

Chapter 1 : Electric Charges and Fields (Notes)

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Notes

GSEB and CBSE Board Exam preparation subjective notes • Class 12 Physics • NCERT Chapter 1

Q.1 What is electric charge? Explain the two kinds of charges with the experiment of rubbing rods. (2 Marks)

Electric charge: Electric charge is an intrinsic property of matter (of elementary particles like electrons and protons) because of which it produces and experiences electric and magnetic effects.

Electrostatics deals with the study of forces, fields and potentials arising from **static charges**.

Experiment (Fig. 1.1):

- Two glass rods rubbed with silk **repel** each other.
- Two plastic rods rubbed with cat's fur **repel** each other.
- A glass rod (rubbed with silk) and a plastic rod (rubbed with fur) **attract** each other.

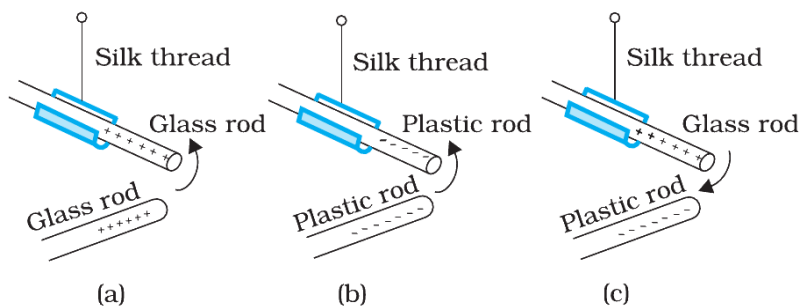


Fig. 1.1: Rods: like charges repel and unlike charges attract each other

Conclusion:

- There are only **two kinds** of charges. (i) **Like charges repel** and (ii) **unlike charges attract** each other.
- The property which differentiates the two kinds of charges is called the **polarity** of charge.
- By convention (Benjamin Franklin), the charge on a glass rod or cat's fur is **positive**, and the charge on a plastic rod or silk is **negative**.

How bodies get charged: When a glass rod is rubbed with silk, some electrons are transferred from the rod to the silk. The rod becomes positively charged (deficit of electrons) and the silk becomes negatively charged (excess of electrons). **No new charge is created** in rubbing.

Q.2 What is a gold-leaf electroscope? Distinguish between conductors and insulators. (2 Marks)

Gold-leaf electroscope: A simple apparatus to detect charge on a body. It consists of a vertical metal rod in a box with two thin gold leaves at its bottom end. When a charged object touches the metal knob, charge flows to the leaves and they **diverge**. The degree of divergence indicates the amount of charge.

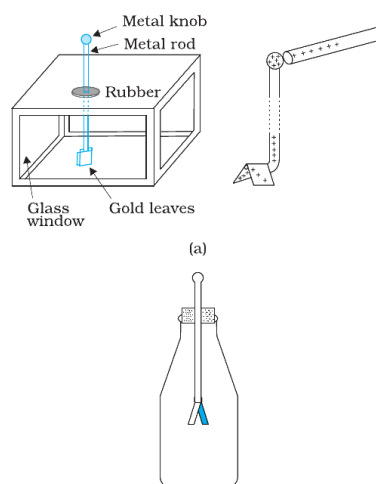


Fig. 1.2: Electroscopes: (a) the gold-leaf electroscope, (b) schematic of a simple electroscope

Conductors	Insulators
Allow electricity to pass through them easily.	Offer high resistance to the passage of electricity.
Have free charges (electrons) that can move inside the material.	Have no free charges.
Charge given to a conductor spreads over its entire surface.	Charge given to an insulator stays at the same place.
Examples: metals, human body, earth.	Examples: glass, porcelain, plastic, nylon, wood.

Key Examination Points for Full Marks:

- A nylon or plastic comb gets electrified on combing dry hair, but a metal spoon does not, because the charge on metal leaks through our body to the ground.
- Semiconductors** are a third category, with resistance between conductors and insulators.

Q.3 Explain the basic properties of electric charge. (3 Marks)

If the sizes of charged bodies are very small compared with the distances between them, they are treated as **point charges**.

(1) Additivity of charges: The total charge of a system is the **algebraic sum** of all individual charges:

$$q = q_1 + q_2 + q_3 + \dots + q_n$$

Charge is a **scalar**. Example: charges +1, +2, -3, +4, -5 give a total of $(+1) + (+2) + (-3) + (+4) + (-5) = -1$ unit. Unlike mass, charge can be positive or negative, so proper signs must be used.

(2) Charge is conserved: The total charge of an **isolated system** is always conserved. Charges may be redistributed, but net charge can neither be created nor destroyed. Example: a neutron turns into a proton and an electron. The proton and electron have equal and opposite charges, so the total charge is zero before and after.

(3) Quantisation of charge: All free charges are **integral multiples** of a basic unit of charge e :

$$q = ne, \quad n = 0, \pm 1, \pm 2, \pm 3, \dots$$

$$e = 1.602192 \times 10^{-19} \text{ C}$$

- Quantisation was first suggested by Faraday's laws of electrolysis and experimentally demonstrated by **Millikan** in 1912.
- If a body contains n_1 electrons and n_2 protons, its charge is $(n_2 - n_1)e$.
- About 6×10^{18} electrons make a charge of -1 C .

Key Examination Points for Full Marks:

- At the macroscopic level**, quantisation has no practical consequence and can be ignored: $1 \mu\text{C}$ contains about 10^{13} times e , so charge appears continuous (like a dotted line seen from far away).
- At the microscopic level**, where charges are a few tens or hundreds of e , quantisation cannot be ignored.

Illustration (NCERT Example 1.1): If 10^9 electrons leave a body every second, the charge given out per second is $1.6 \times 10^{-19} \times 10^9 = 1.6 \times 10^{-10} \text{ C}$. Time to collect $1 \text{ C} = 1/(1.6 \times 10^{-10}) = 6.25 \times 10^9 \text{ s} \approx \mathbf{198 \text{ years}}$. So 1 coulomb is a very large unit.

Q.4 State Coulomb's law. Define one coulomb and write the value of k . (3 Marks)

Statement: The force between two point charges is directly proportional to the product of the magnitudes of the charges, inversely proportional to the square of the distance between them, and acts along the line joining the two charges.

$$F = k \frac{|q_1 q_2|}{r^2}$$

Value of k : In SI units,

$$k = \frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ N m}^2 \text{C}^{-2}$$

where ϵ_0 is the **permittivity of free space**:

$$\epsilon_0 = 8.854 \times 10^{-12} \text{ C}^2 \text{N}^{-1} \text{m}^{-2}$$

So Coulomb's law is

$$F = \frac{1}{4\pi\epsilon_0} \frac{|q_1 q_2|}{r^2}$$

One coulomb: Putting $q_1 = q_2 = 1 \text{ C}$ and $r = 1 \text{ m}$ gives $F = 9 \times 10^9 \text{ N}$. So **1 C is the charge which, placed at a distance of 1 m from an equal charge in vacuum, repels it with a force of $9 \times 10^9 \text{ N}$.**

How Coulomb found it: Coulomb used a **torsion balance**. To vary the charge, he touched a charged sphere (charge q) with an identical uncharged sphere, so each got $q/2$; repeating gave $q/4$, etc.

Key Examination Points for Full Marks:

- 1 C is a very big unit; in practice we use 1 mC or $1 \mu\text{C}$.
- The law holds down to subatomic distances ($r \sim 10^{-10} \text{ m}$).

Q.5 Write Coulomb's law in vector form. Show that it obeys Newton's third law. (3 Marks)

Let charges q_1 and q_2 have position vectors $\vec{r}_1$ and $\vec{r}_2$. The vector from 1 to 2 is

$$\vec{r}_{21} = \vec{r}_2 - \vec{r}_1, \quad \vec{r}_{12} = \vec{r}_1 - \vec{r}_2 = -\vec{r}_{21}$$

Unit vectors: $\hat{r}_{21} = \vec{r}_{21}/r_{21}$ and $\hat{r}_{12} = -\hat{r}_{21}$.

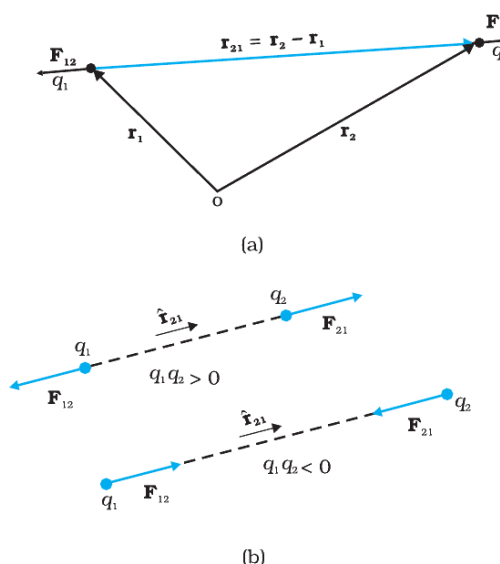


Fig. 1.3: (a) Geometry and (b) forces between charges

Force on q_2 due to q_1 :

$$\vec{F}_{21} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r_{21}^2} \hat{r}_{21}$$

Force on q_1 due to q_2 (interchanging 1 and 2):

$$\vec{F}_{12} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r_{12}^2} \hat{r}_{12} = -\vec{F}_{21}$$

So the two forces are equal and opposite — **Coulomb's law agrees with Newton's third law.**

Key Examination Points for Full Marks:

- The vector form works for any signs of q_1, q_2 . If $q_1 q_2 > 0$, $\vec{F}_{21}$ is along $\hat{r}_{21}$ (**repulsion**). If $q_1 q_2 < 0$, it is along $-\hat{r}_{21}$ (**attraction**).
- The law is for charges **in vacuum**. In a medium of dielectric constant K , the force becomes F/K .

Q.6 Compare electrostatic and gravitational forces.

(2 Marks)

Illustration (NCERT Example 1.3): For an electron and a proton,

$$\left| \frac{F_e}{F_g} \right| = \frac{e^2}{4\pi\epsilon_0 G m_p m_e} = 2.4 \times 10^{39}$$

For two protons, the ratio is $\frac{e^2}{4\pi\epsilon_0 G m_p^2} = 1.3 \times 10^{36}$.

Electrostatic force	Gravitational force
Can be attractive or repulsive.	Always attractive.
Depends on the medium.	Independent of the medium.
Very strong (10^{39} times gravity for e-p).	Very weak.
Acts between charged bodies.	Acts between all masses.

Similarities: Both are central forces, both obey the inverse-square law, both act at a distance and both are conservative.

Acceleration of an electron due to the Coulomb force of a proton $1 \text{ \AA}$ away: $F = 2.3 \times 10^{-8} \text{ N}$, so $a = F/m_e = 2.5 \times 10^{22} \text{ m s}^{-2}$. This is enormous compared with g , so the effect of gravity on the electron is negligible.

Q.7 State the principle of superposition. Find the force on a charge due to many charges. (3 Marks)

Principle of superposition: The force on any charge due to a number of other charges is the **vector sum** of all the forces on that charge due to the other charges, taken one at a time. The individual forces are **unaffected** by the presence of other charges.

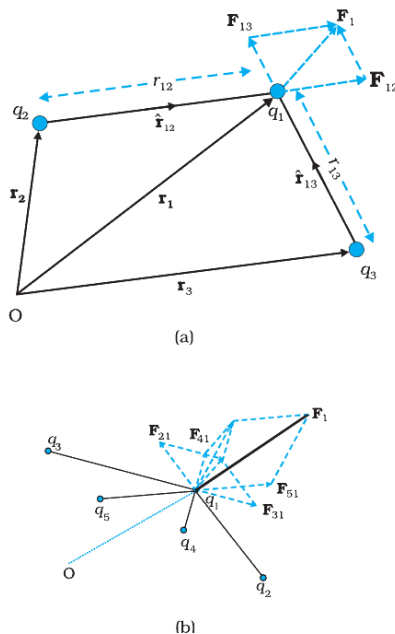


Fig. 1.5: A system of (a) three charges, (b) multiple charges

For three charges: The force on q_1 due to q_2 and q_3 is

$$\vec{F}_1 = \vec{F}_{12} + \vec{F}_{13} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r_{12}^2} \hat{r}_{12} + \frac{1}{4\pi\epsilon_0} \frac{q_1 q_3}{r_{13}^2} \hat{r}_{13}$$

For n charges:

$$\vec{F}_1 = \frac{1}{4\pi\epsilon_0} \left[\frac{q_1 q_2}{r_{12}^2} \hat{r}_{12} + \frac{q_1 q_3}{r_{13}^2} \hat{r}_{13} + \dots + \frac{q_1 q_n}{r_{1n}^2} \hat{r}_{1n} \right] = \frac{q_1}{4\pi\epsilon_0} \sum_{i=2}^n \frac{q_i}{r_{1i}^2} \hat{r}_{1i}$$

The vector sum is found by the **parallelogram law**. All of electrostatics is basically a consequence of Coulomb's law and the superposition principle.

Illustration (NCERT Example 1.5): Three equal charges q at the corners of an equilateral triangle exert three equal forces on a charge Q at the centroid, at 120° to each other. **The net force is zero.**

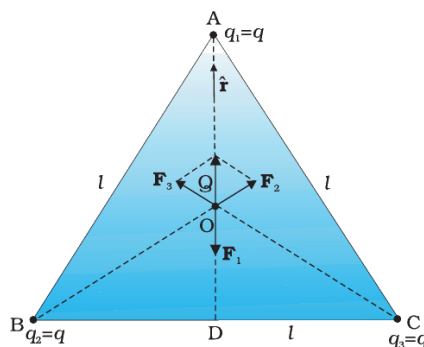


Fig. 1.6: Three equal charges at the vertices of an equilateral triangle

Q.8 Define electric field. Derive the electric field due to a point charge.(3 Marks)

Electric field: The space around a charge in which its electric influence can be experienced. The **electric field at a point** is the force experienced by a **unit positive test charge** placed at that point:

$$\vec{E} = \lim_{q \rightarrow 0} \frac{\vec{F}}{q}$$

The test charge q is made negligibly small so that it does not disturb the source charge Q .

Field due to a point charge: A point charge Q at the origin exerts a force on a test charge q at distance r :

$$\vec{F} = \frac{1}{4\pi\epsilon_0} \frac{Qq}{r^2} \hat{r}$$

$$\vec{E} = \frac{\vec{F}}{q} \Rightarrow \boxed{\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2} \hat{r}}$$

And $\vec{F} = q\vec{E}$.

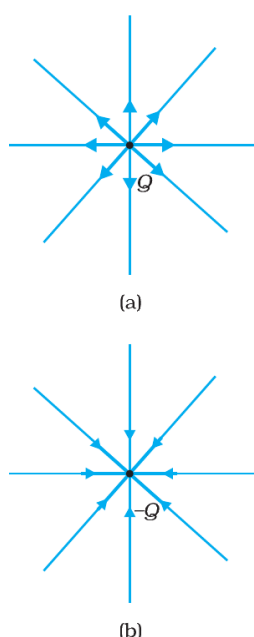


Fig. 1.8: Electric field (a) due to a charge Q , (b) due to a charge $-Q$

SI unit: N C^{-1} (or V m^{-1}). **Dimensions:** $[\text{M L T}^{-3} \text{A}^{-1}]$.

Key Examination Points for Full Marks:

- The field $\mathbf{E}$ due to Q is **independent of the test charge** q , but depends on the position $\vec{r}$.
- For a positive charge, $\mathbf{E}$ points radially **outward**; for a negative charge, radially **inward**.
- E depends only on r , so it has **spherical symmetry**.
- Q is the **source charge**; q is the **test charge**.

Q.9 Obtain the electric field due to a system of charges. What is the physical significance of electric field? (2 Marks)

Field due to a system of charges: By the superposition principle, the field at a point P (position $\vec{r}$) due to charges $q_1, q_2, \dots, q_n$ is the vector sum of their individual fields:

$$\vec{E}(\vec{r}) = \vec{E}_1 + \vec{E}_2 + \dots + \vec{E}_n = \frac{1}{4\pi\epsilon_0} \sum_{i=1}^n \frac{q_i}{r_{iP}^2} \hat{r}_{iP}$$

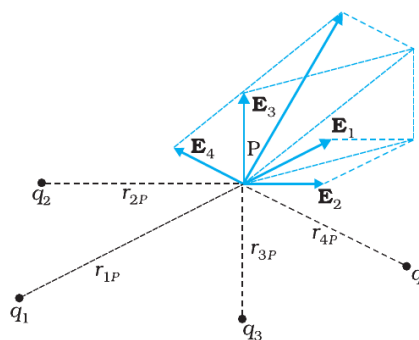


Fig. 1.9: Electric field at a point due to a system of charges is the vector sum of the fields due to individual charges

Physical significance:

- The electric field describes the **electrical environment** of a system of charges. It tells us the force a unit positive charge would experience at any point.
- It is **independent of the test charge**.
- In time-dependent situations, the effect of one moving charge on another cannot be instantaneous; it travels at the speed of light c . **The accelerated charge produces electromagnetic waves, which propagate with speed c and cause a force on the other charge.** The field accounts for this time delay.
- So fields are **physical entities**, not mere mathematical constructs. They carry energy and have their own dynamics. The concept of field was introduced by **Faraday**.

Q.10 What are electric field lines? Write their properties. (3 Marks)

Electric field line: A curve drawn in such a way that the **tangent at each point** on it gives the **direction of the net electric field** at that point. An arrow shows the direction of the field.

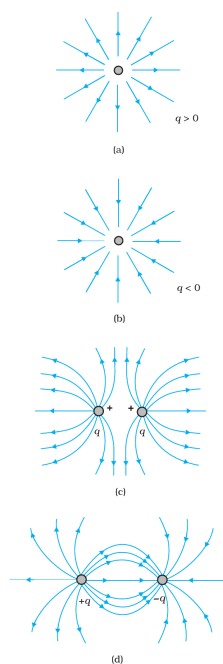


Fig. 1.14: Field lines due to some simple charge configurations

Field strength and density of lines: The relative closeness (density) of lines indicates the strength of the field. Lines **crowd** where the field is strong and spread apart where it is weak. The number of lines through a given solid angle is constant, so the number per unit area falls as $1/r^2$ — consistent with $E \propto 1/r^2$ (Fig. 1.13).

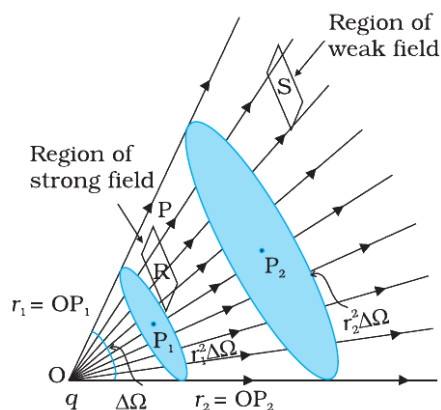


Fig. 1.13: Dependence of electric field strength on the distance and its relation to the number of field lines

Properties:

1. Field lines **start from positive charges and end at negative charges**. For a single charge, they may start or end at infinity.
2. In a charge-free region, field lines are **continuous curves without any breaks**.
3. **Two field lines can never cross** each other. (At the crossing point, the field would have two directions, which is absurd.)
4. Electrostatic field lines **do not form closed loops** (because the electrostatic field is conservative).
5. In a uniform field, field lines are **equally spaced parallel straight lines**.

Note: Faraday called them “lines of force”, but “field lines” is the better term.

Q.11 Define electric flux. Write its SI unit and explain the area vector. (2 Marks)

Area vector: A small planar area element is a vector $\Delta \vec{S} = \Delta S \hat{n}$ whose magnitude is the area and whose direction is along the **normal** to the area. For a **closed surface**, $\hat{n}$ is taken along the **outward normal**.

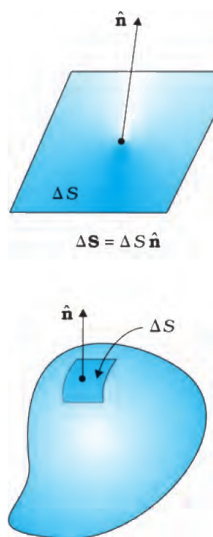


Fig. 1.16: Convention for defining normal $\hat{n}$ and $\Delta \vec{S}$

Electric flux: The electric flux through an area element is

$$\Delta \phi = \vec{E} \cdot \Delta \vec{S} = E \Delta S \cos \theta$$

where θ is the angle between $\vec{E}$ and $\Delta \vec{S}$. It is **proportional to the number of field lines** crossing the area element.

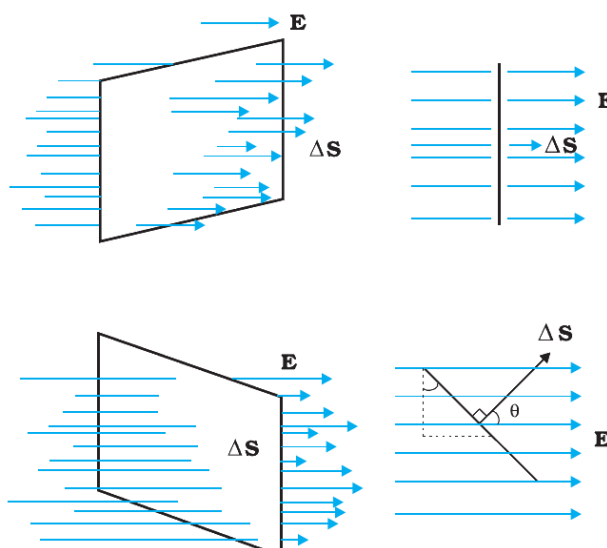


Fig. 1.15: Dependence of flux on the inclination θ between E and $\hat{n}$

Total flux through a surface S :

$$\phi = \sum \vec{E} \cdot \Delta \vec{S} \quad (= \oint \vec{E} \cdot d\vec{S} \text{ for a closed surface})$$

Key Examination Points for Full Marks:

- Flux is a **scalar**. **SI unit:** $\text{N C}^{-1} \text{m}^2$ (or V m). **Dimensions:** $[\text{M L}^3 \text{T}^{-3} \text{A}^{-1}]$.
- Maximum when $\theta = 0^\circ$ ($\phi = E\Delta S$); zero when $\theta = 90^\circ$ (field lines parallel to the surface).

Q.12 What is an electric dipole? Define dipole moment. Give examples of polar molecules. (2 Marks)

Electric dipole: A pair of **equal and opposite point charges** q and $-q$ separated by a distance $2a$. The direction from $-q$ to q is the **direction of the dipole**, and the midpoint is the **centre of the dipole**.

Dipole moment:

$$\vec{p} = q \times 2a \hat{p}$$

It is a **vector** directed from $-q$ to $+q$. **SI unit:** C m . **Dimensions:** $[\text{L T A}]$.

Key Examination Points for Full Marks:

- The total charge of a dipole is zero, but **its field is not zero**, because the two charges are separated.
- At large distances, the dipole field falls off as $1/r^3$ (faster than $1/r^2$ for a single charge).
- Point dipole:** $2a \rightarrow 0$ and $q \rightarrow \infty$ such that $p = q \times 2a$ is finite.

Physical significance:

- Non-polar molecules:** the centres of positive and negative charge coincide, so $p = 0$. Examples: CO_2 , CH_4 . They develop a dipole moment only when an electric field is applied (induced dipole).
- Polar molecules:** the centres do not coincide, so they have a **permanent dipole moment** even without a field. Example: H_2O .

Q.13 Derive the electric field of a dipole at a point on its axis. (3 Marks)

Let P be at distance r from the centre O of the dipole, on the side of q (Fig. 1.17 (a)).

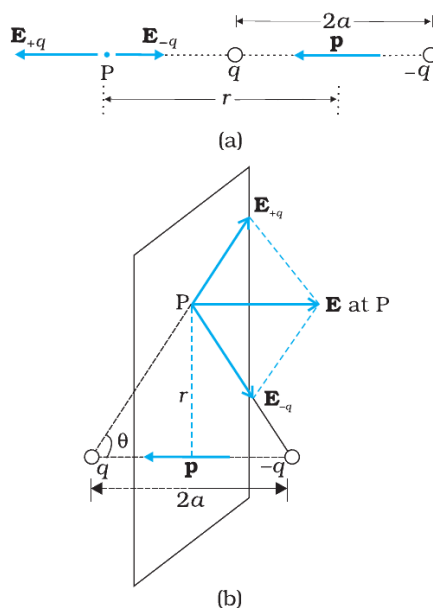


Fig. 1.17: Electric field of a dipole at (a) a point on the axis, (b) a point on the equatorial plane

Field due to $-q$ (at distance $r + a$):

$$\vec{E}_{-q} = -\frac{q}{4\pi\epsilon_0(r+a)^2} \hat{p}$$

Field due to $+q$ (at distance $r - a$):

$$\vec{E}_{+q} = \frac{q}{4\pi\epsilon_0(r-a)^2} \hat{p}$$

Total field at P:

$$\begin{aligned} \vec{E} &= \vec{E}_{+q} + \vec{E}_{-q} = \frac{q}{4\pi\epsilon_0} \left[\frac{1}{(r-a)^2} - \frac{1}{(r+a)^2} \right] \hat{p} \\ &= \frac{q}{4\pi\epsilon_0} \frac{4ar}{(r^2 - a^2)^2} \hat{p} \end{aligned}$$

Since $p = 2aq$:

$$\vec{E} = \frac{2\vec{p}r}{4\pi\epsilon_0(r^2 - a^2)^2}$$

For $r \gg a$:

$$\boxed{\vec{E} = \frac{2\vec{p}}{4\pi\epsilon_0 r^3}}$$

Direction: Along $\vec{p}$ (from $-q$ to $+q$).

Q.14 Derive the electric field of a dipole on its equatorial plane. (3 Marks)

Let P be on the perpendicular bisector of the dipole at distance r from O (Fig. 1.17 (b)). Each charge is at distance $\sqrt{r^2 + a^2}$ from P, so

$$E_{+q} = E_{-q} = \frac{q}{4\pi\epsilon_0} \frac{1}{r^2 + a^2}$$

Resolving: The components **normal** to the dipole axis cancel. The components **along** the axis add up, and point opposite to $\hat{p}$:

$$\vec{E} = -(E_{+q} + E_{-q})\cos\theta \hat{p}, \quad \cos\theta = \frac{a}{\sqrt{r^2 + a^2}}$$

$$\vec{E} = -\frac{2qa}{4\pi\epsilon_0(r^2 + a^2)^{3/2}} \hat{p} = -\frac{\vec{p}}{4\pi\epsilon_0(r^2 + a^2)^{3/2}}$$

For $r \gg a$:

$$\vec{E} = -\frac{\vec{p}}{4\pi\epsilon_0 r^3}$$

Key Examination Points for Full Marks:

- Equatorial field is **antiparallel** to $\vec{p}$.
- $E_{\text{axial}} = 2E_{\text{equatorial}}$ at the same large distance.
- Both fall as $1/r^3$ and depend on $p = q \times 2a$, not on q and a separately. They also depend on the angle between $\vec{r}$ and $\vec{p}$.

Illustration (NCERT Example 1.9): For $\pm 10 \mu\text{C}$, 5.0 mm apart, $p = 5 \times 10^{-8} \text{ C m}$. At 15 cm on the axis, $E = 2.6 \times 10^5 \text{ N C}^{-1}$ (along $\vec{p}$); at 15 cm on the equatorial line, $E = 1.33 \times 10^5 \text{ N C}^{-1}$ (opposite to $\vec{p}$).

Q.15 Derive the torque on an electric dipole in a uniform electric field. What happens in a non-uniform field? (3 Marks)

Uniform field: A dipole of moment $\vec{p}$ is placed in a uniform field $\vec{E}$ at angle θ (Fig. 1.19). Forces: $+q\vec{E}$ on q and $-q\vec{E}$ on $-q$.

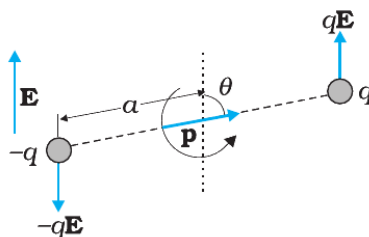


Fig. 1.19: Dipole in a uniform electric field

- **Net force** $= q\vec{E} - q\vec{E} = 0$.
- But the forces act at different points, forming a **couple**. Torque = force $\times$ perpendicular distance between the forces:

$$\tau = qE \times 2a \sin\theta = pE \sin\theta$$

In vector form,

$$\vec{\tau} = \vec{p} \times \vec{E}$$

The torque tends to **align the dipole with the field**. When $\vec{p} \parallel \vec{E}$, $\tau = 0$.

- Maximum torque: $\tau_{\text{max}} = pE$ at $\theta = 90^\circ$.

Non-uniform field: The net force is **not zero**, and in general there is also a torque.

- If $\vec{p} \parallel \vec{E}$: net force is in the direction of **increasing** field.
- If $\vec{p}$ is antiparallel to $\vec{E}$: net force is in the direction of **decreasing** field.

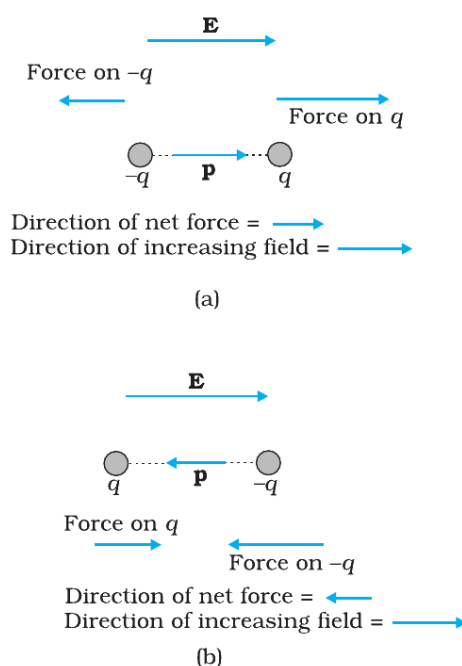


Fig. 1.20: Electric force on a dipole: (a) E parallel to p , (b) E antiparallel to p

Application: A comb run through dry hair attracts pieces of paper. The charged comb **polarises** the paper (induces a dipole moment along the field). The field of the comb is non-uniform, so the paper experiences a net force and moves toward the comb.

Q.16 Define linear, surface and volume charge densities. How is the field of a continuous charge distribution found? (2 Marks)

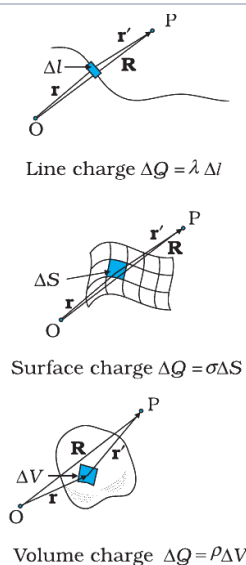


Fig. 1.21: Definition of linear, surface and volume charge densities

Charge density	Definition	SI unit
Linear (λ)	$\lambda = \frac{\Delta Q}{\Delta l}$ — charge per unit length	$C m^{-1}$
Surface (σ)	$\sigma = \frac{\Delta Q}{\Delta S}$ — charge per unit area	$C m^{-2}$
Volume (ρ)	$\rho = \frac{\Delta Q}{\Delta V}$ — charge per unit volume	$C m^{-3}$

The elements Δl , ΔS , ΔV are small on the macroscopic scale but contain a very large number of microscopic charges. So these densities ignore quantisation (a smoothed-out average).

Field of a continuous distribution: The charge in a volume element is $\rho \Delta V$. By Coulomb's law and superposition,

$$\vec{E} \approx \frac{1}{4\pi\epsilon_0} \sum_{\text{all } \Delta V} \frac{\rho \Delta V}{r'^2} \hat{r}'$$

where r' is the distance from the element to the point P. In the limit $\Delta V \rightarrow 0$, the sum becomes an integral.

Q.17 State Gauss's law. Prove it for a point charge enclosed in a sphere. Write its important points. (3 Marks)

Statement: The total electric flux through any **closed surface** S is $\frac{1}{\epsilon_0}$ times the **total charge enclosed** by S:

$$\phi = \oint_S \vec{E} \cdot d\vec{S} = \frac{q}{\epsilon_0}$$

Proof (point charge at the centre of a sphere): Take a sphere of radius r with charge q at its centre (Fig. 1.22). Flux through an area element:

$$\Delta\phi = \vec{E} \cdot \Delta\vec{S} = \frac{q}{4\pi\epsilon_0 r^2} \hat{r} \cdot \Delta\vec{S}$$

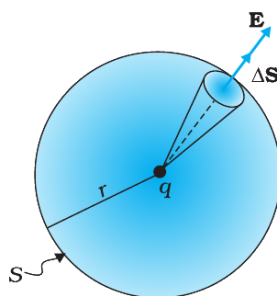


Fig. 1.22: Flux through a sphere enclosing a point charge q at its centre

The normal to the sphere at every point is along $\hat{r}$, so $\hat{r} \cdot \Delta\vec{S} = \Delta S$:

$$\phi = \sum \frac{q}{4\pi\epsilon_0 r^2} \Delta S = \frac{q}{4\pi\epsilon_0 r^2} \sum \Delta S = \frac{q}{4\pi\epsilon_0 r^2} \times 4\pi r^2 = \frac{q}{\epsilon_0}$$

Flux is zero if no charge is enclosed (Fig. 1.23): For a cylinder in a uniform field parallel to its axis, $\phi_1 = -ES$, $\phi_2 = +ES$ and $\phi_3 = 0$ (curved surface). Total $\phi = 0$.

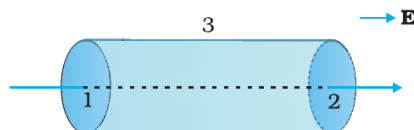


Fig. 1.23: Calculation of the flux of uniform electric field through the surface of a cylinder

Important points:

1. Gauss's law is true for **any closed surface**, whatever its shape or size.
2. q is the sum of **all charges enclosed**, wherever they are inside.
3. The field $\mathbf{E}$ on the surface is due to **all charges** (inside and outside), but q includes **only the charges inside**.
4. The chosen surface is called the **Gaussian surface**. It must **not pass through a discrete charge** ($\mathbf{E}$ is not defined there), but it can pass through a continuous distribution.
5. Gauss's law is very useful when the system has **symmetry**.

6. It is based on the **inverse-square** dependence in Coulomb's law.

Q.18 Using Gauss's law, find the electric field due to an infinitely long straight uniformly charged wire. (3 Marks)

Let the wire have uniform linear charge density λ . By symmetry, the field is **radial** and its magnitude depends only on the radial distance r (Fig. 1.26 (a)).

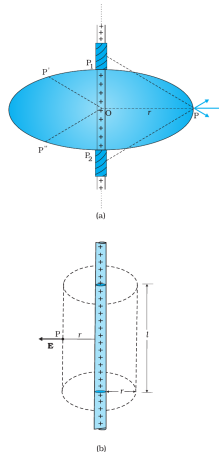


Fig. 1.26: (a) Electric field due to an infinitely long thin straight wire is radial, (b) the Gaussian surface for a long thin wire

Gaussian surface: A cylinder of radius r and length l , coaxial with the wire (Fig. 1.26 (b)).

- Through the two flat ends, flux = 0 (E is parallel to the surface).
- On the curved surface, E is normal to the surface and constant in magnitude.

$$\phi = E \times 2\pi r l$$

Charge enclosed = λl . By Gauss's law,

$$E \times 2\pi r l = \frac{\lambda l}{\epsilon_0}$$

$$\boxed{E = \frac{\lambda}{2\pi\epsilon_0 r}} \quad \text{i.e.} \quad \vec{E} = \frac{\lambda}{2\pi\epsilon_0 r} \hat{n}$$

Key Examination Points for Full Marks:

- $E \propto \frac{1}{r}$. E is outward if $\lambda > 0$ and inward if $\lambda < 0$.
- The field is due to the charge on the **entire wire**, though only the enclosed charge λl appears in Gauss's law.
- The wire must be infinitely long. For a finite wire, the result holds approximately near its middle.

Q.19 Using Gauss's law, find the electric field due to a uniformly charged infinite plane sheet. (3 Marks)

Let σ be the uniform surface charge density. Take the x -axis normal to the sheet. By symmetry, E is along x and does not depend on y and z (Fig. 1.27).

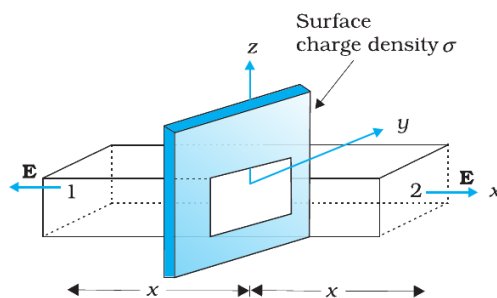


Fig. 1.27: Gaussian surface for a uniformly charged infinite plane sheet

Gaussian surface: A rectangular parallelepiped (or a cylinder) of cross-sectional area A , crossing the sheet.

- Only the two faces 1 and 2 contribute; field lines are parallel to the other faces.
- Flux through each end face = EA . Total flux = $2EA$.

Charge enclosed = σA . By Gauss's law,

$$2EA = \frac{\sigma A}{\epsilon_0}$$

$$\boxed{E = \frac{\sigma}{2\epsilon_0}} \quad \text{i.e.} \quad \vec{E} = \frac{\sigma}{2\epsilon_0} \hat{n}$$

Key Examination Points for Full Marks:

- **E is independent of the distance** from the sheet.
- **E** points away from the sheet if $\sigma > 0$ and toward it if $\sigma < 0$.
- For a finite large sheet, the result holds approximately in the middle region, away from the ends.
- **Two parallel sheets** with $+\sigma$ and $-\sigma$: $E = \frac{\sigma}{\epsilon_0}$ between them and zero outside.

Q.20 Using Gauss's law, find the electric field due to a uniformly charged thin spherical shell (a) outside, (b) inside the shell. (3 Marks)

Let a thin shell of radius R have uniform surface charge density σ . Total charge $q = 4\pi R^2\sigma$. By spherical symmetry, **E** is radial and depends only on r .

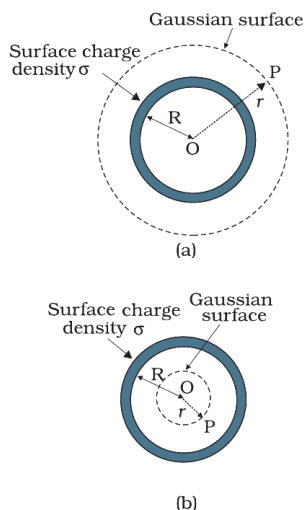


Fig. 1.28: Gaussian surfaces for a point with (a) $r > R$, (b) $r < R$

(i) Outside the shell ($r > R$): Take a spherical Gaussian surface of radius r through P. Flux = $E \times 4\pi r^2$. Charge enclosed = $\sigma \times 4\pi R^2$.

$$E \times 4\pi r^2 = \frac{\sigma}{\epsilon_0} 4\pi R^2$$

$$E = \frac{\sigma R^2}{\epsilon_0 r^2} = \frac{q}{4\pi\epsilon_0 r^2} \Rightarrow \vec{E} = \frac{q}{4\pi\epsilon_0 r^2} \hat{r}$$

The field outside is **as if the entire charge were concentrated at the centre**.

(ii) Inside the shell ($r < R$): The Gaussian surface (radius $r < R$) encloses **no charge**:

$$E \times 4\pi r^2 = 0 \Rightarrow \boxed{E = 0 \quad (r < R)}$$

Key Examination Points for Full Marks:

- The field is **zero everywhere inside** a uniformly charged shell. Its experimental verification confirms the $1/r^2$ dependence in Coulomb's law.
- At the surface ($r = R$): $E = \frac{\sigma}{\epsilon_0}$.
- The same outside result holds for a **solid sphere** of uniform volume charge density.

Formula Sheet — Electric Charges and Fields

Quantity	Formula	SI Unit
Quantisation of charge	$q = ne, e = 1.6 \times 10^{-19} \text{ C}$	C
Coulomb's law	$F = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2}$	N
Coulomb constant	$k = \frac{1}{4\pi\epsilon_0} = 9 \times 10^9$	$\text{N m}^2 \text{ C}^{-2}$
Permittivity of free space	$\epsilon_0 = 8.854 \times 10^{-12}$	$\text{C}^2 \text{ N}^{-1} \text{ m}^{-2}$
Force in a medium	$F_m = \frac{F}{K}$	N
Electric field	$\vec{E} = \vec{F}/q; E = \frac{kQ}{r^2}$	N C^{-1}
Dipole moment	$\vec{p} = q \times 2a$ (from $-q$ to $+q$)	C m
Dipole — axial ($r \gg a$)	$E = \frac{2kp}{r^3}$ (along $\vec{p}$)	N C^{-1}
Dipole — equatorial ($r \gg a$)	$E = \frac{kp}{r^3}$ (opposite $\vec{p}$)	N C^{-1}
Torque on dipole	$\vec{\tau} = \vec{p} \times \vec{E}, \tau = pE \sin \theta$	N m
Electric flux	$\phi = \vec{E} \cdot \vec{S} = ES \cos \theta$	$\text{N m}^2 \text{ C}^{-1}$
Gauss's law	$\oint \vec{E} \cdot d\vec{S} = \frac{q_{enc}}{\epsilon_0}$	$\text{N m}^2 \text{ C}^{-1}$
Infinite line charge	$E = \frac{\lambda}{2\pi\epsilon_0 r} = \frac{2k\lambda}{r}$	N C^{-1}
Infinite plane sheet	$E = \frac{\sigma}{2\epsilon_0}$	N C^{-1}
Two oppositely charged plates	$E = \frac{\sigma}{\epsilon_0}$ (between), 0 (outside)	N C^{-1}
Spherical shell	$E = \frac{kq}{r^2}$ ($r \geq R$); $E = 0$ ($r < R$)	N C^{-1}
Charge densities	$\lambda = Q/l, \sigma = Q/A, \rho = Q/V$	$\text{C m}^{-1}, \text{C m}^{-2}, \text{C m}^{-3}$



Dimensions: E [$M L T^{-3} A^{-1}$]; flux [$M L^3 T^{-3} A^{-1}$]; dipole moment [$L T A$]; ϵ_0 [$M^{-1} L^{-3} T^4 A^2$]; λ [$L^{-1} T A$]; σ [$L^{-2} T A$]; ρ [$L^{-3} T A$]

NEET / JEE Quick Points and Tricks

- Null point ($E = 0$) for two charges:** For **like** charges, it lies **between** them, nearer the smaller charge: $x = \frac{r}{1 + \sqrt{q_2/q_1}}$ from q_1 . For **unlike** charges, it lies **outside**, beyond the smaller charge.
- Maximum force on splitting:** Charge Q split into q and $(Q - q)$ gives maximum repulsion when $q = Q/2$.
- Touching identical spheres:** After contact, each gets $(q_1 + q_2)/2$. Unlike charges: force can drop sharply; equal and opposite charges $\rightarrow$ force becomes zero.
- Symmetric arrangements:** Equal charges at the vertices of a regular polygon give $E = 0$ at the centre. If one charge is removed, E at the centre equals the field of **one** charge.
- Cube flux tricks:** Charge at the **centre** of a cube $\rightarrow q/6\epsilon_0$ per face. At a **corner** $\rightarrow q/8\epsilon_0$ total; $q/24\epsilon_0$ through each of the 3 far faces, zero through the 3 adjacent faces. At the centre of a **face** $\rightarrow q/2\epsilon_0$ total.
- Outside charges give zero net flux** through a closed surface, but they do change E on the surface.
- Dipole:** $E_{axial} : E_{equatorial} = 2 : 1$ at the same distance. Both $\propto 1/r^3$. Uniform field $\rightarrow F = 0$ but $\tau \neq 0$. Non-uniform field $\rightarrow F \neq 0$.
- Stable equilibrium** of a dipole at $\theta = 0^\circ$ ($p \parallel E$); **unstable** at $\theta = 180^\circ$.
- Charged particle in uniform E :** $a = qE/m$. Time to fall a distance h : $t = \sqrt{2hm/qE}$ — heavier particle takes longer (unlike free fall under gravity).
- Oil drop balance (Millikan):** $qE = mg \Rightarrow q = mg/E$.
- Pendulum near a charged sheet:** $\tan\theta = \frac{qE}{mg} = \frac{q\sigma}{2\epsilon_0 mg}$, so $\sigma \propto \tan\theta$.
- E-field vs r graphs:** Shell / conductor: zero inside, jumps to kQ/R^2 at the surface, then $\propto 1/r^2$. Uniform solid sphere: rises linearly inside ($E = kQr/R^3$), then $\propto 1/r^2$ outside.
- Semicircular / ring charge:** Semicircle of radius a : $E = \frac{2k\lambda}{a} = \frac{\lambda}{2\pi\epsilon_0 a}$ at the centre. Full ring: $E = 0$ at the centre.
- Two parallel infinite sheets:** Same $\sigma \rightarrow E = 0$ between, σ/ϵ_0 outside. Opposite $\sigma \rightarrow \sigma/\epsilon_0$ between, 0 outside.