



NEET — JEE MAINS - ADVANCED

Chapter 2 : Electrostatic Potential and Capacitance (Notes)

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Notes

GSEB and CBSE Board Exam preparation subjective notes · Class 12 Physics · NCERT Chapter 2

Q.1 Define electrostatic potential energy and electrostatic potential. Why is the electrostatic force conservative? (3 Marks)

Conservative force: A force for which the work done in moving a body from one point to another depends only on the initial and final points, not on the path. The Coulomb force between stationary charges is conservative, like the gravitational and spring forces. Both Coulomb's law and the law of gravitation have an inverse-square dependence on distance.

Potential energy difference: Let a test charge q be moved from R to P against the repulsive force of a charge Q (Fig. 2.1). An external force $\vec{F}_{ext} = -\vec{F}_E$ is applied, just enough to balance the electric force, so the charge moves without acceleration.

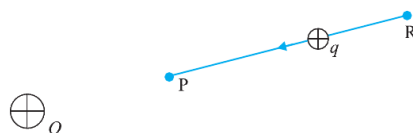


Fig. 2.1: A test charge $q (> 0)$ is moved from the point R to the point P against the repulsive force on it by the charge $Q (> 0)$ placed at the origin

$$W_{RP} = \int_R^P \vec{F}_{ext} \cdot d\vec{r} = - \int_R^P \vec{F}_E \cdot d\vec{r}$$

$$\Delta U = U_P - U_R = W_{RP}$$

Electrostatic potential energy difference between two points is the work done by an external force in moving (without acceleration) a charge q from one point to the other.

Electrostatic potential energy at a point: Taking the potential energy zero at infinity,

$$W_{\infty P} = U_P - U_{\infty} = U_P$$

It is the work done by the external force in bringing the charge q from infinity to that point.

Electrostatic potential (V): Work done per unit test charge. **The work done by an external force in bringing a unit positive charge (without acceleration) from infinity to a point is the electrostatic potential at that point.**

$$V_P - V_R = \frac{U_P - U_R}{q}$$

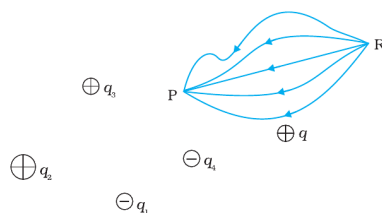


Fig. 2.2: Work done on a test charge q by the electrostatic field due to any given charge configuration is independent of the path, and depends only on its initial and final positions

Key Examination Points for Full Marks:

- Potential energy and potential are defined only up to an **additive constant**. Only **differences** are physically significant.
- **SI unit of potential:** volt (V) = J C⁻¹. **Dimensions:** [M L²T⁻³A⁻¹]. Potential is a **scalar**.
- **1 volt:** the potential at a point is 1 V if 1 J of work is done in bringing 1 C of positive charge from infinity to that point.

Q.2 Derive an expression for the potential due to a point charge. Draw the graph of V and E versus r . (3 Marks)

Let a point charge Q be at the origin O (Fig. 2.3). To find the potential at P (distance r), bring a unit positive charge from infinity to P along the radial direction (work is path-independent).

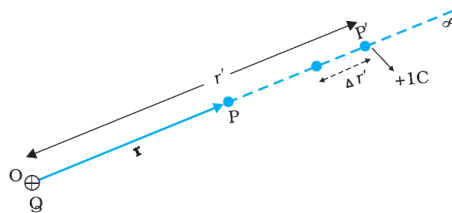


Fig. 2.3: Work done in bringing a unit positive test charge from infinity to the point P , against the repulsive force of charge Q ($Q > 0$), is the potential at P due to the charge Q

At an intermediate point P' (distance r'), the force on the unit positive charge is

$$\vec{F} = \frac{Q \times 1}{4\pi\epsilon_0 r'^2} \hat{r}'$$

Work done against this force from r' to $r' + \Delta r'$:

$$\Delta W = -\frac{Q}{4\pi\epsilon_0 r'^2} \Delta r'$$

(The negative sign appears because $\Delta r' < 0$ while ΔW is positive.) Total work, integrating from $r' = \infty$ to $r' = r$:

$$W = -\int_{\infty}^r \frac{Q}{4\pi\epsilon_0 r'^2} dr' = \frac{Q}{4\pi\epsilon_0 r'} \Big|_{\infty}^r = \frac{Q}{4\pi\epsilon_0 r}$$

$$\boxed{V(r) = \frac{1}{4\pi\epsilon_0} \frac{Q}{r}}$$

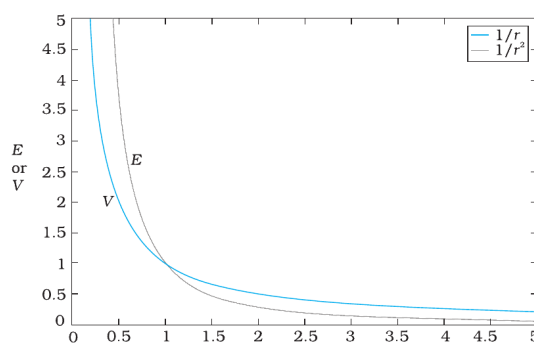


Fig. 2.4: Variation of potential V with r [in units of $(Q/4\pi\epsilon_0) \text{ m}^{-1}$] (blue curve) and field with r [in units of $(Q/4\pi\epsilon_0) \text{ m}^{-2}$] (black curve) for a point charge Q

Key Examination Points for Full Marks:

- The result holds for **any sign** of Q . For $Q < 0$, $V < 0$.
- $V \propto \frac{1}{r}$ while $E \propto \frac{1}{r^2}$ — the field falls off faster.
- The equipotential surfaces of a point charge are spheres centred on the charge.

Illustration (NCERT Example 2.1): For $Q = 4 \times 10^{-7} \text{ C}$ at 9 cm, $V = 9 \times 10^9 \times \frac{4 \times 10^{-7}}{0.09} = 4 \times 10^4 \text{ V}$. Work to bring $2 \times 10^{-9} \text{ C}$ from infinity: $W = qV = 8 \times 10^{-5} \text{ J}$ — independent of the path.

Q.3 Derive an expression for the potential due to an electric dipole at any point. Discuss the special cases. (3 Marks)

Take the origin at the centre of the dipole (charges q and $-q$ separated by $2a$). Let P be at distance r from O , with OP making angle θ with $\vec{p}$ (Fig. 2.5).

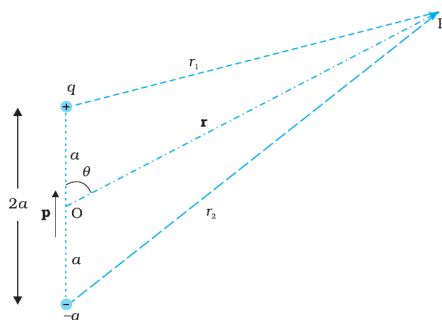


Fig. 2.5: Quantities involved in the calculation of potential due to a dipole

By superposition,

$$V = \frac{1}{4\pi\epsilon_0} \left(\frac{q}{r_1} - \frac{q}{r_2} \right)$$

By geometry,

$$r_1^2 = r^2 + a^2 - 2ar\cos\theta, \quad r_2^2 = r^2 + a^2 + 2ar\cos\theta$$

For $r \gg a$, keeping terms up to first order in a/r :

$$r_1^2 \cong r^2 \left(1 - \frac{2a\cos\theta}{r} \right), \quad r_2^2 \cong r^2 \left(1 + \frac{2a\cos\theta}{r} \right)$$

Using the binomial theorem,

$$\frac{1}{r_1} \cong \frac{1}{r} \left(1 + \frac{a}{r} \cos\theta \right), \quad \frac{1}{r_2} \cong \frac{1}{r} \left(1 - \frac{a}{r} \cos\theta \right)$$

$$V = \frac{q}{4\pi\epsilon_0} \frac{2a\cos\theta}{r^2} = \frac{p\cos\theta}{4\pi\epsilon_0 r^2}$$

Since $p \cos \theta = \vec{p} \cdot \hat{r}$:

$$V = \frac{1}{4\pi\epsilon_0} \frac{\vec{p} \cdot \hat{r}}{r^2} \quad (r \gg a)$$

Special cases:

- **On the axis** ($\theta = 0$ or π): $V = \pm \frac{1}{4\pi\epsilon_0} \frac{p}{r^2}$ (+ for $\theta = 0$, - for $\theta = \pi$).
- **On the equatorial plane** ($\theta = \pi/2$): $V = 0$.

Contrast with a single charge:

1. The dipole potential depends on **r and the angle** between $\vec{r}$ and $\vec{p}$ (it is axially symmetric about $\vec{p}$).
2. It falls off as $\frac{1}{r^2}$, not as $\frac{1}{r}$.

Q.4 Obtain the potential due to a system of charges and due to a charged spherical shell. (2 Marks)

System of charges: For charges $q_1, q_2, \dots, q_n$ at distances $r_{1P}, r_{2P}, \dots, r_{nP}$ from P, by the superposition principle the potential is the **algebraic sum**:

$$V = V_1 + V_2 + \dots + V_n = \frac{1}{4\pi\epsilon_0} \left(\frac{q_1}{r_{1P}} + \frac{q_2}{r_{2P}} + \dots + \frac{q_n}{r_{nP}} \right)$$

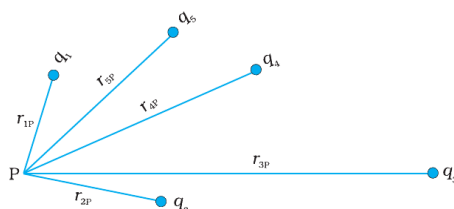


Fig. 2.6: Potential at a point due to a system of charges is the sum of potentials due to individual charges

For a continuous distribution with density $\rho(r)$, divide it into small volume elements Δv carrying $\rho \Delta v$ and add (integrate) their potentials.

Uniformly charged spherical shell (charge q, radius R):

- **Outside** ($r \geq R$): the shell behaves like a point charge at its centre:

$$V = \frac{1}{4\pi\epsilon_0} \frac{q}{r}$$

- **Inside** ($r < R$): $E = 0$, so no work is done in moving a charge inside. The potential is **constant** and equals its value at the surface:

$$V = \frac{1}{4\pi\epsilon_0} \frac{q}{R}$$

Illustration (NCERT Example 2.2): For 3×10^{-8} C and -2×10^{-8} C, 15 cm apart, $V = 0$ at **9 cm** (between) and **45 cm** (outside) from the positive charge, on the side of the negative charge.

Q.5 What are equipotential surfaces? Write their properties with examples.(3 Marks)

Equipotential surface: A surface with a constant value of potential at all points on it.

Examples:

- **Single point charge:** $V = \frac{1}{4\pi\epsilon_0} \frac{q}{r}$ is constant if r is constant, so the equipotential surfaces are **concentric spheres** centred at the charge (Fig. 2.9).
- **Uniform field** (along x): planes normal to the x -axis, i.e. parallel to the y - z plane (Fig. 2.10).
- **Dipole and two identical positive charges:** shown in Fig. 2.11.

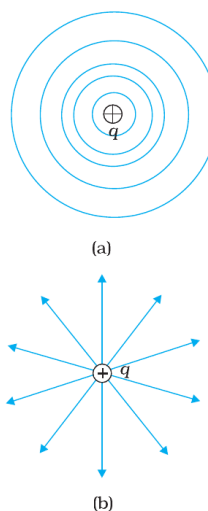


Fig. 2.9: For a single charge q (a) equipotential surfaces are spherical surfaces centred at the charge, and (b) electric field lines are radial, starting from the charge if $q > 0$

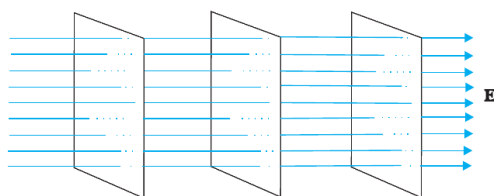


Fig. 2.10: Equipotential surfaces for a uniform electric field

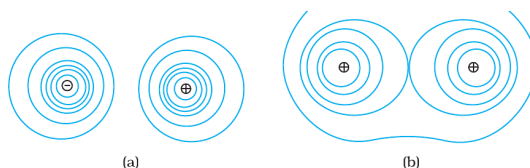


Fig. 2.11: Some equipotential surfaces for (a) a dipole, (b) two identical positive charges

Properties:

1. **No work** is done in moving a test charge on an equipotential surface ($\Delta V = 0$).
2. **The electric field is always normal to the equipotential surface** at every point. (If E had a component along the surface, work would be needed to move a charge along it — contradicting the definition.)
3. Two equipotential surfaces **never intersect** (a point cannot have two potentials).
4. Equipotential surfaces are **closer together** where the field is **stronger**.

Q.6 Derive the relation between electric field and potential. (2 Marks)

Consider two closely spaced equipotential surfaces A and B with potentials V and $V + \delta V$, where δV is the change in V in the direction of $\mathbf{E}$ (Fig. 2.12). Let δl be the perpendicular distance between them.

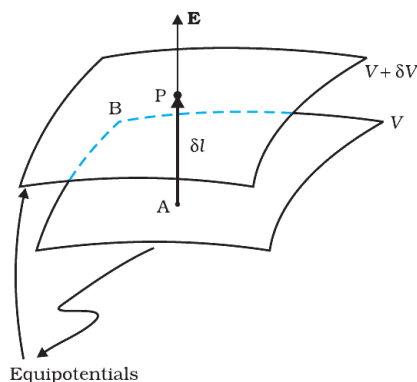


Fig. 2.12: From the potential to the field

Move a unit positive charge along this perpendicular from B to A against the field. Work done = $|\vec{E}| \delta l$, and this equals $V_A - V_B$:

$$|\vec{E}| \delta l = V - (V + \delta V) = -\delta V$$

$$|\vec{E}| = -\frac{\delta V}{\delta l} = +\frac{|\delta V|}{\delta l}$$

Conclusions:

1. The electric field is in the direction in which the potential decreases steepest.
2. Its magnitude is the change in the magnitude of potential per unit displacement normal to the equipotential surface at that point.

Key Examination Points for Full Marks:

- In one dimension, $E = -\frac{dV}{dx}$. The negative sign shows that V decreases along E.
- The SI unit of E can therefore also be written as V m^{-1} ($= \text{N C}^{-1}$).
- If V is constant in a region, $E = 0$ there.

Q.7 Derive the potential energy of a system of two and three point charges.(3 Marks)

Two charges: Bring the charges from infinity one by one.

- Bringing q_1 to $\vec{r}_1$: no field yet, so work = 0.
- Bringing q_2 to $\vec{r}_2$: work = $q_2 \times (\text{potential at } \vec{r}_2 \text{ due to } q_1) = q_2 \cdot \frac{1}{4\pi\epsilon_0} \frac{q_1}{r_{12}}$.

Since the electrostatic force is conservative, this work is stored as the potential energy of the system:

$$U = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r_{12}}$$

Three charges: Bringing q_3 to $\vec{r}_3$ needs work $q_3 V_{1,2}(\vec{r}_3) = \frac{1}{4\pi\epsilon_0} \left(\frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}} \right)$. Adding all steps:

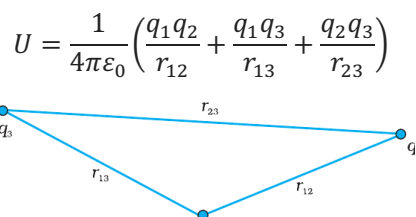


Fig. 2.14: Potential energy of a system of three charges is given by Eq. (2.26), with the notation given in the figure

Key Examination Points for Full Marks:

- U is the same **whatever the order** in which the charges are brought (path independence). It is a property of the present configuration.

- Like charges ($q_1 q_2 > 0$): $U > 0$ (work is done against repulsion).
- Unlike charges ($q_1 q_2 < 0$): $U < 0$ (attraction; work is needed to separate them).
- For n charges, add the term for **every pair** once.

Illustration (NCERT Example 2.4): For $+q, -q, +q, -q$ at the corners of a square of side d , $U = \frac{-q^2}{4\pi\epsilon_0 d} (4 - \sqrt{2})$. No extra work is needed to bring q_0 to the centre, because $V = 0$ there.

Q.8 Obtain the potential energy of (a) a single charge and (b) a system of two charges in an external field. Define electron volt. (3 Marks)

External field: The field $\mathbf{E}$ (and potential V) is produced by sources external to the given charges. The given charge does not disturb those sources.

(a) Single charge: Work done in bringing q from infinity to P is qV . So

$$\text{Potential energy of } q \text{ at } \vec{r} = qV(\vec{r})$$

where $V(\vec{r})$ is the **external** potential at $\vec{r}$.

Electron volt (eV): The energy gained by an electron accelerated through a potential difference of 1 V:

$$1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$$

$$1 \text{ keV} = 10^3 \text{ eV}, 1 \text{ MeV} = 10^6 \text{ eV}, 1 \text{ GeV} = 10^9 \text{ eV}, 1 \text{ TeV} = 10^{12} \text{ eV}.$$

(b) Two charges q_1, q_2 in an external field:

- Bringing q_1 to $\vec{r}_1$: work = $q_1 V(\vec{r}_1)$.
- Bringing q_2 to $\vec{r}_2$: work is done against the external field **and** against the field of q_1 : $q_2 V(\vec{r}_2) + \frac{q_1 q_2}{4\pi\epsilon_0 r_{12}}$.

$$U = q_1 V(\vec{r}_1) + q_2 V(\vec{r}_2) + \frac{q_1 q_2}{4\pi\epsilon_0 r_{12}}$$

Illustration (NCERT Example 2.5): For $7 \mu\text{C}$ and $-2 \mu\text{C}$, 18 cm apart (no external field): $U = 9 \times 10^9 \times \frac{7 \times (-2) \times 10^{-12}}{0.18} = -0.7 \text{ J}$. Work to separate them to infinity = **0.7 J**.

Q.9 Derive the potential energy of a dipole in a uniform electric field.(3 Marks)

A dipole ($+q, -q$, separation $2a$) is placed in a uniform field $\mathbf{E}$ (Fig. 2.16). Net force = 0, but the torque is

$$\vec{\tau} = \vec{p} \times \vec{E}$$

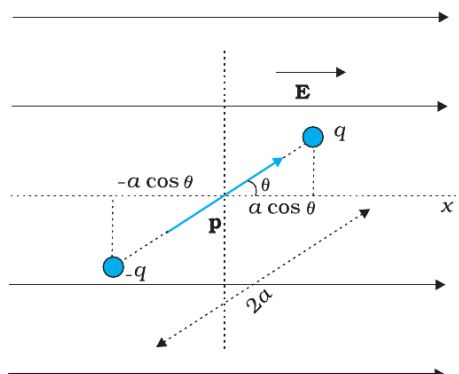


Fig. 2.16: Potential energy of a dipole in a uniform external field

Suppose an external torque τ_{ext} , just neutralising this torque, rotates the dipole from θ_0 to θ_1 at infinitesimal angular speed (no angular acceleration). Work done:

$$W = \int_{\theta_0}^{\theta_1} \tau_{ext}(\theta) d\theta = \int_{\theta_0}^{\theta_1} p E \sin\theta d\theta = pE(\cos\theta_0 - \cos\theta_1)$$

This work is stored as potential energy. Taking $\theta_0 = \pi/2$ (where $U = 0$):

$$U(\theta) = pE \left(\cos \frac{\pi}{2} - \cos\theta \right)$$

$$U(\theta) = -pE \cos\theta = -\vec{p} \cdot \vec{E}$$

Key Examination Points for Full Marks:

Position	θ	U	Equilibrium
$\vec{p}$ parallel to $\vec{E}$	0°	$-pE$ (minimum)	Stable
$\vec{p}$ perpendicular to $\vec{E}$	90°	0	—
$\vec{p}$ antiparallel to $\vec{E}$	180°	$+pE$ (maximum)	Unstable

- **Work done in rotating** a dipole from θ_1 to θ_2 : $W = pE(\cos\theta_1 - \cos\theta_2)$.
- $\theta_0 = \pi/2$ is chosen because there the work done against the external field in bringing $+q$ and $-q$ are equal and opposite and cancel.

Illustration (NCERT Example 2.6): One mole of molecules ($p = 10^{-29}$ C m each, total 6×10^{-6} C m) in 10^6 V m $^{-1}$. Turning the field by 60° changes U from -6 J to -3 J. The **3 J** lost is released as heat.

Q.10 Write the electrostatic properties of a conductor. (3 Marks)

In metals, the outer (valence) electrons are **free** to move within the metal but cannot leave it. In an external field they drift opposite to the field.

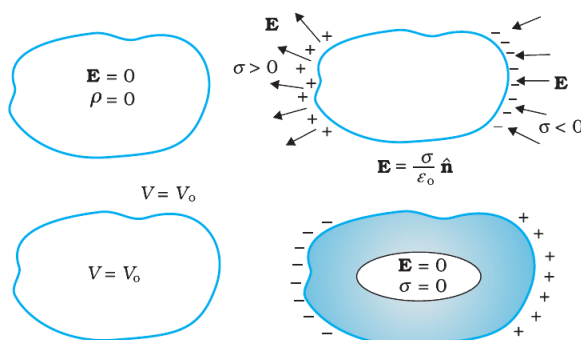


Fig. 2.19: Some important electrostatic properties of a conductor

- 1. Inside a conductor, the electrostatic field is zero.** In the static situation, the free charges distribute themselves so that $E = 0$ everywhere inside. This can be taken as the **defining property** of a conductor.
- 2. At the surface of a charged conductor, E is normal to the surface** at every point. If E had a tangential component, the free surface charges would move.
- 3. The interior of a conductor can have no excess charge.** Take any small volume inside; on its surface $E = 0$, so the flux is zero and by Gauss's law the enclosed charge is zero. **Any excess charge resides on the surface.**
- 4. The potential is constant throughout the volume of a conductor** and has the same value on its surface. ($E = 0$ inside and no tangential component on the surface, so no work is done in moving a charge within or on it.)
- 5. Electric field at the surface of a charged conductor:**

$$\vec{E} = \frac{\sigma}{\epsilon_0} \hat{n}$$

where σ is the surface charge density and $\hat{n}$ is the outward normal.

6. Electrostatic shielding: The field inside a charge-free **cavity** of a conductor is zero, whatever the charge on the conductor or the external field.

Q.11 Derive the electric field at the surface of a charged conductor. What is electrostatic shielding? (2 Marks)

Derivation: Choose a **pill box** (a short cylinder) as the Gaussian surface about a point P on the surface. It is partly inside and partly outside the conductor, with a small area of cross-section δS and negligible height (Fig. 2.17).

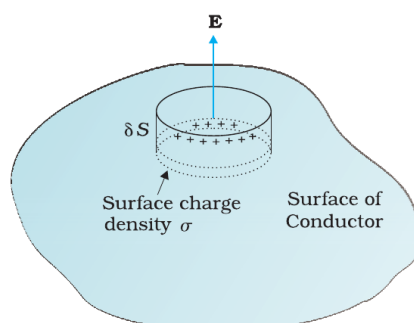


Fig. 2.17: The Gaussian surface (a pill box) chosen to derive Eq. (2.35) for electric field at the surface of a charged conductor

- Just inside the surface, $E = 0$.
- Just outside, E is normal to the surface. Only the outer cross-section contributes to the flux: $\pm E \delta S$.
- Charge enclosed = $\sigma \delta S$.

By Gauss's law,

$$E \delta S = \frac{|\sigma| \delta S}{\epsilon_0} \Rightarrow \boxed{E = \frac{|\sigma|}{\epsilon_0}}$$

For $\sigma > 0$, E is normal to the surface **outward**; for $\sigma < 0$, **inward**.

Electrostatic shielding: The electric field inside a cavity of a conductor (with no charges in the cavity) is **always zero**, whatever the size and shape of the cavity and whatever the charge or external field. All charges reside only on the **outer** surface (Fig. 2.18).

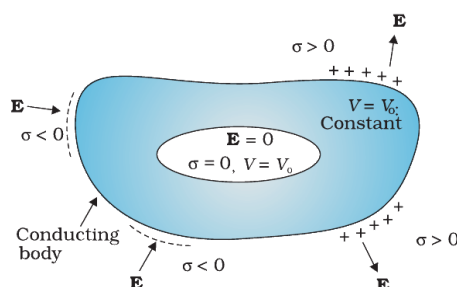


Fig. 2.18: The electric field inside a cavity of any conductor is zero. All charges reside only on the outer surface of a conductor with cavity

Applications: Sensitive instruments are protected from outside electrical influence by enclosing them in metal boxes. During a thunderstorm, it is safest to stay inside a car (a metal body).

Note: Shielding does not work the other way round: charges inside the cavity are **not** shielded from the exterior.

Illustration (NCERT Example 2.7): Aircraft tyres are made slightly conducting, and vehicles carrying inflammable materials have metallic ropes touching the ground, to conduct away static charge before a

spark occurs. A bird on a bare high-power line is safe because there is no potential difference across its body.

Q.12 What are dielectrics? Distinguish between polar and non-polar molecules. Explain polarisation. (3 Marks)

Dielectrics: Non-conducting substances with no (or negligible) free charge carriers.

Conductor vs dielectric in an external field (Fig. 2.20):

- In a **conductor**, free charges move and the induced field **exactly cancels** the external field inside: $E_0 + E_{in} = 0$.
- In a **dielectric**, the field induces dipole moments (by stretching or reorienting molecules). The induced field **opposes** the external field but only **reduces** it: $E_0 + E_{in} \neq 0$.

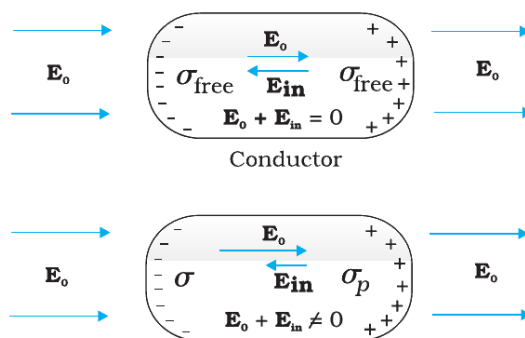


Fig. 2.20: Difference in behaviour of a conductor and a dielectric in an external electric field

Non-polar molecule	Polar molecule
Centres of positive and negative charge coincide .	Centres of positive and negative charge are separated .
No permanent dipole moment.	Has a permanent dipole moment.
In a field, develops an induced dipole moment.	In a field, dipoles tend to align with the field.
Examples: O ₂ , H ₂ , CO ₂ .	Examples: HCl, H ₂ O.

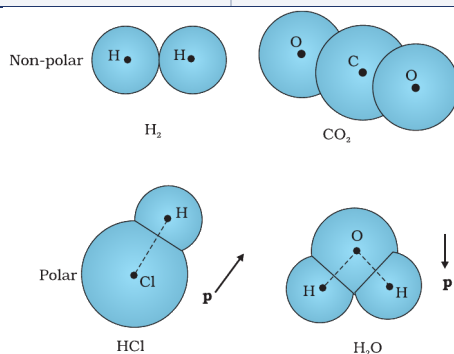


Fig. 2.21: Some examples of polar and non-polar molecules

Polarisation (P): The dipole moment per unit volume. For linear isotropic dielectrics,

$$\vec{P} = \epsilon_0 \chi_e \vec{E}$$

where χ_e is the **electric susceptibility** of the dielectric.

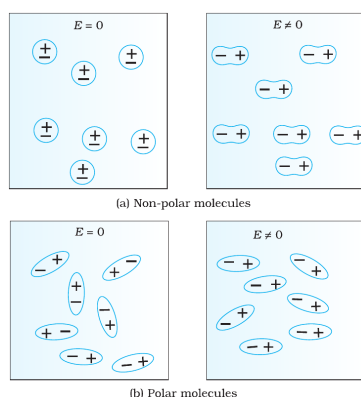


Fig. 2.22: A dielectric develops a net dipole moment in an external electric field. (a) Non-polar molecules, (b) Polar molecules

Polar dielectrics: In the absence of a field, the permanent dipoles are oriented randomly (thermal agitation), so the net dipole moment is zero. In a field, they align partly. The extent depends on the dipole potential energy (aligning) versus thermal energy (disrupting).

Induced surface charges: In a uniformly polarised slab, there is no net volume charge, but the surfaces normal to the field carry induced (bound) charge densities $+\sigma_p$ and $-\sigma_p$. These produce a field opposite to the external field (Fig. 2.23).

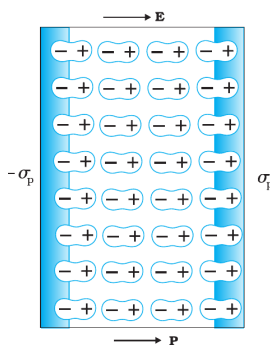


Fig. 2.23: A uniformly polarised dielectric amounts to induced surface charge density, but no volume charge density

Q.13 What is a capacitor? Define capacitance and dielectric strength.(2 Marks)

Capacitor: A system of **two conductors separated by an insulator**. In practice the conductors have charges Q and $-Q$ with potential difference $V = V_1 - V_2$.

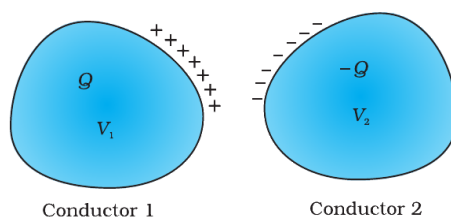


Fig. 2.24: A system of two conductors separated by an insulator forms a capacitor

Capacitance: The field between the conductors is proportional to Q , so V is also proportional to Q . The constant ratio

$$C = \frac{Q}{V}$$

is the **capacitance**. It depends only on the **geometry** (shape, size, separation) of the conductors and on the **dielectric** between them — not on Q or V .

SI unit: farad (F). $1 \text{ F} = 1 \text{ C V}^{-1}$. **Dimensions:** $[M^{-1}L^{-2}T^4A^2]$. 1 F is a very large unit; we use $1 \mu\text{F} = 10^{-6} \text{ F}$, $1 \text{ nF} = 10^{-9} \text{ F}$, $1 \text{ pF} = 10^{-12} \text{ F}$.

Symbol: fixed capacitor —| |—; variable capacitor with an arrow across.

Dielectric strength: The **maximum electric field** that a dielectric medium can withstand without breakdown of its insulating property. For air, it is about $3 \times 10^6 \text{ V m}^{-1}$.

Key Examination Points for Full Marks:

- A large C can store a large charge at a **small V** . A high V means a strong field, which can ionise the surrounding air and cause the charge to **leak**.
- So there is a limit to the charge a capacitor can store without leaking.

Q.14 Derive an expression for the capacitance of a parallel plate capacitor.(3 Marks)

A parallel plate capacitor has two large plane parallel plates, each of area A , separated by a small distance d ($d^2 \ll A$) (Fig. 2.25). The plates carry Q and $-Q$, so the surface charge densities are $\sigma = Q/A$ and $-\sigma$.

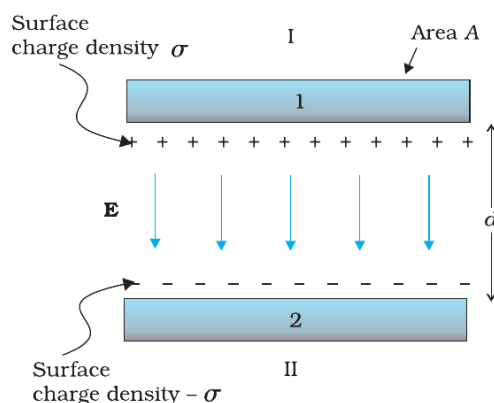


Fig. 2.25: The parallel plate capacitor

Using the field of an infinite plane sheet ($\sigma/2\epsilon_0$):

- **Outer region I** (above plate 1): $E = \frac{\sigma}{2\epsilon_0} - \frac{\sigma}{2\epsilon_0} = 0$
- **Outer region II** (below plate 2): $E = \frac{\sigma}{2\epsilon_0} - \frac{\sigma}{2\epsilon_0} = 0$
- **Between the plates:**

$$E = \frac{\sigma}{2\epsilon_0} + \frac{\sigma}{2\epsilon_0} = \frac{\sigma}{\epsilon_0} = \frac{Q}{\epsilon_0 A}$$

directed from the positive to the negative plate.

The field is uniform, so the potential difference is

$$V = Ed = \frac{1}{\epsilon_0} \frac{Qd}{A}$$

$$\boxed{C = \frac{Q}{V} = \frac{\epsilon_0 A}{d}}$$

Key Examination Points for Full Marks:

- C depends only on the geometry: $C \propto A$ and $C \propto \frac{1}{d}$.
- Near the edges, field lines bend outward (**fringing of the field**); this is ignored for $d^2 \ll A$.
- For $A = 1 \text{ m}^2$, $d = 1 \text{ mm}$: $C = 8.85 \times 10^{-9} \text{ F}$. For $C = 1 \text{ F}$ with $d = 1 \text{ cm}$, $A = 10^9 \text{ m}^2$ (a plate about 30 km on each side!) — so 1 F is a very large unit.

Q.15 Explain the effect of a dielectric on capacitance. Define dielectric constant. (3 Marks)

Without dielectric (vacuum): $E_0 = \frac{\sigma}{\epsilon_0}$, $V_0 = E_0 d$, and

$$C_0 = \frac{Q}{V_0} = \epsilon_0 \frac{A}{d}$$

With a dielectric filling the space: The dielectric is polarised. The effect is equivalent to two charged sheets with induced densities σ_p and $-\sigma_p$. The net surface charge density becomes $\pm(\sigma - \sigma_p)$:

$$E = \frac{\sigma - \sigma_p}{\epsilon_0}, \quad V = Ed = \frac{\sigma - \sigma_p}{\epsilon_0} d$$

For linear dielectrics, $\sigma_p \propto \sigma$, so we write $\sigma - \sigma_p = \frac{\sigma}{K}$ with $K > 1$:

$$V = \frac{\sigma d}{\epsilon_0 K} = \frac{Qd}{A\epsilon_0 K}$$

$$C = \frac{Q}{V} = \frac{\epsilon_0 KA}{d} = KC_0$$

Permittivity of the medium: $\epsilon = \epsilon_0 K$. For vacuum, $K = 1$.

Dielectric constant (K):

$$K = \frac{\epsilon}{\epsilon_0} = \frac{C}{C_0}$$

The dielectric constant is the factor (> 1) by which the capacitance increases from its vacuum value when the dielectric fills the space between the plates. It is dimensionless. This holds for any type of capacitor.

Illustration (NCERT Example 2.8): A slab of dielectric constant K and thickness $3/4 d$ is inserted:

$$V = E_0 \left(\frac{d}{4}\right) + \frac{E_0}{K} \left(\frac{3d}{4}\right) = V_0 \frac{K+3}{4K} \Rightarrow C = \frac{4K}{K+3} C_0$$

In general, for a slab of thickness t : $C = \frac{\epsilon_0 A}{d-t+t/K}$.

Q.16 Derive the effective capacitance for (a) series and (b) parallel combination of capacitors. (3 Marks)

(a) Series combination: The left plate of C_1 and the right plate of C_2 are connected to the battery terminals. The charge Q is the same on each capacitor (Fig. 2.26). The total potential drop is the sum:

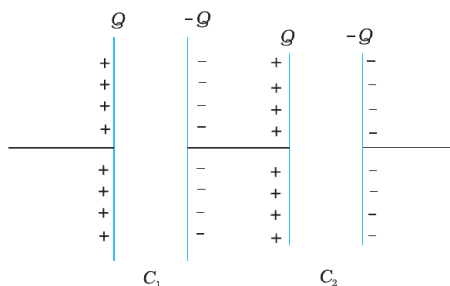


Fig. 2.26: Combination of two capacitors in series

$$V = V_1 + V_2 = \frac{Q}{C_1} + \frac{Q}{C_2}$$

The combination is an effective capacitor $C = Q/V$:

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$

For n capacitors,

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \dots + \frac{1}{C_n}$$

(b) Parallel combination: The **same potential difference V** is applied across each capacitor, but the charges differ (Fig. 2.28):

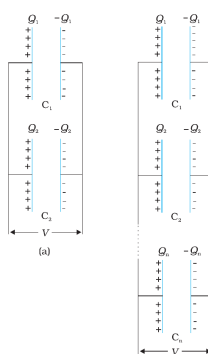


Fig. 2.28: Parallel combination of (a) two capacitors, (b) n capacitors

$$Q_1 = C_1 V, \quad Q_2 = C_2 V, \quad Q = Q_1 + Q_2$$

$$CV = C_1 V + C_2 V \Rightarrow C = C_1 + C_2$$

For n capacitors,

$$C = C_1 + C_2 + \dots + C_n$$

Key Examination Points for Full Marks:

- **Series:** same Q; C is **less** than the smallest capacitance; voltage divides as $V \propto 1/C$.
- **Parallel:** same V; C is the **sum**; charge divides as $Q \propto C$.
- n identical capacitors C: series $\rightarrow C/n$; parallel $\rightarrow nC$.

Q.17 Derive an expression for the energy stored in a capacitor and the energy density of an electric field. (3 Marks)

Start with two uncharged conductors. Transfer charge from conductor 2 to conductor 1 bit by bit, until conductor 1 has Q and conductor 2 has -Q (Fig. 2.30).

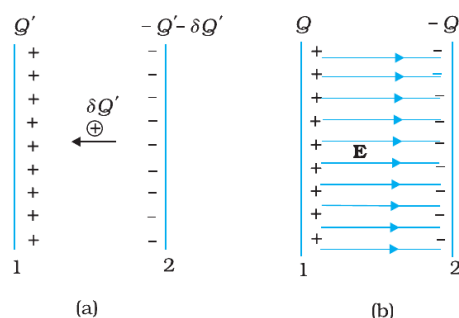


Fig. 2.30: (a) Work done in a small step of building charge on conductor 1 from Q' to $Q' + \delta Q'$. (b) Total work done in charging the capacitor may be viewed as stored in the energy of electric field between the plates

At an intermediate stage, the charges are Q' and $-Q'$, and the potential difference is $V' = Q'/C$. Work done in transferring a further small charge $\delta Q'$:

$$\delta W = V' \delta Q' = \frac{Q'}{C} \delta Q'$$

Total work:

$$W = \int_0^Q \frac{Q'}{C} dQ' = \frac{1}{C} \left[\frac{Q'^2}{2} \right]_0^Q = \frac{Q^2}{2C}$$

This work is stored as electrostatic potential energy:

$$U = \frac{Q^2}{2C} = \frac{1}{2} CV^2 = \frac{1}{2} QV$$

Energy density: For a parallel plate capacitor ($C = \epsilon_0 A/d$, $Q = A\sigma$, $E = \sigma/\epsilon_0$):

$$U = \frac{1}{2} \frac{Q^2}{C} = \frac{(A\sigma)^2}{2} \times \frac{d}{\epsilon_0 A} = \frac{1}{2} \epsilon_0 E^2 \times Ad$$

Ad is the volume between the plates (where the field exists). So the energy per unit volume is

$$u = \frac{1}{2} \epsilon_0 E^2$$

Key Examination Points for Full Marks:

- The energy can be viewed as **stored in the electric field** between the plates.
- The energy density result is **general**, valid for the field of any charge configuration.
- With a dielectric: $u = \frac{1}{2} \epsilon_0 K E^2$.

Illustration (NCERT Example 2.10): A 900 pF capacitor charged to 100 V stores $1/2 CV^2 = 4.5 \times 10^{-6}$ J. Connected to an identical uncharged capacitor, the charge is shared ($V' = 50$ V) and the total energy becomes 2.25×10^{-6} J. **Half the energy is lost** as heat and electromagnetic radiation during the transient current.

Formula Sheet — Electrostatic Potential and Capacitance

Quantity	Formula	SI Unit
Potential difference	$V_P - V_R = \frac{W_{RP}}{q}$	V
Point charge	$V = \frac{kQ}{r}$	V
Dipole ($r \gg a$)	$V = \frac{kpcos\theta}{r^2}$; axial $\frac{kp}{r^2}$; equatorial 0	V
System of charges	$V = k\sum \frac{q_i}{r_i}$	V
Shell (charge q, radius R)	$V = \frac{kq}{r} (r \geq R)$; $\frac{kq}{R} (r < R)$	V
Field and potential	$E = -\frac{dV}{dr}$; uniform: $E = \frac{V}{d}$	V m ⁻¹
PE of two charges	$U = \frac{kq_1q_2}{r}$	J
PE of charge in external V	$U = qV$	J
PE of dipole in uniform E	$U = -pEcos\theta = -\vec{p} \cdot \vec{E}$	J
Work to rotate dipole	$W = pE(cos\theta_1 - cos\theta_2)$	J
E at conductor surface	$E = \frac{\sigma}{\epsilon_0}$	N C ⁻¹
Capacitance	$C = \frac{Q}{V}$	F

Quantity	Formula	SI Unit
Parallel plate (air)	$C_0 = \frac{\epsilon_0 A}{d}$	F
Full dielectric	$C = KC_0 = \frac{K\epsilon_0 A}{d}$	F
Dielectric slab (thickness t)	$C = \frac{\epsilon_0 A}{d - t + t/K}$	F
Metal slab (thickness t)	$C = \frac{\epsilon_0 A}{d - t}$	F
Isolated sphere	$C = 4\pi\epsilon_0 R$	F
Series	$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \dots$	F
Parallel	$C = C_1 + C_2 + \dots$	F
Energy stored	$U = \frac{Q^2}{2C} = \frac{1}{2}CV^2 = \frac{1}{2}QV$	J
Energy density	$u = \frac{1}{2}\epsilon_0 E^2$	J m ⁻³
Common potential (sharing)	$V = \frac{C_1 V_1 + C_2 V_2}{C_1 + C_2}$	V
Energy lost on sharing	$\Delta U = \frac{C_1 C_2 (V_1 - V_2)^2}{2(C_1 + C_2)}$	J

Dimensions: Potential [M L²T⁻³A⁻¹]; capacitance [M⁻¹L⁻²T⁴A²]; polarisation [L⁻²T A]; dielectric constant — dimensionless. **Units:** 1 eV = 1.6 × 10⁻¹⁹ J; 1 V m⁻¹ = 1 N C⁻¹.

NEET / JEE Quick Points and Tricks

- Battery connected vs disconnected (dielectric K inserted):** Connected → V constant; C , Q , U all become K times; E unchanged. Disconnected → Q constant; C becomes $K\times$; V , E and U become $1/K$.
- Changing plate separation:** Battery disconnected (Q constant) → increasing d increases V and U , E unchanged. Battery connected (V constant) → increasing d decreases C , Q and U .
- Charge sharing** between two capacitors always **loses energy** (heat/radiation). Identical capacitors, one charged: half the energy is lost.
- Zero potential points** on the line of two unlike charges: one between them and one outside, beyond the smaller charge. Unlike null-field points, $V = 0$ points can lie between unlike charges.
- $E = 0$ does not mean $V = 0$** , and $V = 0$ does not mean $E = 0$. Example: inside a charged shell, $E = 0$ but $V = kq/R$; on the equatorial line of a dipole, $V = 0$ but $E \neq 0$.
- Work along an equipotential surface is zero.** Work done in moving a charge in a circle around a point charge = 0.
- Drops coalescing:** n identical drops (each radius r , charge q , potential V) form one big drop: $R = n^{1/3}r$, $V_{big} = n^{2/3}V$, $C_{big} = n^{1/3}C$, energy = $n^{5/3}U$.
- Two connected spheres** (by a wire): same potential, so $q \propto R$ and $\sigma \propto \frac{1}{R}$ — charge density is higher at **sharper** points (corona discharge, lightning rods).



9. **Dielectric partially filled side by side** (parallel arrangement of areas): $C = \frac{\epsilon_0}{d} (K_1 A_1 + K_2 A_2)$.
Layered (series arrangement of thicknesses): $C = \frac{\epsilon_0 A}{t_1/K_1 + t_2/K_2}$.
10. **Force between capacitor plates:** $F = \frac{Q^2}{2\epsilon_0 A} = \frac{\sigma^2 A}{2\epsilon_0}$ (not QE , since each plate feels only the other's field).
11. **Kinetic energy gained** by a charge accelerated through V : $K = qV$. An electron through 1 V gains 1 eV; an α -particle ($2e$) through 1 V gains 2 eV.
12. **Capacitor in steady DC circuit:** no current flows through its branch; treat it as an open circuit when finding V across it.
13. **Wheatstone-type capacitor networks:** if $C_1/C_2 = C_3/C_4$, the middle capacitor carries no charge and can be removed.
14. **Isolated sphere:** $C = 4\pi\epsilon_0 R$. The earth ($R = 6400$ km) has $C \approx 711 \mu\text{F}$ only.