



NEET — JEE MAINS - ADVANCED

## Chapter 1 : Electric Charges and Fields (Solved Examples & Exercises)

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### Notes

Step-by-step solutions • NCERT Examples 1.1–1.12 and Exercises 1.1–1.23 • Class 12 Physics

### Part A — NCERT Solved Examples

#### Example 1.1

**Question:** If  $10^9$  electrons move out of a body to another body every second, how much time is required to get a total charge of 1 C on the other body?

**Solution:** Charge given out in one second:

$$q = ne = 10^9 \times 1.6 \times 10^{-19} = 1.6 \times 10^{-10} \text{ C s}^{-1}$$

$$t = \frac{1 \text{ C}}{1.6 \times 10^{-10} \text{ C s}^{-1}} = 6.25 \times 10^9 \text{ s} = \frac{6.25 \times 10^9}{365 \times 24 \times 3600} \text{ years} \approx 198 \text{ years}$$

**Answer:** About **200 years**. So one coulomb is a very large unit for many practical purposes. (A cubic piece of copper of side 1 cm contains about  $2.5 \times 10^{24}$  electrons.)

#### Example 1.2

**Question:** How much positive and negative charge is there in a cup of water?

**Solution:** Take the mass of one cup of water as 250 g. Molecular mass of water = 18 g, so one mole ( $6.02 \times 10^{23}$  molecules) = 18 g.

$$\text{Number of molecules} = \frac{250}{18} \times 6.02 \times 10^{23}$$

Each molecule ( $\text{H}_2\text{O}$ ) has two hydrogen atoms and one oxygen atom, i.e. **10 electrons and 10 protons**.

$$q = \frac{250}{18} \times 6.02 \times 10^{23} \times 10 \times 1.6 \times 10^{-19} \text{ C}$$

**Answer:** Total positive charge = total negative charge =  $1.34 \times 10^7 \text{ C}$ .

#### Example 1.3

**Question:** Coulomb's law for electrostatic force between two point charges and Newton's law for gravitational force between two stationary point masses both have inverse-square dependence on distance.

(a) Compare the strength of these forces by finding the ratio of their magnitudes (i) for an electron and a proton and (ii) for two protons. (b) Estimate the accelerations of the electron and the proton due to the

electrical force of their mutual attraction when they are  $1 \text{ \AA}$  ( $10^{-10} \text{ m}$ ) apart. ( $m_p = 1.67 \times 10^{-27} \text{ kg}$ ,  $m_e = 9.11 \times 10^{-31} \text{ kg}$ )

**Solution:**

**(a) (i)** Electric force  $F_e = -\frac{1}{4\pi\epsilon_0} \frac{e^2}{r^2}$  (attractive). Gravitational force  $F_G = -G \frac{m_p m_e}{r^2}$ .

$$\left| \frac{F_e}{F_G} \right| = \frac{e^2}{4\pi\epsilon_0 G m_p m_e} = 2.4 \times 10^{39}$$

**(ii)** For two protons:

$$\left| \frac{F_e}{F_G} \right| = \frac{e^2}{4\pi\epsilon_0 G m_p^2} = 1.3 \times 10^{36}$$

For two protons, gravity is attractive and the Coulomb force is repulsive. Inside a nucleus (distance  $\sim 10^{-15} \text{ m}$ ),  $F_e \sim 230 \text{ N}$  while  $F_G \sim 1.9 \times 10^{-34} \text{ N}$ .

**(b)** Force between the electron and proton:

$$|F| = \frac{1}{4\pi\epsilon_0} \frac{e^2}{r^2} = \frac{8.987 \times 10^9 \times (1.6 \times 10^{-19})^2}{(10^{-10})^2} = 2.3 \times 10^{-8} \text{ N}$$

$$a_e = \frac{F}{m_e} = \frac{2.3 \times 10^{-8}}{9.11 \times 10^{-31}} = 2.5 \times 10^{22} \text{ m s}^{-2}$$

$$a_p = \frac{F}{m_p} = \frac{2.3 \times 10^{-8}}{1.67 \times 10^{-27}} = 1.4 \times 10^{19} \text{ m s}^{-2}$$

**Answer:** Electric forces are enormously stronger than gravitational forces. Compared with  $g$ , the effect of gravity on the electron's motion is negligible.

## Example 1.4

**Question:** A charged metallic sphere A is suspended by a nylon thread. Another charged metallic sphere B held by an insulating handle is brought close to A such that the distance between their centres is  $10 \text{ cm}$ . The resulting repulsion of A is noted. Spheres A and B are touched by uncharged spheres C and D respectively. C and D are then removed and B is brought closer to A to a distance of  $5.0 \text{ cm}$  between their centres. What is the expected repulsion of A on the basis of Coulomb's law? Spheres A and C, and B and D, have identical sizes. Ignore the sizes of A and B compared with their separation.

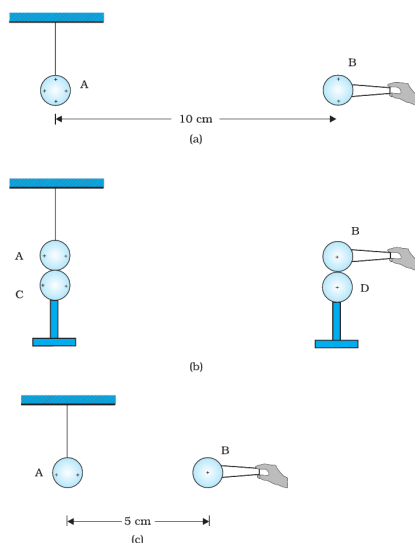


**Fig. 1.4**

**Solution:** Let the original charges be  $q$  on A and  $q'$  on B. At separation  $r$ ,

$$F = \frac{1}{4\pi\epsilon_0} \frac{qq'}{r^2}$$

When identical uncharged C touches A, the charge is shared equally, so A has  $q/2$ . Similarly, B has  $q'/2$ . The separation is halved:

$$F' = \frac{1}{4\pi\epsilon_0} \frac{(q/2)(q'/2)}{(r/2)^2} = \frac{1}{4\pi\epsilon_0} \frac{qq'}{r^2} = F$$

**Answer:** The electrostatic force on A due to B **remains unaltered**.

### Example 1.5

**Question:** Consider three charges  $q_1, q_2, q_3$ , each equal to  $q$ , at the vertices of an equilateral triangle of side  $l$ . What is the force on a charge  $Q$  (with the same sign as  $q$ ) placed at the centroid of the triangle?

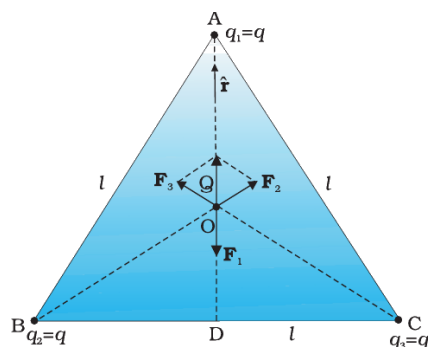


Fig. 1.6

**Solution:** Draw AD perpendicular to BC.  $AD = AC \cos 30^\circ = \frac{\sqrt{3}}{2} l$ . The distance of the centroid O from A is

$$AO = \frac{2}{3} AD = \frac{l}{\sqrt{3}} = BO = CO$$

Each force has magnitude

$$F_1 = F_2 = F_3 = \frac{1}{4\pi\epsilon_0} \frac{Qq}{(l/\sqrt{3})^2} = \frac{3}{4\pi\epsilon_0} \frac{Qq}{l^2}$$

directed along AO, BO and CO respectively. The resultant of  $\vec{F}_2$  and  $\vec{F}_3$  (at  $120^\circ$ ) has the same magnitude  $\frac{3}{4\pi\epsilon_0} \frac{Qq}{l^2}$  and points along OA, opposite to  $\vec{F}_1$ .

$$\vec{F}_{total} = \frac{3}{4\pi\epsilon_0} \frac{Qq}{l^2} (\hat{r} - \hat{r}) = 0$$

**Answer:** The total force on Q is **zero**. This is also clear by symmetry: rotating the system by  $120^\circ$  about O leaves it unchanged, so a non-zero force would have to point in every direction at once.

### Example 1.6

**Question:** Consider the charges  $q, q$  and  $-q$  placed at the vertices of an equilateral triangle. What is the force on each charge?

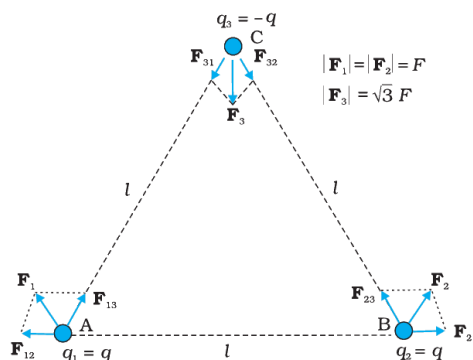


Fig. 1.7

**Solution:** Each pair of charges is at distance  $l$ , so every pairwise force has magnitude

$$F = \frac{q^2}{4\pi\epsilon_0 l^2}$$

- **Charge  $q$  at A:**  $\vec{F}_{12}$  (repulsion, along BA) and  $\vec{F}_{13}$  (attraction, along AC), at  $120^\circ$ . Resultant  $\vec{F}_1 = F \hat{r}_1$ , where  $\hat{r}_1$  is along BC.
- **Charge  $q$  at B:** Similarly,  $\vec{F}_2 = F \hat{r}_2$ , where  $\hat{r}_2$  is along AC.
- **Charge  $-q$  at C:** Two attractive forces of magnitude  $F$  at  $60^\circ$ . Resultant  $\vec{F}_3 = \sqrt{3}F \hat{n}$ , where  $\hat{n}$  is along the bisector of  $\angle BCA$  (toward AB).

**Answer:**  $|\vec{F}_1| = |\vec{F}_2| = F$  and  $|\vec{F}_3| = \sqrt{3}F$ . Also,  $\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 0$ , which is consistent with Newton's third law.

### Example 1.7

**Question:** An electron falls through a distance of 1.5 cm in a uniform electric field of magnitude  $2.0 \times 10^4 \text{ N C}^{-1}$  (field upward). The direction of the field is then reversed keeping its magnitude unchanged, and a proton falls through the same distance. Compute the time of fall in each case. Contrast the situation with that of 'free fall under gravity'.

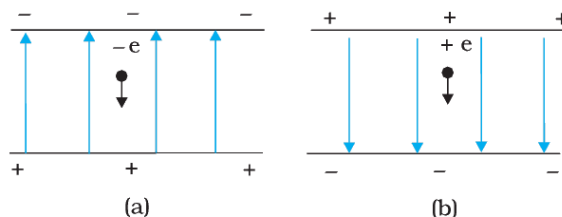


Fig. 1.10

**Solution: Electron:** The field is upward, so the force  $eE$  on the electron is downward.  $a_e = eE/m_e$ . Starting from rest,  $h = 1/2 a_e t^2$ :

$$t_e = \sqrt{\frac{2h}{a_e}} = \sqrt{\frac{2hm_e}{eE}} = \sqrt{\frac{2 \times 1.5 \times 10^{-2} \times 9.11 \times 10^{-31}}{1.6 \times 10^{-19} \times 2.0 \times 10^4}} = 2.9 \times 10^{-9} \text{ s}$$

**Proton:** The field is downward, so the force  $eE$  on the proton is downward.

$$t_p = \sqrt{\frac{2hm_p}{eE}} = \sqrt{\frac{2 \times 1.5 \times 10^{-2} \times 1.67 \times 10^{-27}}{1.6 \times 10^{-19} \times 2.0 \times 10^4}} = 1.3 \times 10^{-7} \text{ s}$$

**Answer:** The heavier proton takes a longer time to fall. This contrasts with free fall under gravity, where the time of fall is independent of mass. Gravity is ignored here because  $a_p = eE/m_p = 1.9 \times 10^{12} \text{ m s}^{-2}$ , which is enormous compared with  $g = 9.8 \text{ m s}^{-2}$ .

## Example 1.8

**Question:** Two point charges  $q_1$  and  $q_2$ , of magnitude  $+10^{-8}$  C and  $-10^{-8}$  C respectively, are placed 0.1 m apart. Calculate the electric fields at points A, B and C shown in Fig. 1.11.

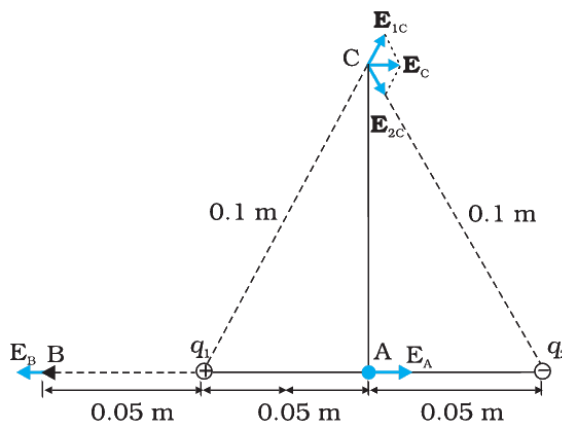


Fig. 1.11

**Solution:**

**At A** (midpoint, 0.05 m from each charge): Both fields point to the right.

$$E_{1A} = E_{2A} = \frac{9 \times 10^9 \times 10^{-8}}{(0.05)^2} = 3.6 \times 10^4 \text{ N C}^{-1}$$

$$E_A = E_{1A} + E_{2A} = 7.2 \times 10^4 \text{ N C}^{-1} \text{ (toward the right)}$$

**At B** (0.05 m left of  $q_1$ , 0.15 m from  $q_2$ ):  $E_{1B}$  points left and  $E_{2B}$  points right.

$$E_{1B} = \frac{9 \times 10^9 \times 10^{-8}}{(0.05)^2} = 3.6 \times 10^4 \text{ N C}^{-1}, \quad E_{2B} = \frac{9 \times 10^9 \times 10^{-8}}{(0.15)^2} = 4 \times 10^3 \text{ N C}^{-1}$$

$$E_B = E_{1B} - E_{2B} = 3.2 \times 10^4 \text{ N C}^{-1} \text{ (toward the left)}$$

**At C** (0.1 m from each charge):

$$E_{1C} = E_{2C} = \frac{9 \times 10^9 \times 10^{-8}}{(0.10)^2} = 9 \times 10^3 \text{ N C}^{-1}$$

The vertical components cancel. The horizontal components add:

$$E_C = E_{1C} \cos \frac{\pi}{3} + E_{2C} \cos \frac{\pi}{3} = 9 \times 10^3 \text{ N C}^{-1} \text{ (toward the right)}$$

**Answer:**  $E_A = 7.2 \times 10^4 \text{ N C}^{-1}$  (right);  $E_B = 3.2 \times 10^4 \text{ N C}^{-1}$  (left);  $E_C = 9 \times 10^3 \text{ N C}^{-1}$  (right).

## Example 1.9

**Question:** Two charges  $\pm 10 \mu\text{C}$  are placed 5.0 mm apart. Determine the electric field at (a) a point P on the axis of the dipole 15 cm away from its centre O on the side of the positive charge, and (b) a point Q, 15 cm away from O on a line passing through O and normal to the axis of the dipole.

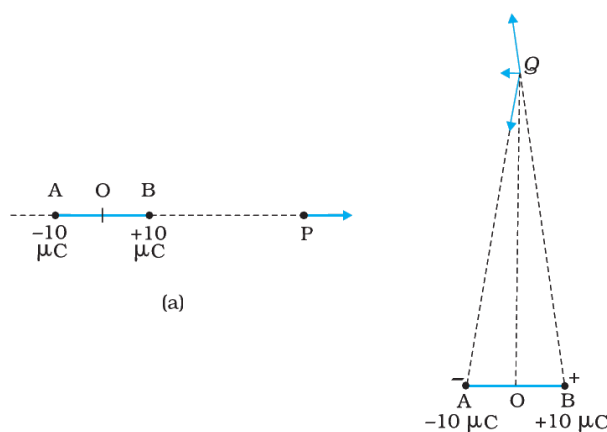


Fig. 1.18

**Solution:**  $a = 0.25$  cm.

**(a)** Field at P due to  $+10 \mu\text{C}$  (distance  $15 - 0.25 = 14.75$  cm):

$$E_+ = \frac{10^{-5}}{4\pi(8.854 \times 10^{-12})} \times \frac{1}{(15 - 0.25)^2 \times 10^{-4}} = 4.13 \times 10^6 \text{ N C}^{-1} \text{ along BP}$$

Field at P due to  $-10 \mu\text{C}$  (distance 15.25 cm):

$$E_- = \frac{10^{-5}}{4\pi(8.854 \times 10^{-12})} \times \frac{1}{(15 + 0.25)^2 \times 10^{-4}} = 3.86 \times 10^6 \text{ N C}^{-1} \text{ along PA}$$

$$E_p = 4.13 \times 10^6 - 3.86 \times 10^6 = 2.7 \times 10^5 \text{ N C}^{-1} \text{ along BP}$$

**Check using the dipole formula** ( $r/a = 60$ , large):  $p = 10^{-5} \times 5 \times 10^{-3} = 5 \times 10^{-8} \text{ C m}$ .

$$E = \frac{2p}{4\pi\epsilon_0 r^3} = \frac{2 \times 5 \times 10^{-8}}{4\pi(8.854 \times 10^{-12})} \times \frac{1}{(15)^3 \times 10^{-6}} = 2.6 \times 10^5 \text{ N C}^{-1}$$

along the dipole moment direction AB — close to the exact result.

**(b)** Each charge is at distance  $\sqrt{15^2 + 0.25^2}$  cm from Q:

$$E_+ = E_- = \frac{10^{-5}}{4\pi(8.854 \times 10^{-12})} \times \frac{1}{[15^2 + (0.25)^2] \times 10^{-4}} = 3.99 \times 10^6 \text{ N C}^{-1}$$

The components along OQ cancel; the components parallel to BA add:

$$E_Q = 2 \times \frac{0.25}{\sqrt{15^2 + (0.25)^2}} \times 3.99 \times 10^6 = 1.33 \times 10^5 \text{ N C}^{-1} \text{ along BA}$$

**Check:**  $E = \frac{p}{4\pi\epsilon_0 r^3} = 1.33 \times 10^5 \text{ N C}^{-1}$ , opposite to  $\vec{p}$ .

**Answer:** (a)  $2.6 \times 10^5 \text{ N C}^{-1}$  along  $\vec{p}$ ; (b)  $1.33 \times 10^5 \text{ N C}^{-1}$  opposite to  $\vec{p}$ .

### Example 1.10

**Question:** The electric field components in Fig. 1.24 are  $E_x = \alpha x^{1/2}$ ,  $E_y = E_z = 0$ , in which  $\alpha = 800 \text{ N C}^{-1} \text{ m}^{-1/2}$ . Calculate (a) the flux through the cube, and (b) the charge within the cube. Assume that  $a = 0.1$  m.

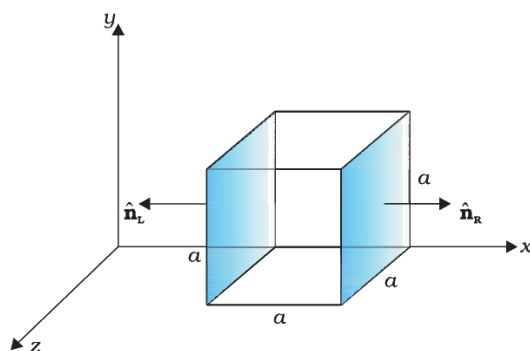


Fig. 1.24

**Solution:**

**(a)**  $E$  has only an  $x$ -component, so the flux is zero through every face except the two faces perpendicular to  $x$  (shaded).

- Left face ( $x = a$ ):  $E_L = \alpha a^{1/2}$ ;  $\phi_L = -E_L a^2$  ( $\theta = 180^\circ$ )
- Right face ( $x = 2a$ ):  $E_R = \alpha(2a)^{1/2}$ ;  $\phi_R = +E_R a^2$  ( $\theta = 0^\circ$ )

$$\begin{aligned}\phi &= \phi_R + \phi_L = a^2(E_R - E_L) = \alpha a^2[(2a)^{1/2} - a^{1/2}] = \alpha a^{5/2}(\sqrt{2} - 1) \\ &= 800 \times (0.1)^{5/2} \times (\sqrt{2} - 1) = 1.05 \text{ N m}^2 \text{ C}^{-1}\end{aligned}$$

**(b)** By Gauss's law,  $q = \phi \epsilon_0$ :

$$q = 1.05 \times 8.854 \times 10^{-12} = 9.27 \times 10^{-12} \text{ C}$$

**Answer:** (a)  $1.05 \text{ N m}^2 \text{ C}^{-1}$ ; (b)  $9.27 \times 10^{-12} \text{ C}$ .

### Example 1.11

**Question:** An electric field is uniform and in the positive  $x$  direction for positive  $x$ , and uniform with the same magnitude but in the negative  $x$  direction for negative  $x$ . It is given that  $\vec{E} = 200 \hat{i} \text{ N/C}$  for  $x > 0$  and  $\vec{E} = -200 \hat{i} \text{ N/C}$  for  $x < 0$ . A right circular cylinder of length 20 cm and radius 5 cm has its centre at the origin and its axis along the  $x$ -axis, so that one face is at  $x = +10 \text{ cm}$  and the other at  $x = -10 \text{ cm}$ . (a) What is the net outward flux through each flat face? (b) What is the flux through the side of the cylinder? (c) What is the net outward flux through the cylinder? (d) What is the net charge inside the cylinder?

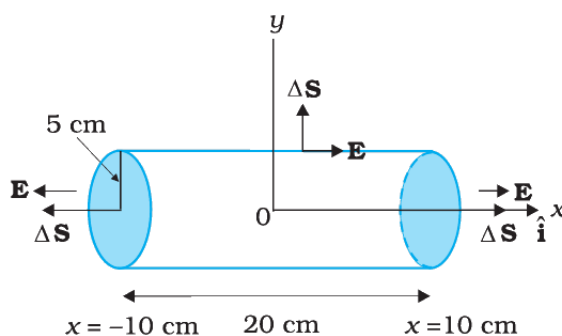


Fig. 1.25

**Solution:**

**(a)** On the left face,  $\vec{E}$  and  $\Delta \vec{S}$  are both along  $-x$  (parallel):

$$\phi_L = \vec{E} \cdot \Delta \vec{S} = (-200 \hat{i}) \cdot (-\Delta S \hat{i}) = +200 \times \pi(0.05)^2 = +1.57 \text{ N m}^2 \text{ C}^{-1}$$

On the right face,  $\vec{E}$  and  $\Delta \vec{S}$  are both along  $+x$ :  $\phi_R = +1.57 \text{ N m}^2 \text{ C}^{-1}$ .

**(b)** On the side,  $\vec{E}$  is perpendicular to  $\Delta \vec{S}$ , so the flux is **zero**.

(c)  $\phi = 1.57 + 1.57 + 0 = 3.14 \text{ N m}^2 \text{ C}^{-1}$ .

(d)  $q = \epsilon_0 \phi = 3.14 \times 8.854 \times 10^{-12} = 2.78 \times 10^{-11} \text{ C}$ .

**Answer:** (a)  $+1.57 \text{ N m}^2 \text{ C}^{-1}$  each; (b) 0; (c)  $3.14 \text{ N m}^2 \text{ C}^{-1}$ ; (d)  $2.78 \times 10^{-11} \text{ C}$ .

### Example 1.12

**Question:** An early model of an atom considered it to have a positively charged point nucleus of charge  $Ze$ , surrounded by a uniform density of negative charge up to a radius  $R$ . The atom as a whole is neutral. For this model, what is the electric field at a distance  $r$  from the nucleus?

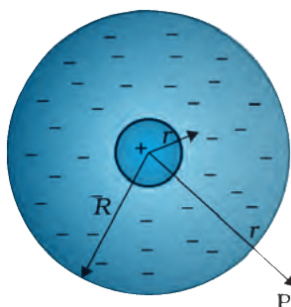


Fig. 1.29

**Solution:** The atom is neutral, so the total negative charge is  $-Ze$ :

$$\frac{4\pi R^3}{3} \rho = -Ze \Rightarrow \rho = -\frac{3Ze}{4\pi R^3}$$

Take a spherical Gaussian surface of radius  $r$  centred on the nucleus.

(i)  $r < R$ : Flux  $\phi = E(r) \times 4\pi r^2$ . Charge enclosed:

$$q = Ze + \frac{4\pi r^3}{3} \rho = Ze - Ze \frac{r^3}{R^3}$$

By Gauss's law,

$$E(r) = \frac{Ze}{4\pi\epsilon_0} \left( \frac{1}{r^2} - \frac{r}{R^3} \right); \quad r < R$$

directed radially outward.

(ii)  $r > R$ : The atom is neutral, so the enclosed charge is zero:  $E(r) \times 4\pi r^2 = 0$ , i.e.  $E(r) = 0$ .

**Answer:**  $E = \frac{Ze}{4\pi\epsilon_0} \left( \frac{1}{r^2} - \frac{r}{R^3} \right)$  for  $r < R$ , and  $E = 0$  for  $r > R$ . At  $r = R$ , both give  $E = 0$ .

## Part B — NCERT Exercises

### Exercise 1.1

**Question:** What is the force between two small charged spheres having charges of  $2 \times 10^{-7} \text{ C}$  and  $3 \times 10^{-7} \text{ C}$  placed 30 cm apart in air?

**Solution:**

$$F = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} = \frac{9 \times 10^9 \times 2 \times 10^{-7} \times 3 \times 10^{-7}}{(0.30)^2} = \frac{5.4 \times 10^{-4}}{0.09}$$

**Answer:**  $F = 6 \times 10^{-3} \text{ N}$ . The charges are alike, so the force is **repulsive**.

## Exercise 1.2

**Question:** The electrostatic force on a small sphere of charge  $0.4 \mu\text{C}$  due to another small sphere of charge  $-0.8 \mu\text{C}$  in air is  $0.2 \text{ N}$ . (a) What is the distance between the two spheres? (b) What is the force on the second sphere due to the first?

**Solution:**

(a)

$$r^2 = \frac{kq_1q_2}{F} = \frac{9 \times 10^9 \times 0.4 \times 10^{-6} \times 0.8 \times 10^{-6}}{0.2} = 0.0144 \Rightarrow r = 0.12 \text{ m}$$

(b) By Newton's third law, the force on the second sphere is equal and opposite: **0.2 N**, attractive (toward the first sphere).

**Answer:** (a)  $0.12 \text{ m} = 12 \text{ cm}$ ; (b)  $0.2 \text{ N}$ , attractive.

## Exercise 1.3

**Question:** Check that the ratio  $ke^2/Gm_em_p$  is dimensionless. Look up a Table of Physical Constants and determine the value of this ratio. What does the ratio signify?

**Solution:**  $ke^2/r^2$  is a force and  $Gm_em_p/r^2$  is also a force, so the ratio is (force)/(force), i.e. **dimensionless**.

In units:  $\frac{(\text{N m}^2\text{C}^{-2})(\text{C}^2)}{(\text{N m}^2\text{kg}^{-2})(\text{kg})(\text{kg})} = 1$ .

$$\frac{ke^2}{Gm_em_p} = \frac{9 \times 10^9 \times (1.6 \times 10^{-19})^2}{6.67 \times 10^{-11} \times 9.1 \times 10^{-31} \times 1.67 \times 10^{-27}} \approx 2.3 \times 10^{39}$$

**Answer:** About  $2.3 \times 10^{39}$ . It is the ratio of the electric force to the gravitational force between an electron and a proton at the same distance. It shows that the **electric force is about  $10^{39}$  times stronger** than gravity.

## Exercise 1.4

**Question:** (a) Explain the meaning of the statement 'electric charge of a body is quantised'. (b) Why can one ignore quantisation of electric charge when dealing with macroscopic, i.e. large-scale, charges?

**Answer:**

(a) The charge on any body is always an **integral multiple** of the basic charge  $e$ :  $q = ne$ , where  $n = 0, \pm 1, \pm 2, \dots$  and  $e = 1.6 \times 10^{-19} \text{ C}$ . A body cannot have a charge like  $1.5e$ ; charge can change only in steps of  $e$ .

(b) Macroscopic charges are huge compared with  $e$ . For example,  $1 \mu\text{C} \approx 6 \times 10^{12}e$ . At this scale, a change of one  $e$  is negligible, so charge appears continuous and quantisation can be ignored.

## Exercise 1.5

**Question:** When a glass rod is rubbed with a silk cloth, charges appear on both. A similar phenomenon is observed with many other pairs of bodies. Explain how this observation is consistent with the law of conservation of charge.

**Answer:** Rubbing does not create charge; it only **transfers electrons** from one body to the other. Electrons pass from the glass rod to the silk, so the rod becomes positive and the silk becomes equally negative. Before rubbing, the total charge of the system (rod + silk) is zero. After rubbing, it is  $(+q) + (-q) = 0$ . The total charge of the isolated system is unchanged, so charge is conserved.

## Exercise 1.6

**Question:** Four point charges  $q_A = 2 \mu\text{C}$ ,  $q_B = -5 \mu\text{C}$ ,  $q_C = 2 \mu\text{C}$  and  $q_D = -5 \mu\text{C}$  are located at the corners of a square ABCD of side 10 cm. What is the force on a charge of  $1 \mu\text{C}$  placed at the centre of the square?

**Solution:** The centre O is equidistant from all four corners ( $r = 5\sqrt{2} \text{ cm}$ ).

- $q_A$  and  $q_C$  are equal and at opposite corners. They exert equal and opposite forces on the charge at O, so these cancel.
- $q_B$  and  $q_D$  are also equal and at opposite corners, so their forces cancel too.

**Answer:** The net force on the  $1 \mu\text{C}$  charge is **zero**.

## Exercise 1.7

**Question:** (a) An electrostatic field line is a continuous curve. That is, a field line cannot have sudden breaks. Why not? (b) Explain why two field lines never cross each other at any point.

**Answer:**

**(a)** A field line traces the path a small positive test charge tends to follow. The charge moves continuously and cannot jump from one point to another, so the field line has no sudden breaks. Also, the electric field exists at every point in a charge-free region, so the line must be continuous.

**(b)** The tangent to a field line gives the direction of  $\mathbf{E}$  at that point. If two lines crossed, there would be two tangents (two directions of the field) at the crossing point. This is impossible, because the field at a point has only one direction.

## Exercise 1.8

**Question:** Two point charges  $q_A = 3 \mu\text{C}$  and  $q_B = -3 \mu\text{C}$  are located 20 cm apart in vacuum. (a) What is the electric field at the midpoint O of the line AB joining the two charges? (b) If a negative test charge of magnitude  $1.5 \times 10^{-9} \text{ C}$  is placed at this point, what is the force experienced by the test charge?

**Solution:**

**(a)** Each charge is 0.1 m from O. The field due to  $q_A$  points away from A (toward B); the field due to  $q_B$  points toward B. Both are along AB:

$$E = 2 \times \frac{9 \times 10^9 \times 3 \times 10^{-6}}{(0.1)^2} = 2 \times 2.7 \times 10^6 = 5.4 \times 10^6 \text{ N C}^{-1}$$

**(b)**  $F = qE = 1.5 \times 10^{-9} \times 5.4 \times 10^6 = 8.1 \times 10^{-3} \text{ N}$ . The charge is negative, so the force is opposite to  $\mathbf{E}$ , i.e. along **OA** (toward A).

**Answer:** (a)  $5.4 \times 10^6 \text{ N C}^{-1}$  along OB; (b)  $8.1 \times 10^{-3} \text{ N}$  along OA.

## Exercise 1.9

**Question:** A system has two charges  $q_A = 2.5 \times 10^{-7} \text{ C}$  and  $q_B = -2.5 \times 10^{-7} \text{ C}$  located at points A: (0, 0, -15 cm) and B: (0, 0, +15 cm), respectively. What are the total charge and electric dipole moment of the system?

**Solution:** Total charge =  $2.5 \times 10^{-7} - 2.5 \times 10^{-7} = 0$ .

Separation  $2a = 30 \text{ cm} = 0.30 \text{ m}$ .

$$p = q \times 2a = 2.5 \times 10^{-7} \times 0.30 = 7.5 \times 10^{-8} \text{ C m}$$

The dipole moment points from the negative charge (B, at  $z = +15 \text{ cm}$ ) to the positive charge (A, at  $z = -15 \text{ cm}$ ), i.e. along the **negative z-axis**.

**Answer:** Total charge = 0;  $p = 7.5 \times 10^{-8} \text{ C m}$  along  $-z$ .

### Exercise 1.10

**Question:** An electric dipole with dipole moment  $4 \times 10^{-9} \text{ C m}$  is aligned at  $30^\circ$  with the direction of a uniform electric field of magnitude  $5 \times 10^4 \text{ N C}^{-1}$ . Calculate the magnitude of the torque acting on the dipole.

**Solution:**

$$\tau = pE \sin \theta = 4 \times 10^{-9} \times 5 \times 10^4 \times \sin 30^\circ = 2 \times 10^{-4} \times 0.5$$

**Answer:**  $\tau = 10^{-4} \text{ N m}$ .

### Exercise 1.11

**Question:** A polythene piece rubbed with wool is found to have a negative charge of  $3 \times 10^{-7} \text{ C}$ . (a) Estimate the number of electrons transferred (from which to which?). (b) Is there a transfer of mass from wool to polythene?

**Solution:**

(a)

$$n = \frac{q}{e} = \frac{3 \times 10^{-7}}{1.6 \times 10^{-19}} = 1.875 \times 10^{12}$$

The polythene is negative, so the electrons are transferred **from the wool to the polythene**.

(b) Yes. Electrons have mass, so a tiny mass is transferred:

$$m = n m_e = 1.875 \times 10^{12} \times 9.1 \times 10^{-31} = 1.7 \times 10^{-18} \text{ kg}$$

This is negligibly small.

**Answer:** (a) About  $2 \times 10^{12}$  electrons, from wool to polythene; (b) Yes, but only about  $1.7 \times 10^{-18} \text{ kg}$ .

### Exercise 1.12

**Question:** (a) Two insulated charged copper spheres A and B have their centres separated by a distance of 50 cm. What is the mutual force of electrostatic repulsion if the charge on each is  $6.5 \times 10^{-7} \text{ C}$ ? The radii of A and B are negligible compared with the distance of separation. (b) What is the force of repulsion if each sphere is charged double the above amount, and the distance between them is halved?

**Solution:**

(a)

$$F = \frac{9 \times 10^9 \times (6.5 \times 10^{-7})^2}{(0.5)^2} = \frac{9 \times 10^9 \times 4.225 \times 10^{-13}}{0.25} = 1.5 \times 10^{-2} \text{ N}$$

(b)  $F \propto \frac{q_1 q_2}{r^2}$ . Doubling both charges multiplies  $F$  by 4; halving  $r$  multiplies it by 4 again:

$$F' = 16F = 16 \times 1.52 \times 10^{-2} = 0.24 \text{ N}$$

**Answer:** (a)  $1.5 \times 10^{-2} \text{ N}$ ; (b)  $0.24 \text{ N}$ .

### Exercise 1.13

**Question:** Figure 1.30 shows tracks of three charged particles in a uniform electrostatic field. Give the signs of the three charges. Which particle has the highest charge to mass ratio?

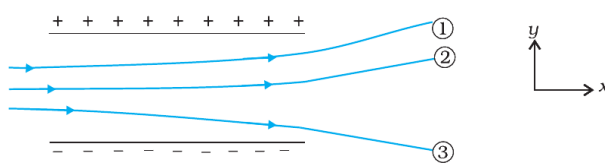


Fig. 1.30

**Answer:**

- Particles **1 and 2** bend toward the **positive plate**, so they are **negatively charged**.
- Particle **3** bends toward the **negative plate**, so it is **positively charged**.

The deflection is  $y = \frac{1}{2} \frac{qE}{m} \left(\frac{x}{v}\right)^2$ , so for the same speed, deflection  $\propto q/m$ . **Particle 3** shows the largest deflection, so it has the **highest charge to mass ratio**.

### Exercise 1.14

**Question:** Consider a uniform electric field  $\vec{E} = 3 \times 10^3 \hat{i}$  N/C. (a) What is the flux of this field through a square of 10 cm on a side whose plane is parallel to the  $yz$  plane? (b) What is the flux through the same square if the normal to its plane makes a  $60^\circ$  angle with the  $x$ -axis?

**Solution:** Area  $S = (0.1)^2 = 10^{-2} \text{ m}^2$ .

(a) The plane is parallel to the  $yz$  plane, so its normal is along  $x$  ( $\theta = 0^\circ$ ):

$$\phi = ES \cos 0^\circ = 3 \times 10^3 \times 10^{-2} = 30 \text{ N m}^2 \text{C}^{-1}$$

(b)  $\theta = 60^\circ$ :

$$\phi = ES \cos 60^\circ = 30 \times 0.5 = 15 \text{ N m}^2 \text{C}^{-1}$$

**Answer:** (a)  $30 \text{ N m}^2/\text{C}$ ; (b)  $15 \text{ N m}^2/\text{C}$ .

### Exercise 1.15

**Question:** What is the net flux of the uniform electric field of Exercise 1.14 through a cube of side 20 cm oriented so that its faces are parallel to the coordinate planes?

**Answer:** The field is uniform, so the number of field lines entering the cube equals the number leaving it. The flux through the face at the lower  $x$  is  $-ES$  and through the opposite face is  $+ES$ ; the other four faces have zero flux. Also, the cube encloses no charge. So the **net flux is zero**.

### Exercise 1.16

**Question:** Careful measurement of the electric field at the surface of a black box indicates that the net outward flux through the surface of the box is  $8.0 \times 10^3 \text{ N m}^2/\text{C}$ . (a) What is the net charge inside the box? (b) If the net outward flux through the surface of the box were zero, could you conclude that there were no charges inside the box? Why or why not?

**Solution:**

(a) By Gauss's law,

$$q = \epsilon_0 \phi = 8.854 \times 10^{-12} \times 8.0 \times 10^3 = 7.08 \times 10^{-8} \text{ C} \approx 0.07 \mu\text{C}$$

(b) **No.** Zero net flux only means the **net** charge inside is zero. The box may contain equal amounts of positive and negative charges.

**Answer:** (a)  $7.08 \times 10^{-8} \text{ C}$ ; (b) No, only the net charge must be zero.

### Exercise 1.17

**Question:** A point charge  $+10\ \mu\text{C}$  is at a distance 5 cm directly above the centre of a square of side 10 cm, as shown in Fig. 1.31. What is the magnitude of the electric flux through the square? (Hint: Think of the square as one face of a cube with edge 10 cm.)

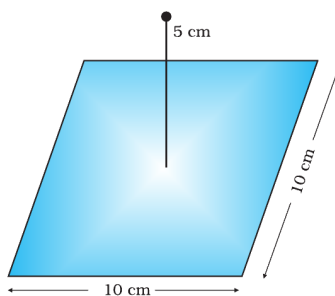


Fig. 1.31

**Solution:** The charge is at the centre of an imaginary cube of edge 10 cm, of which the square is one face. By Gauss's law, the total flux through the cube is  $q/\epsilon_0$ . By symmetry, each of the 6 faces gets an equal share:

$$\phi = \frac{q}{6\epsilon_0} = \frac{10 \times 10^{-6}}{6 \times 8.854 \times 10^{-12}}$$

**Answer:**  $\phi = 1.88 \times 10^5\ \text{N m}^2\ \text{C}^{-1}$ .

### Exercise 1.18

**Question:** A point charge of  $2.0\ \mu\text{C}$  is at the centre of a cubic Gaussian surface 9.0 cm on edge. What is the net electric flux through the surface?

**Solution:**

$$\phi = \frac{q}{\epsilon_0} = \frac{2.0 \times 10^{-6}}{8.854 \times 10^{-12}}$$

The size of the cube does not matter.

**Answer:**  $\phi = 2.26 \times 10^5\ \text{N m}^2\ \text{C}^{-1}$ .

### Exercise 1.19

**Question:** A point charge causes an electric flux of  $-1.0 \times 10^3\ \text{N m}^2/\text{C}$  to pass through a spherical Gaussian surface of 10.0 cm radius centred on the charge. (a) If the radius of the Gaussian surface were doubled, how much flux would pass through the surface? (b) What is the value of the point charge?

**Solution:**

**(a)** Flux depends only on the charge enclosed, not on the size of the surface. So the flux is still  $-1.0 \times 10^3\ \text{N m}^2/\text{C}$ .

**(b)**

$$q = \epsilon_0 \phi = 8.854 \times 10^{-12} \times (-1.0 \times 10^3) = -8.854 \times 10^{-9}\ \text{C}$$

**Answer:** (a)  $-1.0 \times 10^3\ \text{N m}^2/\text{C}$ ; (b)  $-8.8\ \text{nC}$ .

### Exercise 1.20

**Question:** A conducting sphere of radius 10 cm has an unknown charge. If the electric field 20 cm from the centre of the sphere is  $1.5 \times 10^3\ \text{N/C}$  and points radially inward, what is the net charge on the sphere?

**Solution:** Outside the sphere,  $E = \frac{kq}{r^2}$ , with  $r = 0.20$  m:

$$q = \frac{Er^2}{k} = \frac{1.5 \times 10^3 \times (0.20)^2}{9 \times 10^9} = 6.67 \times 10^{-9} \text{ C}$$

The field points radially inward, so the charge is **negative**.

**Answer:**  $q = -6.67 \times 10^{-9} \text{ C} = -6.67 \text{ nC}$ .

## Exercise 1.21

**Question:** A uniformly charged conducting sphere of 2.4 m diameter has a surface charge density of  $80.0 \mu\text{C}/\text{m}^2$ . (a) Find the charge on the sphere. (b) What is the total electric flux leaving the surface of the sphere?

**Solution:**  $R = 1.2$  m.

(a)

$$q = \sigma \times 4\pi R^2 = 80.0 \times 10^{-6} \times 4 \times 3.14 \times (1.2)^2 = 1.45 \times 10^{-3} \text{ C}$$

(b)

$$\phi = \frac{q}{\epsilon_0} = \frac{1.45 \times 10^{-3}}{8.854 \times 10^{-12}} = 1.6 \times 10^8 \text{ N m}^2 \text{ C}^{-1}$$

**Answer:** (a)  $1.45 \times 10^{-3} \text{ C}$ ; (b)  $1.6 \times 10^8 \text{ N m}^2 \text{ C}^{-1}$ .

## Exercise 1.22

**Question:** An infinite line charge produces a field of  $9 \times 10^4 \text{ N/C}$  at a distance of 2 cm. Calculate the linear charge density.

**Solution:**

$$E = \frac{\lambda}{2\pi\epsilon_0 r} = \frac{2k\lambda}{r} \Rightarrow \lambda = \frac{Er}{2k} = \frac{9 \times 10^4 \times 0.02}{2 \times 9 \times 10^9}$$

**Answer:**  $\lambda = 10^{-7} \text{ C/m} = 0.1 \mu\text{C/m}$ .

## Exercise 1.23

**Question:** Two large, thin metal plates are parallel and close to each other. On their inner faces, the plates have surface charge densities of opposite signs and of magnitude  $17.0 \times 10^{-22} \text{ C/m}^2$ . What is **E**: (a) in the outer region of the first plate, (b) in the outer region of the second plate, and (c) between the plates?

**Solution:** Each plate (as a sheet) produces  $\sigma/2\epsilon_0$ .

- **Outside** the plates, the fields of  $+\sigma$  and  $-\sigma$  are equal and opposite, so they cancel.
- **Between** the plates, they are in the same direction and add:

$$E = \frac{\sigma}{2\epsilon_0} + \frac{\sigma}{2\epsilon_0} = \frac{\sigma}{\epsilon_0} = \frac{17.0 \times 10^{-22}}{8.854 \times 10^{-12}} = 1.92 \times 10^{-10} \text{ N C}^{-1}$$

**Answer:** (a) zero; (b) zero; (c)  $1.92 \times 10^{-10} \text{ N/C}$ , directed from the positive plate to the negative plate.