



## Chapter 6 : Systems of Particles and Rotational Motion (Solved Examples & Exercises)

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### Notes

Step-by-step solutions · NCERT Examples 6.1–6.12 and Exercises 6.1–6.17 · Class 11 Physics

### Part A — NCERT Solved Examples

#### Example 6.1

**Question:** Find the centre of mass of three particles at the vertices of an equilateral triangle. The masses of the particles are 100 g, 150 g, and 200 g respectively. Each side of the equilateral triangle is 0.5 m long.

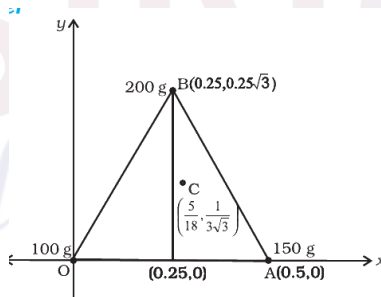


Fig. 6.9

**Solution:** With the axes as shown, the coordinates of O, A and B are (0, 0), (0.5, 0) and  $(0.25, 0.25\sqrt{3})$ . The masses 100 g, 150 g and 200 g are at O, A and B.

$$X = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3} = \frac{100(0) + 150(0.5) + 200(0.25)}{450} \text{ m} = \frac{75 + 50}{450} = \frac{125}{450} = \frac{5}{18} \text{ m}$$

$$Y = \frac{100(0) + 150(0) + 200(0.25\sqrt{3})}{450} \text{ m} = \frac{50\sqrt{3}}{450} = \frac{\sqrt{3}}{9} = \frac{1}{3\sqrt{3}} \text{ m}$$

The centre of mass C is shown in the figure. It is **not the geometric centre** of triangle OAB because the masses are unequal.

**Answer:**  $\left(\frac{5}{18} \text{ m}, \frac{1}{3\sqrt{3}} \text{ m}\right)$

#### Example 6.2

**Question:** Find the centre of mass of a triangular lamina.

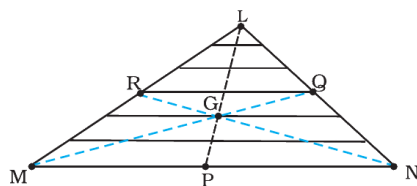


Fig. 6.10

Fig. 6.10

**Solution:** The lamina ( $\triangle LMN$ ) may be subdivided into narrow strips, each parallel to the base  $MN$ . By symmetry, each strip has its centre of mass at its midpoint. Joining the midpoints of all strips gives the median  $LP$ . The centre of mass of the triangle as a whole therefore lies on the median  $LP$ . Similarly, it lies on the medians  $MQ$  and  $NR$ .

**Answer:** The centre of mass lies at the point of concurrence of the medians, i.e. the **centroid G** of the triangle.

### Example 6.3

**Question:** Find the centre of mass of a uniform L-shaped lamina (a thin flat plate) with dimensions as shown. The mass of the lamina is 3 kg.

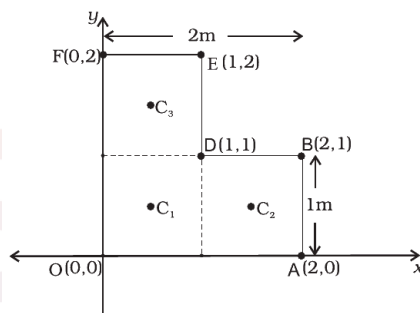


Fig. 6.11

Fig. 6.11

**Solution:** The L-shape consists of 3 squares, each of side 1 m and mass 1 kg (uniform lamina). By symmetry, their centres  $C_1$ ,  $C_2$ ,  $C_3$  are at  $(1/2, 1/2)$ ,  $(3/2, 1/2)$  and  $(1/2, 3/2)$ . Treating each square as a point mass at its centre:

$$X = \frac{1(1/2) + 1(3/2) + 1(1/2)}{1 + 1 + 1} m = \frac{5}{6} m$$

$$Y = \frac{1(1/2) + 1(1/2) + 1(3/2)}{1 + 1 + 1} m = \frac{5}{6} m$$

The CM lies on the line  $OD$  — the line of symmetry of the L-shape, so we could have guessed this without calculation.

**Answer:**  $\left(\frac{5}{6} m, \frac{5}{6} m\right)$

### Example 6.4

**Question:** Find the scalar and vector products of the two vectors  $\vec{a} = (3\hat{i} - 4\hat{j} + 5\hat{k})$  and  $\vec{b} = (-2\hat{i} + \hat{j} - 3\hat{k})$ .

**Solution:**

$$\vec{a} \cdot \vec{b} = (3)(-2) + (-4)(1) + (5)(-3) = -6 - 4 - 15 = -25$$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -4 & 5 \\ -2 & 1 & -3 \end{vmatrix} = (12 - 5)\hat{i} - (-9 + 10)\hat{j} + (3 - 8)\hat{k} = 7\hat{i} - \hat{j} - 5\hat{k}$$

Note:  $\vec{b} \times \vec{a} = -7\hat{i} + \hat{j} + 5\hat{k}$ .

**Answer:**  $\vec{a} \cdot \vec{b} = -25$ ;  $\vec{a} \times \vec{b} = 7\hat{i} - \hat{j} - 5\hat{k}$

### Example 6.5

**Question:** Find the torque of a force  $7\hat{i} + 3\hat{j} - 5\hat{k}$  about the origin. The force acts on a particle whose position vector is  $\hat{i} - \hat{j} + \hat{k}$ .

**Solution:**  $\vec{r} = \hat{i} - \hat{j} + \hat{k}$  and  $\vec{F} = 7\hat{i} + 3\hat{j} - 5\hat{k}$ .

$$\vec{\tau} = \vec{r} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ 7 & 3 & -5 \end{vmatrix} = (5 - 3)\hat{i} - (-5 - 7)\hat{j} + (3 + 7)\hat{k}$$

**Answer:**  $\vec{\tau} = 2\hat{i} + 12\hat{j} + 10\hat{k}$

### Example 6.6

**Question:** Show that the angular momentum about any point of a single particle moving with constant velocity remains constant throughout the motion.

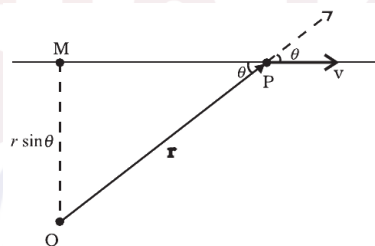


Fig. 6.19

Fig. 6.19

**Solution:** Let the particle with velocity  $\vec{v}$  be at point P at some instant t. Its angular momentum about an arbitrary point O is

$$\vec{l} = \vec{r} \times m\vec{v}, \quad l = mvr \sin \theta$$

where  $\theta$  is the angle between  $\vec{r}$  and  $\vec{v}$ . Although the particle changes position with time, its line of motion remains the same, so the perpendicular distance  $OM = r \sin \theta$  is constant. Hence  $l = mv (OM)$  is constant in magnitude.

The direction of  $\vec{l}$  is perpendicular to the plane of  $\vec{r}$  and  $\vec{v}$  (into the page), which also does not change with time.

Thus  $\vec{l}$  is constant in magnitude and direction — it is **conserved**. There is no external torque on the particle.

### Example 6.7

**Question:** Show that the moment of a couple does not depend on the point about which you take the moments.

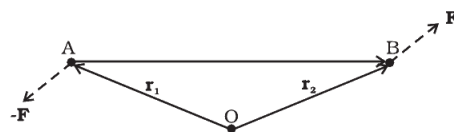


Fig. 6.22

Fig. 6.22

**Solution:** Consider a couple acting on a rigid body: forces  $\vec{F}$  and  $-\vec{F}$  act at points B and A, with position vectors  $\vec{r}_2$  and  $\vec{r}_1$  about the origin O. The moment of the couple is the sum of the moments of the two forces:

$$\vec{r}_1 \times (-\vec{F}) + \vec{r}_2 \times \vec{F} = \vec{r}_2 \times \vec{F} - \vec{r}_1 \times \vec{F} = (\vec{r}_2 - \vec{r}_1) \times \vec{F}$$

Since  $\vec{r}_1 + \vec{AB} = \vec{r}_2$ , we have  $\vec{AB} = \vec{r}_2 - \vec{r}_1$ . The moment of the couple is therefore

$$\vec{AB} \times \vec{F}$$

which is **independent of the origin O** about which the moments are taken.

### Example 6.8

**Question:** A metal bar 70 cm long and 4.00 kg in mass is supported on two knife-edges placed 10 cm from each end. A 6.00 kg load is suspended at 30 cm from one end. Find the reactions at the knife-edges. (Assume the bar to be of uniform cross section and homogeneous.)

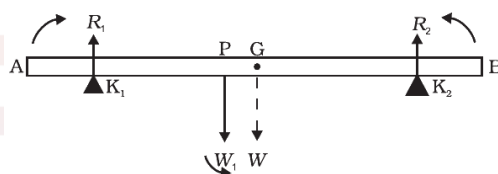


Fig. 6.26

Fig. 6.26

**Solution:** The bar is uniform, so its weight  $W$  acts at its centre  $G$ .  $AB = 70$  cm,  $AG = 35$  cm,  $AP = 30$  cm,  $PG = 5$  cm,  $AK_1 = BK_2 = 10$  cm and  $K_1G = K_2G = 25$  cm.  $W = 4.00g$  N and  $W_1 = 6.00g$  N;  $R_1$  and  $R_2$  are the reactions at the knife-edges.

**Translational equilibrium:**

$$R_1 + R_2 - W_1 - W = 0 \Rightarrow R_1 + R_2 = 10.00g = 98.00 \text{ N} \quad (\text{i})$$

**Rotational equilibrium** (moments about  $G$ ; anticlockwise positive):

$$-R_1 (K_1G) + W_1 (PG) + R_2 (K_2G) = 0 \Rightarrow -0.25R_1 + 0.05W_1 + 0.25R_2 = 0$$

$$R_1 - R_2 = 1.2g = 11.76 \text{ N} \quad (\text{ii})$$

From (i) and (ii):  $R_1 = 54.88$  N,  $R_2 = 43.12$  N.

**Answer:** About 55 N at  $K_1$  and 43 N at  $K_2$ .

### Example 6.9

**Question:** A 3 m long ladder weighing 20 kg leans on a frictionless wall. Its feet rest on the floor 1 m from the wall as shown in Fig. 6.27. Find the reaction forces of the wall and the floor.

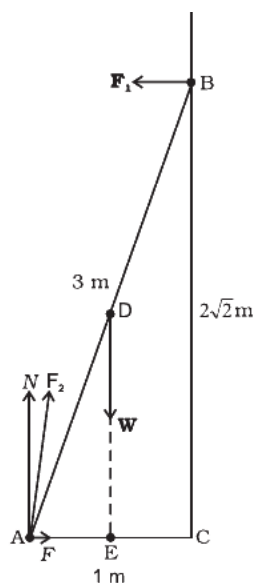


Fig. 6.27

Fig. 6.27

**Solution:** The ladder AB is 3 m long and its foot A is AC = 1 m from the wall. By Pythagoras, BC =  $2\sqrt{2}$  m. The forces are: weight W at the centre of gravity D; the wall's reaction  $F_1$  (perpendicular to the frictionless wall); and the floor's reaction  $F_2$ , with components N (normal) and F (friction, towards the wall).

Vertical forces:  $N - W = 0 \dots (i)$

Horizontal forces:  $F - F_1 = 0 \dots (ii)$

Moments about A:  $2\sqrt{2} F_1 - \frac{1}{2}W = 0 \dots (iii)$

With  $W = 20 \times 9.8 = 196.0$  N:

- From (i):  $N = 196.0$  N
- From (iii):  $F_1 = \frac{W}{4\sqrt{2}} = \frac{196.0}{4\sqrt{2}} = 34.6$  N
- From (ii):  $F = 34.6$  N

$$F_2 = \sqrt{F^2 + N^2} = 199.0 \text{ N}, \quad \tan \alpha = \frac{N}{F} = 4\sqrt{2} \Rightarrow \alpha \approx 80^\circ$$

**Answer:** Wall reaction 34.6 N (horizontal); floor reaction 199 N at about  $80^\circ$  to the horizontal.

### Example 6.10

**Question:** Obtain Eq. (6.36),  $\omega = \omega_0 + \alpha t$ , from first principles.

**Solution:** The angular acceleration is uniform:

$$\frac{d\omega}{dt} = \alpha = \text{constant}$$

Integrating:  $\omega = \int \alpha dt + c = \alpha t + c$  (as  $\alpha$  is constant).

At  $t = 0$ ,  $\omega = \omega_0$  (given), so  $c = \omega_0$ . Thus

$$\omega = \omega_0 + \alpha t$$

With  $\omega = d\theta/dt$ , integrating again gives  $\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$ .

### Example 6.11

**Question:** The angular speed of a motor wheel is increased from 1200 rpm to 3120 rpm in 16 seconds. (i) What is its angular acceleration, assuming the acceleration to be uniform? (ii) How many revolutions does the engine

make during this time?

**Solution:**

(i) Using  $\omega = \omega_0 + \alpha t$ :

$$\omega_0 = \frac{2\pi \times 1200}{60} = 40\pi \text{ rad/s}, \quad \omega = \frac{2\pi \times 3120}{60} = 104\pi \text{ rad/s}$$

$$\alpha = \frac{\omega - \omega_0}{t} = \frac{64\pi}{16} = 4\pi \text{ rad/s}^2$$

(ii)

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2 = 40\pi \times 16 + \frac{1}{2} \times 4\pi \times 16^2 = 640\pi + 512\pi = 1152\pi \text{ rad}$$

$$\text{Number of revolutions} = \frac{1152\pi}{2\pi} = 576$$

**Answer:** (i)  $4\pi \text{ rad/s}^2$  (ii) 576 revolutions

### Example 6.12

**Question:** A cord of negligible mass is wound round the rim of a flywheel of mass 20 kg and radius 20 cm. A steady pull of 25 N is applied on the cord as shown in Fig. 6.31. The flywheel is mounted on a horizontal axle with frictionless bearings. (a) Compute the angular acceleration of the wheel. (b) Find the work done by the pull, when 2 m of the cord is unwound. (c) Find also the kinetic energy of the wheel at this point. Assume that the wheel starts from rest. (d) Compare answers to parts (b) and (c).

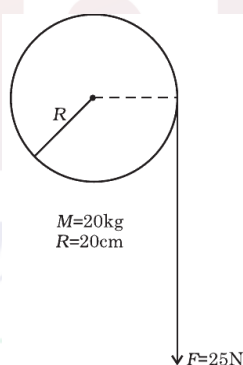


**Fig. 6.31**

**Solution:**

(a)  $\tau = FR = 25 \times 0.20 = 5.0 \text{ N m}$ . Moment of inertia of the flywheel (a disc):

$$I = \frac{MR^2}{2} = \frac{20.0 \times (0.2)^2}{2} = 0.4 \text{ kg m}^2$$

$$\alpha = \frac{\tau}{I} = \frac{5.0}{0.4} = 12.5 \text{ s}^{-2}$$

(b) Work done by the pull:  $W = 25 \text{ N} \times 2 \text{ m} = 50 \text{ J}$ .

(c) Angular displacement  $\theta = \text{length unwound} / \text{radius} = 2/0.2 = 10 \text{ rad}$ . Starting from rest ( $\omega_0 = 0$ ):

$$\omega^2 = 2\alpha\theta = 2 \times 12.5 \times 10.0 = 250 \text{ (rad/s)}^2$$

$$K = \frac{1}{2}I\omega^2 = \frac{1}{2} \times 0.4 \times 250 = 50 \text{ J}$$

(d) The answers are the same: the kinetic energy gained by the wheel equals the work done by the force. There is no loss of energy due to friction.

## Part B — NCERT Exercises

### Exercise 6.1

**Question:** Give the location of the centre of mass of a (i) sphere, (ii) cylinder, (iii) ring, and (iv) cube, each of uniform mass density. Does the centre of mass of a body necessarily lie inside the body?

**Answer:** For uniform bodies, the centre of mass coincides with the **geometric centre**:

- i. Sphere — at its centre.
- ii. Cylinder — at the midpoint of its axis.
- iii. Ring — at the centre of the ring.
- iv. Cube — at the point where its body diagonals intersect.

**No**, the centre of mass need not lie inside the body. For a ring, a hollow sphere or a hollow cylinder, the CM is at a point where there is no material.

### Exercise 6.2

**Question:** In the HCl molecule, the separation between the nuclei of the two atoms is about  $1.27 \text{ \AA}$  ( $1 \text{ \AA} = 10^{-10} \text{ m}$ ). Find the approximate location of the CM of the molecule, given that a chlorine atom is about 35.5 times as massive as a hydrogen atom and nearly all the mass of an atom is concentrated in its nucleus.

**Solution:** Take the H nucleus at the origin and the Cl nucleus at  $x = 1.27 \text{ \AA}$ . Let the mass of H be  $m$ , so that of Cl is  $35.5m$ .

$$X = \frac{m(0) + 35.5m(1.27)}{m + 35.5m} = \frac{35.5 \times 1.27}{36.5} = 1.235 \text{ \AA}$$

**Answer:** The CM lies on the line joining the nuclei, about  **$1.24 \text{ \AA}$  from the H atom** (about  $0.035 \text{ \AA}$  from the Cl atom).

### Exercise 6.3

**Question:** A child sits stationary at one end of a long trolley moving uniformly with a speed  $V$  on a smooth horizontal floor. If the child gets up and runs about on the trolley in any manner, what is the speed of the CM of the (trolley + child) system?

**Answer:** The forces between the child and the trolley are **internal** to the system. The floor is smooth, so there is no external horizontal force. The velocity of the CM therefore does not change: it remains  **$V$** .

### Exercise 6.4

**Question:** Show that the area of the triangle contained between the vectors  $\vec{a}$  and  $\vec{b}$  is one half of the magnitude of  $\vec{a} \times \vec{b}$ .

**Solution:** Let  $\vec{a}$  and  $\vec{b}$  be represented by OP and OQ, with angle  $\theta$  between them. The height of the triangle OPQ, with OP as base, is  $b \sin \theta$ .

$$\text{Area} = \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} a (b \sin \theta) = \frac{1}{2} |\vec{a} \times \vec{b}|$$

### Exercise 6.5

**Question:** Show that  $\vec{a} \cdot (\vec{b} \times \vec{c})$  is equal in magnitude to the volume of the parallelepiped formed on the three vectors  $\vec{a}$ ,  $\vec{b}$  and  $\vec{c}$ .

**Solution:**  $\vec{b} \times \vec{c}$  is a vector of magnitude  $bc \sin \theta$  — the **area of the base** (the parallelogram formed by  $\vec{b}$  and  $\vec{c}$ ) — directed along the normal  $\hat{n}$  to the base.

$$\vec{a} \cdot (\vec{b} \times \vec{c}) = (bc \sin \theta) (a \cos \phi)$$

where  $\phi$  is the angle between  $\vec{a}$  and  $\hat{n}$ . Here  $a \cos \phi$  is the **height** of the parallelepiped. Hence

$$\left| \vec{a} \cdot (\vec{b} \times \vec{c}) \right| = \text{base area} \times \text{height} = \text{volume of the parallelepiped}$$

### Exercise 6.6

**Question:** Find the components along the x, y, z axes of the angular momentum  $\vec{l}$  of a particle, whose position vector is  $\vec{r}$  with components x, y, z and momentum is  $\vec{p}$  with components  $p_x$ ,  $p_y$  and  $p_z$ . Show that if the particle moves only in the x-y plane the angular momentum has only a z-component.

**Solution:**

$$\vec{l} = \vec{r} \times \vec{p} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & y & z \\ p_x & p_y & p_z \end{vmatrix}$$

$$l_x = yp_z - zp_y, \quad l_y = zp_x - xp_z, \quad l_z = xp_y - yp_x$$

If the particle moves only in the x-y plane, then  $z = 0$  and  $p_z = 0$ . Hence  $l_x = 0$  and  $l_y = 0$ , and only  $l_z = xp_y - yp_x$  remains. The angular momentum has **only a z-component** (perpendicular to the plane of motion).

### Exercise 6.7

**Question:** Two particles, each of mass  $m$  and speed  $v$ , travel in opposite directions along parallel lines separated by a distance  $d$ . Show that the angular momentum vector of the two particle system is the same whatever be the point about which the angular momentum is taken.

**Solution:** Take any point O. Let its perpendicular distances from the two lines be  $r_1$  and  $r_2$ ; there are two cases.

- **O between the lines:**  $r_1 + r_2 = d$ . Both particles turn the same way about O, so their angular momenta add:  
 $L = mvr_1 + mvr_2 = mvd$ .
- **O outside the lines:**  $|r_1 - r_2| = d$ . The particles now turn opposite ways about O:  
 $L = mvr_1 - mvr_2 = mvd$ .

In both cases, the direction of  $\vec{L}$  is perpendicular to the plane of motion, in the same sense.

**Result:**  $\left| \vec{L} \right| = mvd$  in a fixed direction, for **any point** O. (This is because the total linear momentum of the system is zero.)

### Exercise 6.8

**Question:** A non-uniform bar of weight  $W$  is suspended at rest by two strings of negligible weight as shown in Fig. 6.33. The angles made by the strings with the vertical are  $36.9^\circ$  and  $53.1^\circ$  respectively. The bar is 2 m long. Calculate the distance  $d$  of the centre of gravity of the bar from its left end.

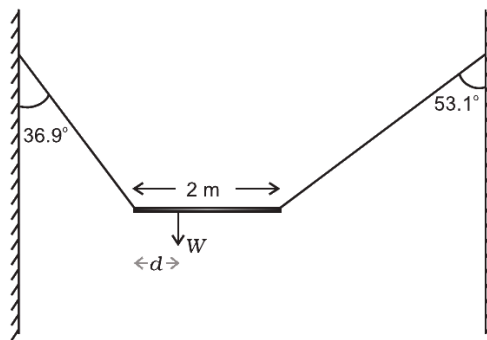


Fig. 6.33

Fig. 6.33

**Solution:** Let  $T_1$  (left) and  $T_2$  (right) be the tensions. Use  $\sin 36.9^\circ = 0.6$ ,  $\cos 36.9^\circ = 0.8$ ,  $\sin 53.1^\circ = 0.8$ ,  $\cos 53.1^\circ = 0.6$ .

Horizontal equilibrium:

$$T_1 \sin 36.9^\circ = T_2 \sin 53.1^\circ \Rightarrow 0.6T_1 = 0.8T_2 \Rightarrow T_1 = \frac{4}{3}T_2$$

Vertical equilibrium:

$$T_1 \cos 36.9^\circ + T_2 \cos 53.1^\circ = W \Rightarrow \frac{4}{3}T_2 (0.8) + 0.6T_2 = W \Rightarrow T_2 = 0.6W$$

Moments about the left end:

$$Wd = (T_2 \cos 53.1^\circ) \times 2 = 0.6W \times 0.6 \times 2 \Rightarrow d = 0.72 \text{ m}$$

**Answer:**  $d = 0.72 \text{ m}$  from the left end

### Exercise 6.9

**Question:** A car weighs 1800 kg. The distance between its front and back axles is 1.8 m. Its centre of gravity is 1.05 m behind the front axle. Determine the force exerted by the level ground on each front wheel and each back wheel.

**Solution:**  $W = 1800 \times 9.8 = 17640 \text{ N}$ . Let  $R_f$  and  $R_b$  be the total reactions on the two front and two back wheels. The CG is 1.05 m behind the front axle and 0.75 m in front of the back axle.

Vertical equilibrium:  $R_f + R_b = 17640 \text{ N}$

Moments about the CG:  $R_f (1.05) = R_b (0.75)$

$$R_f = \frac{17640 \times 0.75}{1.8} = 7350 \text{ N}, \quad R_b = \frac{17640 \times 1.05}{1.8} = 10290 \text{ N}$$

**Answer:** Each front wheel: **3675 N**; each back wheel: **5145 N**.

### Exercise 6.10

**Question:** Torques of equal magnitude are applied to a hollow cylinder and a solid sphere, both having the same mass and radius. The cylinder is free to rotate about its standard axis of symmetry, and the sphere is free to rotate about an axis passing through its centre. Which of the two will acquire a greater angular speed after a given time?

**Solution:**  $I_{cyl} = MR^2$  and  $I_{sphere} = \frac{2}{5}MR^2$ . With the same torque,  $\alpha = \tau/I$ :

$$\frac{\alpha_{sphere}}{\alpha_{cyl}} = \frac{I_{cyl}}{I_{sphere}} = \frac{5}{2}$$

Since  $\omega = \alpha t$  (from rest), the **solid sphere** acquires the greater angular speed (2.5 times that of the cylinder).

### Exercise 6.11

**Question:** A solid cylinder of mass 20 kg rotates about its axis with angular speed  $100 \text{ rad s}^{-1}$ . The radius of the cylinder is 0.25 m. What is the kinetic energy associated with the rotation of the cylinder? What is the magnitude of angular momentum of the cylinder about its axis?

**Solution:**

$$I = \frac{1}{2}MR^2 = \frac{1}{2} \times 20 \times (0.25)^2 = 0.625 \text{ kg m}^2$$

$$K = \frac{1}{2}I\omega^2 = \frac{1}{2} \times 0.625 \times (100)^2 = 3125 \text{ J}$$

$$L = I\omega = 0.625 \times 100 = 62.5 \text{ J s}$$

**Answer:**  $K = 3125 \text{ J}$ ;  $L = 62.5 \text{ J s}$

### Exercise 6.12

**Question:** (a) A child stands at the centre of a turntable with his two arms outstretched. The turntable is set rotating with an angular speed of 40 rev/min. How much is the angular speed of the child if he folds his hands back and thereby reduces his moment of inertia to  $2/5$  times the initial value? Assume that the turntable rotates without friction. (b) Show that the child's new kinetic energy of rotation is more than the initial kinetic energy of rotation. How do you account for this increase in kinetic energy?

**Solution:**

(a) No external torque acts, so angular momentum is conserved:

$$I_1\omega_1 = I_2\omega_2 \Rightarrow \omega_2 = \frac{I_1}{I_2}\omega_1 = \frac{5}{2} \times 40 = 100 \text{ rev/min}$$

(b)

$$\frac{K_2}{K_1} = \frac{\frac{1}{2}I_2\omega_2^2}{\frac{1}{2}I_1\omega_1^2} = \frac{2}{5} \times \left(\frac{5}{2}\right)^2 = \frac{5}{2} = 2.5$$

The kinetic energy **increases** 2.5 times. The increase comes from the child's **internal energy** — the child does work with his muscles in pulling his arms inwards.

### Exercise 6.13

**Question:** A rope of negligible mass is wound round a hollow cylinder of mass 3 kg and radius 40 cm. What is the angular acceleration of the cylinder if the rope is pulled with a force of 30 N? What is the linear acceleration of the rope? Assume that there is no slipping.

**Solution:**

$$I = MR^2 = 3 \times (0.4)^2 = 0.48 \text{ kg m}^2, \quad \tau = FR = 30 \times 0.4 = 12 \text{ N m}$$

$$\alpha = \frac{\tau}{I} = \frac{12}{0.48} = 25 \text{ rad s}^{-2}$$

$$a = \alpha R = 25 \times 0.4 = 10 \text{ m s}^{-2}$$

**Answer:**  $\alpha = 25 \text{ rad s}^{-2}$ ;  $a = 10 \text{ m s}^{-2}$

### Exercise 6.14

**Question:** To maintain a rotor at a uniform angular speed of  $200 \text{ rad s}^{-1}$ , an engine needs to transmit a torque of 180 N m. What is the power required by the engine? (Note: uniform angular velocity in the absence of friction implies zero torque. In practice, applied torque is needed to counter frictional torque.) Assume that the engine is 100% efficient.

**Solution:**

$$P = \tau\omega = 180 \times 200 = 36000 \text{ W} = 36 \text{ kW}$$

**Answer:** 36 kW

### Exercise 6.15

**Question:** From a uniform disk of radius  $R$ , a circular hole of radius  $R/2$  is cut out. The centre of the hole is at  $R/2$  from the centre of the original disc. Locate the centre of gravity of the resulting flat body.

**Solution:** Let the mass of the full disc be  $M$ , with its centre  $O$  at the origin. The hole has  $1/4$  the area, so its mass is  $M/4$ , centred at  $x = R/2$ . The remaining body has mass  $3M/4$ , with its CG at  $x$ . Treating the hole as a negative mass:

$$x = \frac{M(0) - \frac{M}{4} \cdot \frac{R}{2}}{M - \frac{M}{4}} = \frac{-MR/8}{3M/4} = -\frac{R}{6}$$

**Answer:** The CG is at  $R/6$  from the centre of the original disc, on the side **opposite** to the hole.

### Exercise 6.16

**Question:** A metre stick is balanced on a knife edge at its centre. When two coins, each of mass 5 g are put one on top of the other at the 12.0 cm mark, the stick is found to be balanced at 45.0 cm. What is the mass of the metre stick?

**Solution:** Let the mass of the stick be  $m$ , acting at its centre (50 cm). The coins (10 g) are at 12 cm. Taking moments about the new balance point (45 cm):

$$10 \times (45 - 12) = m \times (50 - 45) \Rightarrow 330 = 5m \Rightarrow m = 66 \text{ g}$$

**Answer:**  $m = 66 \text{ g}$

### Exercise 6.17

**Question:** The oxygen molecule has a mass of  $5.30 \times 10^{-26} \text{ kg}$  and a moment of inertia of  $1.94 \times 10^{-46} \text{ kg m}^2$  about an axis through its centre perpendicular to the lines joining the two atoms. Suppose the mean speed of such a molecule in a gas is 500 m/s and that its kinetic energy of rotation is two thirds of its kinetic energy of translation. Find the average angular velocity of the molecule.

**Solution:**

$$K_{trans} = \frac{1}{2}mv^2 = \frac{1}{2} \times 5.30 \times 10^{-26} \times (500)^2 = 6.625 \times 10^{-21} \text{ J}$$

$$K_{rot} = \frac{2}{3}K_{trans} = 4.417 \times 10^{-21} \text{ J} = \frac{1}{2}I\omega^2$$

$$\omega = \sqrt{\frac{2 \times 4.417 \times 10^{-21}}{1.94 \times 10^{-46}}} = 6.75 \times 10^{12} \text{ rad s}^{-1}$$

**Answer:**  $\omega \approx 6.75 \times 10^{12} \text{ rad s}^{-1}$