22000 \text{ N}$$

The motor must supply enough power to balance this force:

$$P = \vec{F} \cdot \vec{v} = 22000 \times 2 = 44000 \text{ W} = \frac{44000}{746} \approx 59 \text{ hp}$$

Answer: $44000 \text{ W} \approx 59 \text{ hp}$

Example 5.11

Question: *Slowing down of neutrons:* In a nuclear reactor a neutron of high speed (typically 10^7 m s^{-1}) must be slowed to 10^3 m s^{-1} so that it can have a high probability of interacting with isotope ^{235}U and causing it to fission. Show that a neutron can lose most of its kinetic energy in an elastic collision with a light nucleus like deuterium or carbon which has a mass of only a few times the neutron mass. The material making up the light nuclei, usually heavy water (D_2O) or graphite, is called a moderator.

Solution: Initial KE of the neutron: $K_{1i} = \frac{1}{2}m_1v_{1i}^2$. Final KE (from $v_{1f} = \frac{m_1 - m_2}{m_1 + m_2}v_{1i}$):

$$K_{1f} = \frac{1}{2}m_1 \left(\frac{m_1 - m_2}{m_1 + m_2} \right)^2 v_{1i}^2$$

Fraction of KE retained by the neutron:

$$f_1 = \frac{K_{1f}}{K_{1i}} = \left(\frac{m_1 - m_2}{m_1 + m_2} \right)^2$$

Fraction gained by the moderating nucleus:

$$f_2 = 1 - f_1 = \frac{4m_1m_2}{(m_1 + m_2)^2}$$

- **Deuterium** ($m_2 = 2m_1$): $f_1 = 1/9$, $f_2 = 8/9$ — almost 90% of the neutron's energy is transferred.
- **Carbon** ($m_2 = 12m_1$): $f_1 = 71.6\%$, $f_2 = 28.4\%$.

In practice the numbers are smaller since head-on collisions are rare.

Example 5.12

Question: Consider the collision depicted in Fig. 5.10 to be between two billiard balls with equal masses $m_1 = m_2$. The first ball is called the cue while the second ball is called the target. The billiard player wants to 'sink' the target ball in a corner pocket, which is at an angle $\theta_2 = 37^\circ$. Assume that the collision is elastic and that friction and rotational motion are not important. Obtain θ_1 .

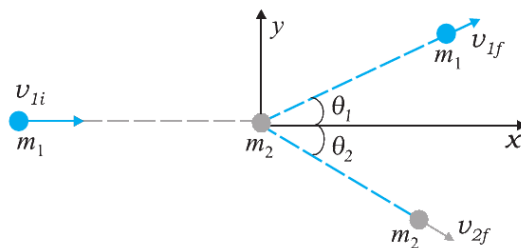


Fig. 5.10

Solution: From momentum conservation, since the masses are equal: $\vec{v}_{1i} = \vec{v}_{1f} + \vec{v}_{2f}$.

$$v_{1i}^2 = (\vec{v}_{1f} + \vec{v}_{2f}) \cdot (\vec{v}_{1f} + \vec{v}_{2f}) = v_{1f}^2 + v_{2f}^2 + 2v_{1f}v_{2f} \cos(\theta_1 + 37^\circ)$$

Since the collision is elastic and $m_1 = m_2$: $v_{1i}^2 = v_{1f}^2 + v_{2f}^2$. Comparing:

$$\cos(\theta_1 + 37^\circ) = 0 \Rightarrow \theta_1 + 37^\circ = 90^\circ \Rightarrow \theta_1 = 53^\circ$$

Result: When two equal masses undergo a glancing elastic collision with one of them at rest, after the collision they move at right angles to each other.

Part B — NCERT Exercises

Exercise 5.1

Question: The sign of work done by a force on a body is important to understand. State carefully if the following quantities are positive or negative: (a) work done by a man in lifting a bucket out of a well by means of a rope tied to the bucket. (b) work done by gravitational force in the above case. (c) work done by friction on a body sliding down an inclined plane. (d) work done by an applied force on a body moving on a rough horizontal plane with uniform velocity. (e) work done by the resistive force of air on a vibrating pendulum in bringing it to rest.

Answer:

- (a) **Positive** — the force (upward) and displacement (upward) are in the same direction.
- (b) **Negative** — gravity acts downward while the bucket moves up.
- (c) **Negative** — friction acts opposite to the motion (up the incline).
- (d) **Positive** — the applied force is along the motion (it balances friction).
- (e) **Negative** — the resistive force always opposes the motion.

Exercise 5.2

Question: A body of mass 2 kg initially at rest moves under the action of an applied horizontal force of 7 N on a table with coefficient of kinetic friction = 0.1. Compute the (a) work done by the applied force in 10 s, (b) work done by friction in 10 s, (c) work done by the net force on the body in 10 s, (d) change in kinetic energy of the body in 10 s, and interpret your results.

Solution: Friction $f = \mu mg = 0.1 \times 2 \times 9.8 = 1.96$ N. Net force = $7 - 1.96 = 5.04$ N.

$$a = \frac{5.04}{2} = 2.52 \text{ m s}^{-2}, \quad s = \frac{1}{2}at^2 = \frac{1}{2} \times 2.52 \times 10^2 = 126 \text{ m}$$

- (a) $W_F = 7 \times 126 = 882$ J
- (b) $W_f = -1.96 \times 126 = -247$ J
- (c) $W_{net} = 5.04 \times 126 = 635$ J
- (d) $v = at = 25.2$ m/s; $\Delta K = \frac{1}{2} \times 2 \times 25.2^2 - 0 = 635$ J

Interpretation: The change in kinetic energy equals the work done by the net force — the work-energy theorem.

Exercise 5.3

Question: Given in Fig. 5.11 are examples of some potential energy functions in one dimension. The total energy of the particle is indicated by a cross on the ordinate axis. In each case, specify the regions, if any, in which the particle cannot be found for the given energy. Also, indicate the minimum total energy the particle must have in each case. Think of simple physical contexts for which these potential energy shapes are relevant.

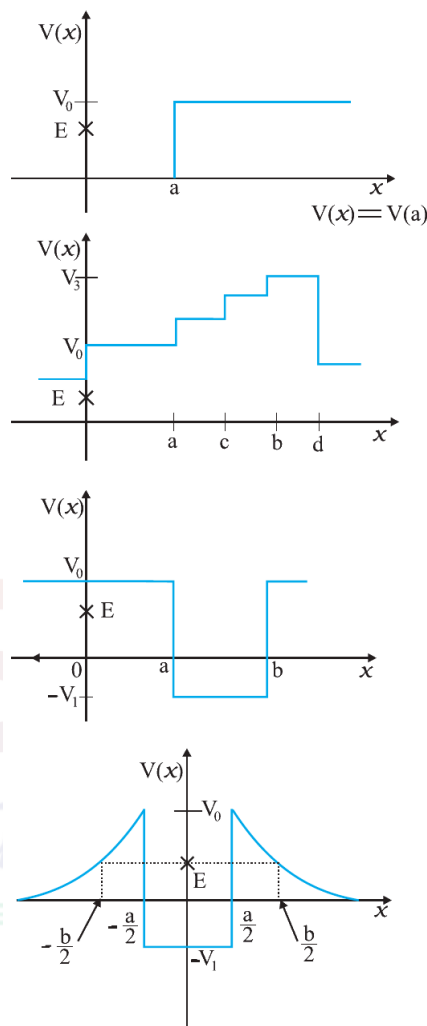


Fig. 5.11

Solution: Since $K = E - V \geq 0$, the particle cannot be in any region where $V > E$.

- (a) For $x > a$, $V_0 > E$. The particle **cannot be found in $x > a$** . Minimum total energy = 0.
- (b) $V > E$ everywhere, so the particle **cannot be found anywhere**. Minimum total energy = V_1 (the lowest value of V on the left).
- (c) For $x < a$ and $x > b$, $V_0 > E$. The particle can only be in $a < x < b$. Minimum total energy = $-V_1$.
- (d) For $-b/2 < x < -a/2$ and $a/2 < x < b/2$, $V > E$. The particle **cannot be found** in these regions. Minimum total energy = $-V_1$.

Physical contexts: (a) a particle meeting a step or wall; (c) a particle trapped in a potential well (e.g. an electron in a metal); (d) a well with walls, like a molecule's binding potential.

Exercise 5.4

Question: The potential energy function for a particle executing linear simple harmonic motion is given by $V(x) = kx^2/2$, where k is the force constant of the oscillator. For $k = 0.5 \text{ N m}^{-1}$, the graph of $V(x)$ versus x is

shown in Fig. 5.12. Show that a particle of total energy 1 J moving under this potential must 'turn back' when it reaches $x = \pm 2$ m.

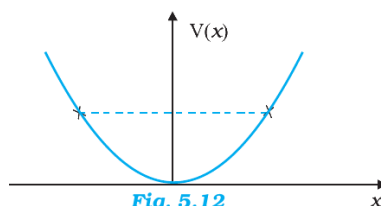


Fig. 5.12
Fig. 5.12

Solution: The particle turns back where its KE becomes zero, i.e. where all its energy is potential:

$$E = \frac{1}{2}kx^2 \Rightarrow 1 = \frac{1}{2} \times 0.5 \times x^2 \Rightarrow x^2 = 4 \Rightarrow x = \pm 2 \text{ m}$$

Beyond these points $V > E$, so the particle must turn back.

Exercise 5.5

Question: Answer the following: (a) The casing of a rocket in flight burns up due to friction. At whose expense is the heat energy required for burning obtained? The rocket or the atmosphere? (b) Comets move around the sun in highly elliptical orbits. The gravitational force on the comet due to the sun is not normal to the comet's velocity in general. Yet the work done by the gravitational force over every complete orbit of the comet is zero. Why? (c) An artificial satellite orbiting the earth in very thin atmosphere loses its energy gradually due to dissipation against atmospheric resistance, however small. Why then does its speed increase progressively as it comes closer and closer to the earth? (d) In Fig. 5.13(i) the man walks 2 m carrying a mass of 15 kg on his hands. In Fig. 5.13(ii), he walks the same distance pulling the rope behind him. The rope goes over a pulley, and a mass of 15 kg hangs at its other end. In which case is the work done greater?

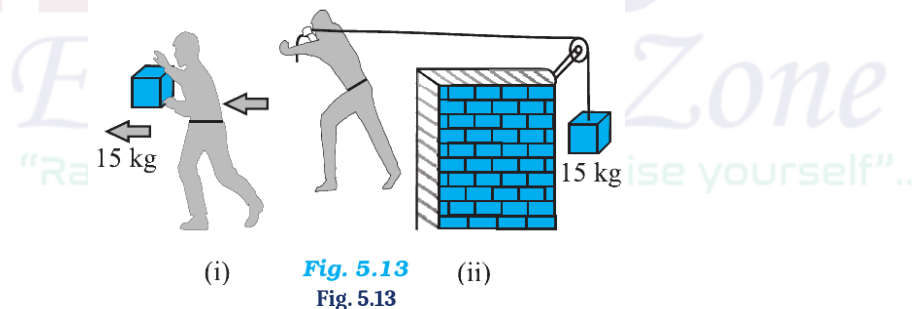


Fig. 5.13
Fig. 5.13

Answer:

- (a) At the expense of the **rocket**. The work done against friction comes from the rocket's mechanical energy.
- (b) Gravity is a **conservative force**, so the work done over a closed path (a complete orbit) is zero.
- (c) As the satellite comes closer, its potential energy decreases sharply. Part of this loss appears as an increase in kinetic energy ($KE = -\frac{1}{2} PE$ for an orbit), and only a small part is lost to friction. So the total energy decreases, but the speed increases.
- (d) In (i) the force on the mass is vertical and the displacement horizontal: $W = 0$. In (ii) the mass rises 2 m: $W = mgh = 15 \times 9.8 \times 2 = 294 \text{ J}$. **Work done is greater in case (ii).**

Exercise 5.6

Question: Underline the correct alternative: (a) When a conservative force does positive work on a body, the potential energy of the body increases/decreases/remains unaltered. (b) Work done by a body against friction always results in a loss of its kinetic/potential energy. (c) The rate of change of total momentum of a many-particle system is proportional to the external force/sum of the internal forces on the system. (d) In an inelastic

collision of two bodies, the quantities which do not change after the collision are the total kinetic energy/total linear momentum/total energy of the system of two bodies.

Answer:

- (a) **decreases** ($W = -\Delta V$)
- (b) **kinetic energy**
- (c) **external force** (internal forces cancel in pairs)
- (d) **total linear momentum** (and total energy, counting all its forms)

Exercise 5.7

Question: State if each of the following statements is true or false. Give reasons for your answer. (a) In an elastic collision of two bodies, the momentum and energy of each body is conserved. (b) Total energy of a system is always conserved, no matter what internal and external forces on the body are present. (c) Work done in the motion of a body over a closed loop is zero for every force in nature. (d) In an inelastic collision, the final kinetic energy is always less than the initial kinetic energy of the system.

Answer:

- (a) **False.** The **total** momentum and total KE of the system are conserved, not those of each body.
- (b) **False.** External forces can do work on the system and change its total energy. (The total energy of an isolated system is conserved.)
- (c) **False.** This holds only for conservative forces. For friction, the work over a closed loop is not zero.
- (d) **True** (usually). In an inelastic collision, part of the KE is converted into other forms (heat, sound, deformation), so the final KE is less than the initial KE.

Exercise 5.8

Question: Answer carefully, with reasons: (a) In an elastic collision of two billiard balls, is the total kinetic energy conserved during the short time of collision of the balls (i.e. when they are in contact)? (b) Is the total linear momentum conserved during the short time of an elastic collision of two balls? (c) What are the answers to (a) and (b) for an inelastic collision? (d) If the potential energy of two billiard balls depends only on the separation distance between their centres, is the collision elastic or inelastic? (Note, we are talking here of potential energy corresponding to the force during collision, not gravitational potential energy.)

Answer:

- (a) **No.** During contact, the balls are deformed and part of the KE is temporarily stored as elastic PE. KE is conserved only between the states before and after the collision.
- (b) **Yes.** The forces between the balls are internal, so the total linear momentum is conserved at every instant.
- (c) For an inelastic collision: KE is **not** conserved; total linear momentum **is** conserved.
- (d) **Elastic.** A force derived from a PE that depends only on separation is conservative, so no KE is lost after the balls separate.

Exercise 5.9

Question: A body is initially at rest. It undergoes one-dimensional motion with constant acceleration. The power delivered to it at time t is proportional to (i) $t^{1/2}$ (ii) t (iii) $t^{3/2}$ (iv) t^2

Solution: $F = ma$ is constant and $v = at$, so

$$P = Fv = ma \cdot at = ma^2t \propto t$$

Answer: (ii) t

Exercise 5.10

Question: A body is moving unidirectionally under the influence of a source of constant power. Its displacement in time t is proportional to (i) $t^{1/2}$ (ii) t (iii) $t^{3/2}$ (iv) t^2

Solution: With constant power, $\frac{1}{2}mv^2 = Pt \Rightarrow v = \sqrt{\frac{2Pt}{m}} \propto t^{1/2}$.

$$x = \int v dt \propto \int t^{1/2} dt \propto t^{3/2}$$

Answer: (iii) $t^{3/2}$

Exercise 5.11

Question: A body constrained to move along the z-axis of a coordinate system is subject to a constant force $\vec{F}$ given by $\vec{F} = -\hat{i} + 2\hat{j} + 3\hat{k}$ N, where $\hat{i}, \hat{j}, \hat{k}$ are unit vectors along the x-, y- and z-axis of the system respectively. What is the work done by this force in moving the body a distance of 4 m along the z-axis?

Solution: $\vec{d} = 4\hat{k}$ m.

$$W = \vec{F} \cdot \vec{d} = (-\hat{i} + 2\hat{j} + 3\hat{k}) \cdot 4\hat{k} = 12 \text{ J}$$

Answer: 12 J

Exercise 5.12

Question: An electron and a proton are detected in a cosmic ray experiment, the first with kinetic energy 10 keV, and the second with 100 keV. Which is faster, the electron or the proton? Obtain the ratio of their speeds. (electron mass = 9.11×10^{-31} kg, proton mass = 1.67×10^{-27} kg, 1 eV = 1.60×10^{-19} J)

Solution: $v = \sqrt{2K/m}$.

$$v_e = \sqrt{\frac{2 \times 10^4 \times 1.6 \times 10^{-19}}{9.11 \times 10^{-31}}} = 5.93 \times 10^7 \text{ m/s}$$

$$v_p = \sqrt{\frac{2 \times 10^5 \times 1.6 \times 10^{-19}}{1.67 \times 10^{-27}}} = 4.38 \times 10^6 \text{ m/s}$$

$$\frac{v_e}{v_p} = 13.5$$

Answer: The electron is faster; $v_e : v_p \approx 13.5 : 1$.

Exercise 5.13

Question: A rain drop of radius 2 mm falls from a height of 500 m above the ground. It falls with decreasing acceleration (due to viscous resistance of the air) until at half its original height, it attains its maximum (terminal) speed, and moves with uniform speed thereafter. What is the work done by the gravitational force on the drop in the first and second half of its journey? What is the work done by the resistive force in the entire journey if its speed on reaching the ground is 10 m s^{-1} ?

Solution: Mass of the drop (density of water 10^3 kg m^{-3}):

$$m = \frac{4}{3}\pi r^3 \rho = \frac{4}{3}\pi (2 \times 10^{-3})^3 \times 10^3 = 3.35 \times 10^{-5} \text{ kg}$$

Work by gravity in each half ($h = 250 \text{ m}$):

$$W_g = mgh = 3.35 \times 10^{-5} \times 9.8 \times 250 = 0.082 \text{ J}$$

This is the same for both halves — gravity does not depend on the speed.

Total work by gravity = 0.164 J. By the WE theorem, $W_g + W_r = \Delta K$:

$$\Delta K = \frac{1}{2} \times 3.35 \times 10^{-5} \times 10^2 = 1.68 \times 10^{-3} \text{ J}$$

$$W_r = 1.68 \times 10^{-3} - 0.164 = -0.162 \text{ J}$$

Answer: 0.082 J in each half; work by the resistive force = -0.162 J

Exercise 5.14

Question: A molecule in a gas container hits a horizontal wall with speed 200 m s^{-1} and angle 30° with the normal, and rebounds with the same speed. Is momentum conserved in the collision? Is the collision elastic or inelastic?

Answer:

- **Momentum:** The momentum of the molecule alone changes (its normal component reverses). But the total momentum of the **molecule + wall** system is conserved — the wall (with the Earth) takes up the change, though its recoil is negligible.
- The molecule rebounds with the **same speed**, so its KE is unchanged and the wall's KE is negligible. The collision is **elastic**.

Exercise 5.15

Question: A pump on the ground floor of a building can pump up water to fill a tank of volume 30 m^3 in 15 min. If the tank is 40 m above the ground, and the efficiency of the pump is 30%, how much electric power is consumed by the pump?

Solution: Mass of water = $30 \times 10^3 \text{ kg}$; $t = 900 \text{ s}$.

$$P_{out} = \frac{mgh}{t} = \frac{30 \times 10^3 \times 9.8 \times 40}{900} = 1.307 \times 10^4 \text{ W}$$

$$P_{in} = \frac{P_{out}}{0.30} = 4.36 \times 10^4 \text{ W} \approx 43.6 \text{ kW}$$

Answer: $\approx 43.6 \text{ kW}$

Exercise 5.16

Question: Two identical ball bearings in contact with each other and resting on a frictionless table are hit head-on by another ball bearing of the same mass moving initially with a speed V . If the collision is elastic, which of the following (Fig. 5.14) is a possible result after collision?

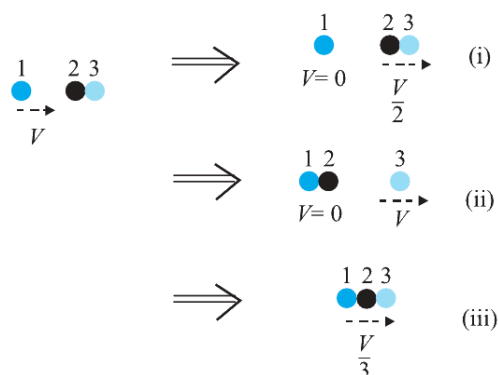


Fig. 5.14

Solution: Initial momentum = mV ; initial KE = $\frac{1}{2}mV^2$.

- (i) Ball 1 at rest, balls 2 and 3 move with $V/2$: momentum = $2m (V/2) = mV$ ✓; KE = $2 \times \frac{1}{2}m (V/2)^2 = \frac{1}{4}mV^2$ ✗
- (ii) Balls 1 and 2 at rest, ball 3 moves with V : momentum = mV ✓; KE = $\frac{1}{2}mV^2$ ✓
- (iii) All three move with $V/3$: momentum = mV ✓; KE = $3 \times \frac{1}{2}m (V/3)^2 = \frac{1}{6}mV^2$ ✗

Answer: Case (ii) is the only possible result (KE and momentum both conserved).

Exercise 5.17

Question: The bob A of a pendulum released from 30° to the vertical hits another bob B of the same mass at rest on a table as shown in Fig. 5.15. How high does the bob A rise after the collision? Neglect the size of the bobs and assume the collision to be elastic.

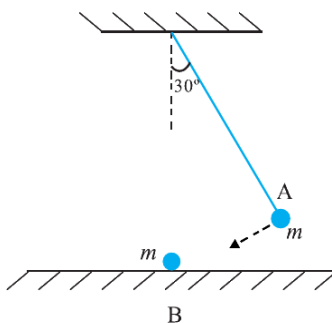


Fig. 5.15

Solution: In an elastic collision between equal masses with one at rest, the velocities are exchanged. Bob A comes to rest and bob B moves off with A's velocity.

Answer: Bob A does not rise at all after the collision (height = 0).

Exercise 5.18

Question: The bob of a pendulum is released from a horizontal position. If the length of the pendulum is 1.5 m, what is the speed with which the bob arrives at the lowermost point, given that it dissipated 5% of its initial energy against air resistance?

Solution: The bob falls through $h = l = 1.5$ m. 95% of the PE becomes KE:

$$\frac{1}{2}mv^2 = 0.95 mgh \Rightarrow v = \sqrt{2 \times 0.95 \times 9.8 \times 1.5} = 5.28 \text{ m s}^{-1}$$

Answer: $\approx 5.3 \text{ m s}^{-1}$

Exercise 5.19

Question: A trolley of mass 300 kg carrying a sandbag of 25 kg is moving uniformly with a speed of 27 km/h on a frictionless track. After a while, sand starts leaking out of a hole on the floor of the trolley at the rate of 0.05 kg s^{-1} . What is the speed of the trolley after the entire sand bag is empty?

Solution: The sand leaks out vertically, carrying its own horizontal velocity (the same as the trolley's). No external horizontal force acts on the trolley, so its speed does not change.

Answer: 27 km/h (unchanged)

Exercise 5.20

Question: A body of mass 0.5 kg travels in a straight line with velocity $v = ax^{3/2}$ where $a = 5 \text{ m}^{-1/2} \text{ s}^{-1}$. What is the work done by the net force during its displacement from $x = 0$ to $x = 2$ m?

Solution: At $x = 0$, $v = 0$. At $x = 2$ m: $v = 5 \times 2^{3/2} = 10\sqrt{2}$ m/s.

$$W = \Delta K = \frac{1}{2} \times 0.5 \times (10\sqrt{2})^2 - 0 = 50 \text{ J}$$

Answer: 50 J

Exercise 5.21

Question: The blades of a windmill sweep out a circle of area A . (a) If the wind flows at a velocity v perpendicular to the circle, what is the mass of the air passing through it in time t ? (b) What is the kinetic energy of the air? (c) Assume that the windmill converts 25% of the wind's energy into electrical energy, and that $A = 30 \text{ m}^2$, $v = 36 \text{ km/h}$ and the density of air is 1.2 kg m^{-3} . What is the electrical power produced?

Solution:

(a) Volume of air through the circle in time $t = Avt$, so mass $m = \rho Avt$.

(b) $K = \frac{1}{2}mv^2 = \frac{1}{2}\rho Av^3t$

(c) $v = 36 \text{ km/h} = 10 \text{ m/s}$. Wind power $= \frac{1}{2}\rho Av^3 = \frac{1}{2} \times 1.2 \times 30 \times 10^3 = 18000 \text{ W}$.

$$P_{\text{electrical}} = 0.25 \times 18000 = 4500 \text{ W} = 4.5 \text{ kW}$$

Answer: (a) ρAvt (b) $\frac{1}{2}\rho Av^3t$ (c) 4.5 kW

Exercise 5.22

Question: A person trying to lose weight (dieter) lifts a 10 kg mass, one thousand times, to a height of 0.5 m each time. Assume that the potential energy lost each time she lowers the mass is dissipated. (a) How much work does she do against the gravitational force? (b) Fat supplies $3.8 \times 10^7 \text{ J}$ of energy per kilogram which is converted to mechanical energy with a 20% efficiency rate. How much fat will the dieter use up?

Solution:

(a) $W = n mgh = 1000 \times 10 \times 9.8 \times 0.5 = 4.9 \times 10^4 \text{ J}$

(b) Useful mechanical energy per kg of fat $= 0.2 \times 3.8 \times 10^7 = 7.6 \times 10^6 \text{ J}$.

$$\text{Fat used} = \frac{4.9 \times 10^4}{7.6 \times 10^6} = 6.45 \times 10^{-3} \text{ kg} \approx 6.5 \text{ g}$$

Answer: (a) 49 kJ (b) $\approx 6.45 \times 10^{-3} \text{ kg}$

Exercise 5.23

Question: A family uses 8 kW of power. (a) Direct solar energy is incident on the horizontal surface at an average rate of 200 W per square meter. If 20% of this energy can be converted to useful electrical energy, how large an area is needed to supply 8 kW? (b) Compare this area to that of the roof of a typical house.

Solution:

(a) Useful power per $\text{m}^2 = 0.2 \times 200 = 40 \text{ W m}^{-2}$.

$$A = \frac{8000}{40} = 200 \text{ m}^2$$

(b) 200 m^2 is about a $14 \text{ m} \times 14 \text{ m}$ square — comparable to the roof of a large house (a typical house roof is of the order of $100\text{--}200 \text{ m}^2$).

Answer: (a) 200 m^2 (b) about the roof area of a large house