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Chapter 5 : Work, Energy and Power (Notes)

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Notes

GSEB and CBSE Board Exam preparation subjective notes · Class 11 Physics · NCERT Chapter 5

Q.1 Define the scalar (dot) product of two vectors. State its properties. (2 Marks)

Scalar product: The scalar or dot product of two vectors **A** and **B** is

$$\vec{A} \cdot \vec{B} = AB \cos \theta$$

where θ is the angle between the two vectors. The result is a **scalar**.

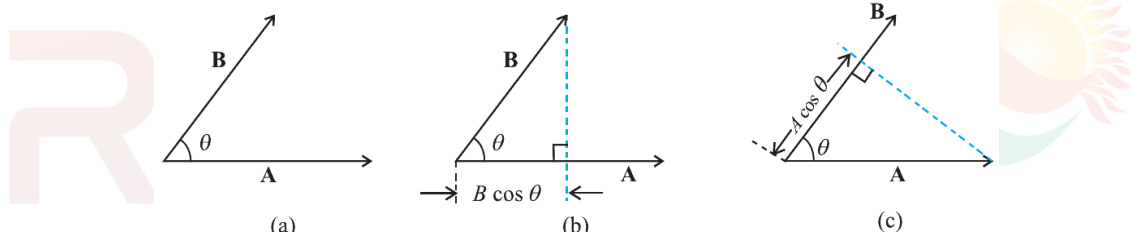


Fig. 5.1: (a) The scalar product $AB = AB \cos \theta$. (b) $B \cos \theta$ is the projection of **B** onto **A**. (c) $A \cos \theta$ is the projection of **A** onto **B**

Geometrical meaning: $\vec{A} \cdot \vec{B} = A (B \cos \theta) = B (A \cos \theta)$ — the product of the magnitude of one vector and the component (projection) of the other along it.

In component form: For $\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$ and $\vec{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k}$:

$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$$

Properties:

- **Commutative:** $\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$
- **Distributive:** $\vec{A} \cdot (\vec{B} + \vec{C}) = \vec{A} \cdot \vec{B} + \vec{A} \cdot \vec{C}$
- $\vec{A} \cdot (\lambda \vec{B}) = \lambda (\vec{A} \cdot \vec{B})$
- $\hat{i} \cdot \hat{i} = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = 1$ and $\hat{i} \cdot \hat{j} = \hat{j} \cdot \hat{k} = \hat{k} \cdot \hat{i} = 0$
- $\vec{A} \cdot \vec{A} = A^2 = A_x^2 + A_y^2 + A_z^2$
- $\vec{A} \cdot \vec{B} = 0$ if $\vec{A} \perp \vec{B}$.

Key Examination Points for Full Marks:

- Angle between two vectors: $\cos \theta = \frac{\vec{A} \cdot \vec{B}}{AB}$.
- **Example 5.1:** $\vec{F} = 3\hat{i} + 4\hat{j} - 5\hat{k}$, $\vec{d} = 5\hat{i} + 4\hat{j} + 3\hat{k}$; $\vec{F} \cdot \vec{d} = 15 + 16 - 15 = 16$; $F = d = \sqrt{50}$;
 $\cos \theta = \frac{16}{50} = 0.32$, so $\theta = \cos^{-1} 0.32$. Projection of **F** on **d** = $\frac{16}{\sqrt{50}} = 2.26$ unit.

Q.2 Define work. When is work positive, negative or zero? Give its unit and dimensions. (3 Marks)

Work: The work done by a constant force is the product of the component of the force in the direction of the displacement and the magnitude of the displacement:

$$W = (F \cos \theta) d = \vec{F} \cdot \vec{d}$$

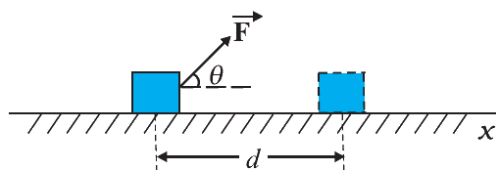


Fig. 5.2: An object undergoes a displacement d under the influence of the force F

Unit: joule (J); $1 \text{ J} = 1 \text{ N m}$. **Dimensions:** $[M L^2 T^{-2}]$. Work is a **scalar**.

Positive work ($0^\circ \leq \theta < 90^\circ$): the force has a component along the displacement. Example: work done by gravity on a falling body.

Negative work ($90^\circ < \theta \leq 180^\circ$): the force opposes the displacement. Example: work done by friction ($\theta = 180^\circ$, $\cos \theta = -1$).

Zero work is done if:

- i. **the displacement is zero** — pushing a rigid wall; a weightlifter holding 150 kg steadily on his shoulder for 30 s;
- ii. **the force is zero** — a block moving on a smooth horizontal table with no horizontal force;
- iii. **the force and displacement are perpendicular** ($\theta = 90^\circ$) — gravity on a block moving on a horizontal table; the Earth's gravitational force on the Moon in a circular orbit.

Alternative units of work/energy:

Unit	Value in joule
erg	10^{-7} J
electron volt (eV)	$1.6 \times 10^{-19} \text{ J}$
calorie (cal)	4.186 J
kilowatt hour (kWh)	$3.6 \times 10^6 \text{ J}$

Key Examination Points for Full Marks:

- In physics, work needs a displacement. You get tired pushing a wall, but no work is done on the wall.
- **Example 5.3:** A cyclist skids to a stop in 10 m with a road force of 200 N opposing motion. Work by the road on the cycle $= 200 \times 10 \times \cos 180^\circ = -2000 \text{ J}$. Work by the cycle on the road $= 0$ (the road does not move). So the work done on A by B need not be equal and opposite to the work done on B by A.

Q.3 Define kinetic energy. State and prove the work–energy theorem for a constant force. (3 Marks)

Kinetic energy: The energy possessed by a body by virtue of its motion:

$$K = \frac{1}{2} m \vec{v} \cdot \vec{v} = \frac{1}{2} m v^2$$

It is a scalar and always positive. It measures the work a body can do by virtue of its motion. Unit: joule.

Work-energy theorem: The change in kinetic energy of a particle is equal to the work done on it by the net force.

$$K_f - K_i = W$$

Proof (constant force): For uniformly accelerated motion, $v^2 - u^2 = 2as$. Multiplying both sides by $m/2$:

$$\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = mas = Fs$$

In three dimensions, $v^2 - u^2 = 2\vec{a} \cdot \vec{d}$, so

$$\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = m\vec{a} \cdot \vec{d} = \vec{F} \cdot \vec{d} = W$$

Key Examination Points for Full Marks:

- The theorem lets us find the work of an unknown force. **Example 5.2:** A 1.00 g drop falls 1.00 km and hits the ground at 50.0 m/s. $\Delta K = \frac{1}{2} \times 10^{-3} \times 50^2 = 1.25 \text{ J}$; $W_g = mgh = 10.0 \text{ J}$; so the resistive force does $W_r = 1.25 - 10 = -8.75 \text{ J}$.
- Example 5.4:** A 50 g bullet at 200 m/s ($K = 1000 \text{ J}$) emerges from plywood with 10% of its KE (100 J): $v_f = \sqrt{2 \times 100 / 0.05} = 63.2 \text{ m/s}$ — the **speed** drops by about 68% (not 90%).

Q.4 How is work done by a variable force calculated?

(2 Marks)

For a force $F(x)$ that varies with position, take a small displacement Δx over which $F(x)$ is nearly constant. The work done is $\Delta W = F(x) \Delta x$ — the area of a thin rectangle (Fig. 5.3(a)).

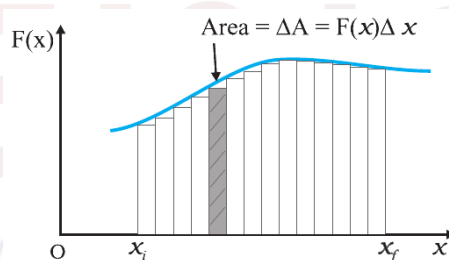


Fig. 5.3(a): The shaded rectangle represents the work done by $F(x)$ over a small displacement Δx

Adding all such rectangles from x_i to x_f : $W \approx \sum F(x) \Delta x$. In the limit $\Delta x \rightarrow 0$:

$$W = \lim_{\Delta x \rightarrow 0} \sum_{x_i}^{x_f} F(x) \Delta x = \int_{x_i}^{x_f} F(x) dx$$

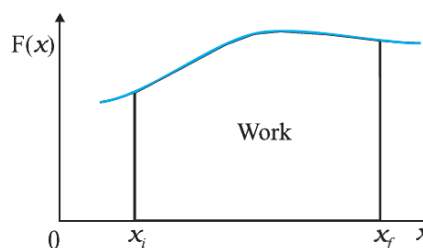


Fig. 5.3(b): For $\Delta x \rightarrow 0$, the area under the curve is exactly the work done by $F(x)$

Work = area under the force–displacement graph.

Key Examination Points for Full Marks:

- Area below the x-axis counts as **negative** work.
- Example 5.5:** A woman pushes a trunk with 100 N for 10 m, then her force falls linearly to 50 N over the next 10 m; friction is 50 N.

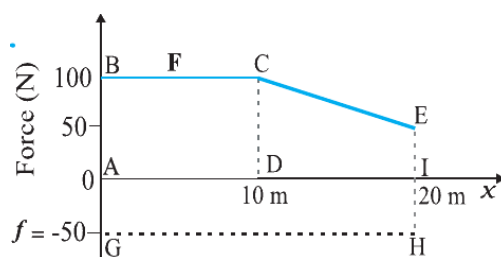


Fig. 5.4: Force applied by the woman (F) and the opposing friction (f) against displacement

$$W_F = 100 \times 10 + \frac{1}{2} (100 + 50) \times 10 = 1750 \text{ J}; \quad W_f = -50 \times 20 = -1000 \text{ J}$$

Q.5 Prove the work–energy theorem for a variable force.

(2 Marks)

In one dimension, the rate of change of kinetic energy is

$$\frac{dK}{dt} = \frac{d}{dt} \left(\frac{1}{2} m v^2 \right) = m \frac{dv}{dt} v = Fv = F \frac{dx}{dt}$$

So $dK = F dx$. Integrating from x_i to x_f :

$$\int_{K_i}^{K_f} dK = \int_{x_i}^{x_f} F dx \Rightarrow K_f - K_i = W$$

Key Examination Points for Full Marks:

- The WE theorem is an **integral form** (over space) of Newton's second law, and is a **scalar** relation; the time and direction information of Newton's law is not explicit in it.
- Example 5.6:** A 1 kg block at 2 m/s crosses a rough patch from $x = 0.1$ m to 2.01 m with $F_r = -k/x$, $k = 0.5$ J: $K_f = 2 - 0.5 \ln(20.1) = 0.5$ J, $v_f = 1$ m/s.

Q.6 What is potential energy? Derive the expression for gravitational potential energy.

(2 Marks)

Potential energy is the energy “stored” in a body by virtue of its **position or configuration**. Examples: a stretched bow-string, a compressed spring, fault lines in the Earth's crust.

Gravitational PE near the Earth's surface: To raise a ball of mass m through height h ($h \ll R_E$) at constant speed, an external agency does work mgh against gravity. This work is stored as potential energy:

$$V(h) = mgh$$

The gravitational force is the negative derivative of $V(h)$:

$$F = -\frac{d}{dh} V(h) = -mg$$

(the negative sign shows that the force is downward). When released, the ball reaches the ground with $v^2 = 2gh$, i.e. $\frac{1}{2}mv^2 = mgh$ — the PE is converted into KE.

General definition (one dimension): A potential energy $V(x)$ exists if the force can be written as

$$F(x) = -\frac{dV}{dx} \Rightarrow \int_{x_i}^{x_f} F(x) dx = V_i - V_f$$

Key Examination Points for Full Marks:

- Dimensions of PE: $[ML^2T^{-2}]$; unit: joule.
- The change in PE equals the **negative** of the work done by a conservative force: $\Delta V = -F(x) \Delta x$.
- The **zero of PE is arbitrary** (only differences matter), but once chosen it must be kept throughout a problem.

Q.7 What are conservative and non-conservative forces? Give examples. (2 Marks)

Conservative force: A force $F(x)$ is conservative if

- i. it can be derived from a scalar potential: $F(x) = -\frac{dV}{dx}$; **or**
- ii. the work done by it depends **only on the end points**, not on the path:
 $W = K_f - K_i = V(x_i) - V(x_f)$; **or**
- iii. the work done by it in a **closed path is zero**.

Examples: gravitational force, spring force, electrostatic force.

Non-conservative force: The work done depends on the path, and the work done in a closed path is not zero.

Examples: friction, viscous force, air resistance.

Key Examination Points for Full Marks:

- Example: a body sliding from rest down a smooth incline of height h reaches the bottom with speed $\sqrt{2gh}$, **whatever the angle** of the incline — gravity is conservative.
- No potential energy can be associated with friction.

Q.8 State and prove the principle of conservation of mechanical energy (3 Marks) for a freely falling body.

Statement: The total mechanical energy of a system is conserved if the forces doing work on it are conservative.

General proof (1-D): By the WE theorem, $\Delta K = F(x) \Delta x$. For a conservative force, $\Delta V = -F(x) \Delta x$. Hence

$$\Delta K + \Delta V = 0 \Rightarrow \Delta(K + V) = 0 \Rightarrow K_i + V(x_i) = K_f + V(x_f)$$

Freely falling body: A ball of mass m is dropped from rest from a cliff of height H (Fig. 5.5).

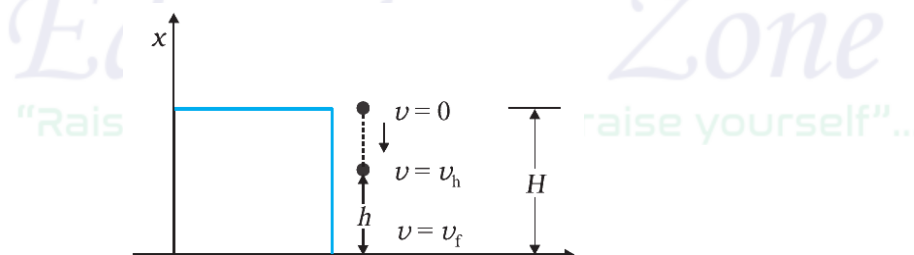


Fig. 5.5: Conversion of potential energy to kinetic energy for a ball of mass m dropped from a height H

- At height H : $E_H = mgH + 0 = mgH$
- At height h : $E_h = mgh + \frac{1}{2}mv_h^2$, with $v_h^2 = 2g(H - h)$, so $E_h = mgh + mg(H - h) = mgH$
- At the ground: $E_0 = 0 + \frac{1}{2}mv_f^2$, with $v_f^2 = 2gH$, so $E_0 = mgH$

$$E_H = E_h = E_0 = mgH$$

The energy is purely potential at H , partly kinetic at h , and fully kinetic at ground level — the total remains constant.

Key Examination Points for Full Marks:

- With non-conservative forces: $E_f - E_i = W_{nc}$ (work done by the non-conservative forces), which depends on the path.

Q.9 A bob is given a horizontal velocity at the lowest point so that it just completes a vertical circle. Find the speeds at the top and at the horizontal position. (3 Marks)

A bob of mass m hangs on a light string of length L . It is given a horizontal velocity v_0 at the lowest point A so that the string just becomes slack at the top C (Fig. 5.6).

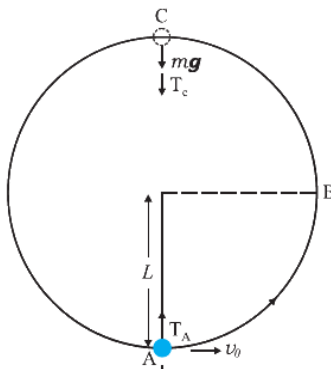


Fig. 5.6: A bob moving in a vertical circle (Example 5.7)

Tension does no work (it is always normal to the displacement), so mechanical energy is conserved. Take PE = 0 at A.

At A: $E = \frac{1}{2}mv_0^2$; also $T_A - mg = \frac{mv_0^2}{L}$.

At C (top): The string just slackens, so $T_C = 0$:

$$mg = \frac{mv_C^2}{L} \Rightarrow v_C = \sqrt{gL}; \quad E = \frac{1}{2}mv_C^2 + 2mgL = \frac{5}{2}mgL$$

(i) Speed at A: $\frac{5}{2}mgL = \frac{1}{2}mv_0^2 \Rightarrow v_0 = \sqrt{5gL}$

(ii) Speed at B (string horizontal, height L):

$$\frac{1}{2}mv_B^2 + mgL = \frac{5}{2}mgL \Rightarrow v_B = \sqrt{3gL}$$

(iii) $\frac{K_B}{K_C} = \frac{3gL}{gL} = \frac{3}{1}$.

Key Examination Points for Full Marks:

- If the string is cut at C, the bob moves as a projectile with horizontal velocity $\sqrt{gL}$.
- Minimum speed at the lowest point to complete the circle: $\sqrt{5gL}$. Tension at the lowest point then is $6mg$;
 $T_{max} - T_{min} = 6mg$.

Q.10 Derive the potential energy of a spring. Show that the spring force is conservative. (3 Marks)

Hooke's law: For an ideal spring, the restoring force is proportional to the displacement x from the equilibrium position:

$$F_s = -kx$$

k is the **spring constant** (N m^{-1}); a stiff spring has a large k .

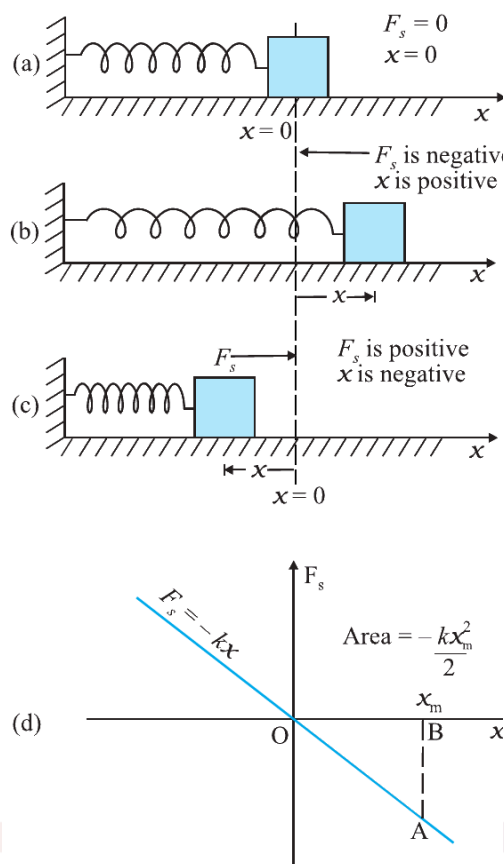


Fig. 5.7: (a) Spring force is zero at equilibrium. (b) Stretched spring: $x > 0$, $F_s < 0$. (c) Compressed spring: $x < 0$, $F_s > 0$. (d) Plot of F_s against x ; the shaded triangle gives the work done by the spring force

Work done by the spring force when the block is pulled out to x_m :

$$W_s = \int_0^{x_m} F_s dx = - \int_0^{x_m} kx dx = -\frac{kx_m^2}{2}$$

(The external pulling force does $+\frac{kx_m^2}{2}$.) The same result holds for compression.

From x_i to x_f : $W_s = \frac{kx_i^2}{2} - \frac{kx_f^2}{2}$ — it depends **only on the end points**. In a cyclic process ($x_f = x_i$), $W_s = 0$.

Hence **the spring force is conservative**.

Potential energy: Taking $V = 0$ at $x = 0$:

$$V(x) = \frac{1}{2}kx^2$$

(Check: $-\frac{dV}{dx} = -kx = \text{spring force}$.)

Energy of oscillation: If the block is released from x_m , then at any x : $\frac{1}{2}kx_m^2 = \frac{1}{2}kx^2 + \frac{1}{2}mv^2$. The speed is maximum at $x = 0$:

$$v_m = \sqrt{\frac{k}{m}} x_m$$

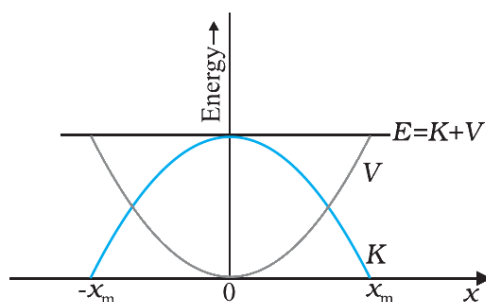


Fig. 5.8: Parabolic plots of PE (V) and KE (K) of a block attached to a spring. Total $E = K + V$ remains constant

Key Examination Points for Full Marks:

- **Example 5.8:** A 1000 kg car at 18 km/h (5 m/s) hits a spring of $k = 5.25 \times 10^3 \text{ N m}^{-1}$. $\text{KE} = 1.25 \times 10^4 \text{ J}$
 $= \frac{1}{2} k x_m^2 \Rightarrow x_m = 2.18 \text{ m}$. (The textbook prints 2.00 m, which corresponds to $k = 6.25 \times 10^3 \text{ N m}^{-1}$.)
- **36 km/h = 10 m/s** is a useful conversion.

Q.11 How is the conservation of energy modified when friction acts? (2 Marks) Explain with an example.

When both a conservative force F_c and a non-conservative force F_{nc} act, the WE theorem gives

$$(F_c + F_{nc}) \Delta x = \Delta K. \text{ Since } F_c \Delta x = -\Delta V:$$

$$\Delta(K + V) = F_{nc} \Delta x \Rightarrow E_f - E_i = W_{nc}$$

W_{nc} is the total work done by the non-conservative forces, and it depends on the path.

Example 5.9 (car and spring with friction, $\mu = 0.5$): Both the spring force and friction oppose the compression.

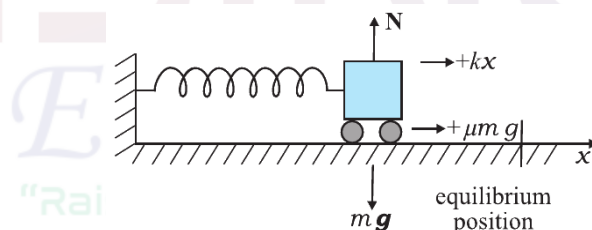


Fig. 5.9: The forces acting on the car

$$\frac{1}{2} m v^2 = \frac{1}{2} k x_m^2 + \mu m g x_m \Rightarrow k x_m^2 + 2 \mu m g x_m - m v^2 = 0$$

With $\mu m g = 5 \times 10^3 \text{ N}$: $x_m = \frac{-\mu m g + \sqrt{\mu^2 m^2 g^2 + m k v^2}}{k} = 1.43 \text{ m}$ — less than without friction, as expected.
 (The textbook prints 1.35 m.)

Key Examination Points for Full Marks:

- Information about **time** is absent from energy methods; Newton's second law is needed for that.

Q.12 Define average and instantaneous power. Show that $P = \mathbf{F} \cdot \mathbf{v}$. What is a kilowatt hour? (2 Marks)

Power is the time rate at which work is done or energy is transferred.

Average power: $P_{av} = \frac{W}{t}$

Instantaneous power: $P = \frac{dW}{dt}$

Since $dW = \vec{F} \cdot d\vec{r}$:

$$P = \vec{F} \cdot \frac{d\vec{r}}{dt} = \vec{F} \cdot \vec{v}$$

Unit: watt (W); $1 \text{ W} = 1 \text{ J s}^{-1}$. **Dimensions:** $[M L^2 T^{-3}]$. Power is a **scalar**.

Horse-power: $1 \text{ hp} = 746 \text{ W}$.

Kilowatt hour (kWh) is a unit of **energy**, not power:

$$1 \text{ kWh} = 10^3 \text{ W} \times 3600 \text{ s} = 3.6 \times 10^6 \text{ J}$$

A 100 W bulb on for 10 hours uses 1 kWh.

Key Examination Points for Full Marks:

- Example 5.10:** An elevator (1800 kg) moves up at 2 m/s against friction 4000 N: $F = mg + f = 22000 \text{ N}$;
 $P = Fv = 44000 \text{ W} = 59 \text{ hp}$.

Q.13 What is a collision? Distinguish between elastic and inelastic collisions. (2 Marks)

A **collision** is an event in which two bodies interact strongly for a short time, exchanging momentum and energy.

In every collision, total linear momentum is conserved. During contact, $\Delta \vec{p}_1 = \vec{F}_{12} \Delta t$ and $\Delta \vec{p}_2 = \vec{F}_{21} \Delta t$. By Newton's third law $\vec{F}_{12} = -\vec{F}_{21}$, so $\Delta \vec{p}_1 + \Delta \vec{p}_2 = 0$.

Elastic collision	Inelastic collision
Total momentum is conserved	Total momentum is conserved
Total kinetic energy is conserved	Kinetic energy is not conserved (part changes to heat, sound, deformation)
Deformation is fully relieved	Deformation is not (fully) relieved
Example: collisions of atoms, ideal billiard balls	Example: a bullet embedding in a block

A collision in which the bodies **move together** afterwards is a **completely (perfectly) inelastic collision**.

Key Examination Points for Full Marks:

- Even in an elastic collision, KE is **not** conserved during the short contact time; KE conservation applies to the states before and after.
- A one-dimensional collision is also called a **head-on** collision.

Q.14 Find the loss of kinetic energy in a completely inelastic collision in one dimension. (3 Marks)

A mass m_1 moving with v_{1i} hits m_2 at rest; they stick together and move with v_f .

Momentum conservation:

$$m_1 v_{1i} = (m_1 + m_2) v_f \Rightarrow v_f = \frac{m_1}{m_1 + m_2} v_{1i}$$

Loss in KE:

$$\begin{aligned} \Delta K &= \frac{1}{2} m_1 v_{1i}^2 - \frac{1}{2} (m_1 + m_2) v_f^2 = \frac{1}{2} m_1 v_{1i}^2 - \frac{1}{2} \frac{m_1^2}{m_1 + m_2} v_{1i}^2 \\ &= \frac{1}{2} m_1 v_{1i}^2 \left[1 - \frac{m_1}{m_1 + m_2} \right] = \frac{1}{2} \frac{m_1 m_2}{m_1 + m_2} v_{1i}^2 \end{aligned}$$

which is always positive.

Key Examination Points for Full Marks:

- If both bodies move initially: $\Delta K = \frac{1}{2} \frac{m_1 m_2}{m_1 + m_2} (u_1 - u_2)^2$.
- Fraction of KE lost when m_2 is at rest: $\frac{m_2}{m_1 + m_2}$.

Q.15 Derive the final velocities in a one-dimensional elastic collision. (3 Marks)
Discuss the special cases.

Mass m_1 with velocity v_{1i} collides elastically with m_2 at rest.

Momentum: $m_1 v_{1i} = m_1 v_{1f} + m_2 v_{2f} \dots (1)$

Kinetic energy: $m_1 v_{1i}^2 = m_1 v_{1f}^2 + m_2 v_{2f}^2 \dots (2)$

From (1): $m_1 (v_{1i} - v_{1f}) = m_2 v_{2f}$. From (2): $m_1 (v_{1i} - v_{1f}) (v_{1i} + v_{1f}) = m_2 v_{2f}^2$. Dividing:

$$v_{2f} = v_{1i} + v_{1f}$$

(The relative speed of separation equals the relative speed of approach.) Substituting in (1):

$$v_{1f} = \frac{(m_1 - m_2)}{m_1 + m_2} v_{1i}, \quad v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i}$$

Special cases:

- **Case I — equal masses** ($m_1 = m_2$): $v_{1f} = 0, v_{2f} = v_{1i}$. The first mass stops and the second moves off with the initial speed — **velocities are exchanged**.
- **Case II — very heavy target** ($m_2 \gg m_1$): $v_{1f} \approx -v_{1i}, v_{2f} \approx 0$. The light body rebounds with the same speed; the heavy body stays at rest.

Key Examination Points for Full Marks:

- **Example 5.11 (moderator):** Fraction of KE lost by a neutron: $f_1 = \left(\frac{m_1 - m_2}{m_1 + m_2} \right)^2$ is retained; transferred $f_2 = \frac{4m_1 m_2}{(m_1 + m_2)^2}$. Deuterium ($m_2 = 2m_1$): $f_1 = 1/9, f_2 = 8/9$ ($\approx 90\%$ transferred). Carbon: $f_1 = 71.6\%, f_2 = 28.4\%$. Heavy water and graphite are used as **moderators**.

Q.16 Discuss an elastic collision in two dimensions. Show that two equal masses move at right angles after a glancing elastic collision. (3 Marks)

Mass m_1 with velocity v_{1i} hits m_2 at rest. After the collision they move at angles θ_1 and θ_2 to the original direction, in a plane (Fig. 5.10).

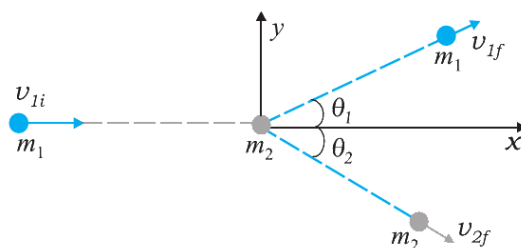


Fig. 5.10: Collision of mass m_1 with a stationary mass m_2

Momentum conservation:

- **x:** $m_1 v_{1i} = m_1 v_{1f} \cos \theta_1 + m_2 v_{2f} \cos \theta_2$
- **y:** $0 = m_1 v_{1f} \sin \theta_1 - m_2 v_{2f} \sin \theta_2$

For an elastic collision, in addition: $\frac{1}{2} m_1 v_{1i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$.

There are four unknowns ($v_{1f}, v_{2f}, \theta_1, \theta_2$) and three equations, so **one quantity (say θ_1) must be measured.**

Equal masses (Example 5.12): Momentum: $\vec{v}_{1i} = \vec{v}_{1f} + \vec{v}_{2f}$. Squaring:

$$v_{1i}^2 = v_{1f}^2 + v_{2f}^2 + 2\vec{v}_{1f} \cdot \vec{v}_{2f}$$

Energy: $v_{1i}^2 = v_{1f}^2 + v_{2f}^2$. Comparing: $\vec{v}_{1f} \cdot \vec{v}_{2f} = 0$, i.e. $\cos(\theta_1 + \theta_2) = 0$:

$$\theta_1 + \theta_2 = 90^\circ$$

For $\theta_2 = 37^\circ, \theta_1 = 53^\circ$.

Key Examination Points for Full Marks:

- When two equal masses undergo a glancing elastic collision with one at rest, they move at right angles to each other after the collision.
- **Scattering:** interaction at a distance (e.g. α -particle and nucleus) without physical contact is also a collision.

Formula Sheet — Work, Energy and Power

Quantity	Formula	SI Unit
Scalar product	$\vec{A} \cdot \vec{B} = AB \cos \theta = A_x B_x + A_y B_y + A_z B_z$	—
Work (constant force)	$W = \vec{F} \cdot \vec{d} = Fd \cos \theta$	J
Work (variable force)	$W = \int_{x_i}^{x_f} F(x) dx = \text{area under F-x graph}$	J
Kinetic energy	$K = \frac{1}{2}mv^2 = \frac{p^2}{2m}$	J
Work-energy theorem	$W_{net} = K_f - K_i$	J
Gravitational PE	$V = mgh$	J
Force from PE	$F = -\frac{dV}{dx}$	N
Spring force	$F_s = -kx$	N
Spring PE	$V = \frac{1}{2}kx^2$	J
Max speed (spring)	$v_m = \sqrt{\frac{k}{m}} x_m$	m s^{-1}
Mechanical energy	$E = K + V = \text{constant (conservative forces)}$	J
With friction	$E_f - E_i = W_{nc}$	J
Power	$P_{av} = \frac{W}{t}; P = \frac{dW}{dt} = \vec{F} \cdot \vec{v}$	W
Perfectly inelastic	$v_f = \frac{m_1 v_{1i}}{m_1 + m_2}; \Delta K = \frac{m_1 m_2}{2(m_1 + m_2)} v_{1i}^2$	—
Elastic (1-D, m_2 at rest)	$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i}; v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i}$	m s^{-1}
Coefficient of restitution	$e = \frac{v_{2f} - v_{1f}}{v_{1i} - v_{2i}}$ (e = 1 elastic, 0 perfectly inelastic)	—
Vertical circle	$v_{bottom} = \sqrt{5gL}, v_{side} = \sqrt{3gL}, v_{top} = \sqrt{gL} \text{ (minimum)}$	m s^{-1}

Units: 1 kWh = 3.6×10^6 J; 1 eV = 1.6×10^{-19} J; 1 cal = 4.186 J; 1 erg = 10^{-7} J; 1 hp = 746 W.

Dimensions: work, energy $[ML^2 T^{-2}]$; power $[ML^2 T^{-3}]$; spring constant $[M T^{-2}]$.

NEET / JEE Quick Points and Tricks

- K and p:** $K = \frac{p^2}{2m}$. For the same momentum, the lighter body has more KE; for the same KE, the heavier body has more momentum ($p = \sqrt{2mK}$).
- Percentage changes:** if p increases by a small $x\%$, K increases by about $2x\%$. If p doubles, K becomes 4 times.
- Bullet penetrating planks:** with constant resistance, distance $\propto$ loss of KE. If v halves in distance d , it stops after a further $d/3$.
- Constant power from rest:** $v \propto t^{1/2}$, $s \propto t^{3/2}$, $F \propto t^{-1/2}$. **Constant force from rest:** $P \propto t$.
- Pump/lift power:** $P = (mg + f)v$; for a pump lifting water, $P = \frac{mgh}{t}$ (divide by efficiency for input power).
- Spring cut into n equal parts:** each piece has spring constant nk . Same force: $U = \frac{F^2}{2k}$ (softer spring stores more); same extension: $U = \frac{1}{2}kx^2$ (stiffer spring stores more).
- Vertical circle (string):** minimum speed at bottom $\sqrt{5gR}$; $T_{\text{bottom}} - T_{\text{top}} = 6mg$; at the horizontal position, $T = \frac{mv^2}{R}$. For a **rod** (or a bead on a ring), the minimum speed at the bottom is $\sqrt{4gR}$.
- Loop the loop from height h** (smooth track): $h_{\text{min}} = \frac{5}{2}R$.
- Elastic, equal masses (1-D):** velocities are exchanged. **Glancing (2-D) with one at rest:** they move at 90° to each other.
- Energy transfer** to a target at rest in a head-on elastic collision: $\frac{4m_1m_2}{(m_1+m_2)^2}$; it is 100% when $m_1 = m_2$.
- Coefficient of restitution:** a ball dropped from h rebounds to e^2h ; after n bounces, $h_n = e^{2n}h$. Total distance $= h \frac{1+e^2}{1-e^2}$.
- Perfectly inelastic (2-D):** find the common velocity by vector addition of momenta, then
$$\Delta K = K_i - \frac{p^2}{2(m_1+m_2)}.$$
- Ballistic pendulum:** $v = \frac{m+M}{m} \sqrt{2gh}$.
- Work by gravity, spring force or any conservative force in a closed path is zero.** Friction always does negative work on the sliding body (relative to the surface).