



NEET — JEE MAINS - ADVANCED

Chapter 2 : Electrostatic Potential and Capacitance (Solved Examples & Exercises)

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Notes

Step-by-step solutions · NCERT Examples 2.1–2.10 and Exercises 2.1–2.11 · Class 12 Physics

Part A — NCERT Solved Examples

Example 2.1

Question: (a) Calculate the potential at a point P due to a charge of 4×10^{-7} C located 9 cm away. (b) Hence obtain the work done in bringing a charge of 2×10^{-9} C from infinity to the point P. Does the answer depend on the path along which the charge is brought?

Solution:

(a)

$$V = \frac{1}{4\pi\epsilon_0} \frac{Q}{r} = 9 \times 10^9 \text{ N m}^2 \text{C}^{-2} \times \frac{4 \times 10^{-7} \text{ C}}{0.09 \text{ m}} = 4 \times 10^4 \text{ V}$$

(b)

$$W = qV = 2 \times 10^{-9} \text{ C} \times 4 \times 10^4 \text{ V} = 8 \times 10^{-5} \text{ J}$$

Answer: (a) 4×10^4 V; (b) 8×10^{-5} J. No, the work is **path independent**. Any small path can be split into a displacement along $\vec{r}$ and one perpendicular to $\vec{r}$; the work for the perpendicular part is zero.

Example 2.2

Question: Two charges 3×10^{-8} C and -2×10^{-8} C are located 15 cm apart. At what point on the line joining the two charges is the electric potential zero? Take the potential at infinity to be zero.



Fig. 2.7

Solution: Take the origin O at the positive charge and the x -axis along the line, with the negative charge at A ($x = 15$ cm). Clearly x must be positive (for $x < 0$, the potentials cannot cancel, since the positive charge is always nearer).

If P lies between O and A:

$$\frac{1}{4\pi\epsilon_0} \left[\frac{3 \times 10^{-8}}{x \times 10^{-2}} - \frac{2 \times 10^{-8}}{(15 - x) \times 10^{-2}} \right] = 0 \Rightarrow \frac{3}{x} = \frac{2}{15 - x} \Rightarrow x = 9 \text{ cm}$$

If P lies on the extended line OA:

$$\frac{3}{x} - \frac{2}{x-15} = 0 \Rightarrow 3x - 45 = 2x \Rightarrow x = 45 \text{ cm}$$

Answer: The potential is zero at **9 cm** and **45 cm** from the positive charge, on the side of the negative charge.

Example 2.3

Question: Figures 2.8 (a) and (b) show the field lines of a positive and a negative point charge respectively. (a) Give the signs of the potential differences $V_P - V_Q$ and $V_B - V_A$. (b) Give the sign of the potential energy difference of a small negative charge between the points Q and P; A and B. (c) Give the sign of the work done by the field in moving a small positive charge from Q to P. (d) Give the sign of the work done by the external agency in moving a small negative charge from B to A. (e) Does the kinetic energy of a small negative charge increase or decrease in going from B to A?

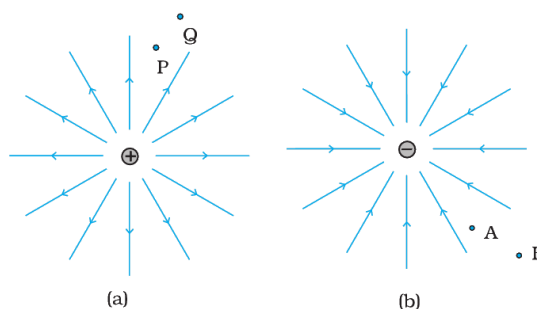


Fig. 2.8

Solution:

(a) $V \propto \frac{1}{r}$. P is nearer the positive charge, so $V_P > V_Q$: $(V_P - V_Q)$ is **positive**. For the negative charge, V_B is less negative than V_A (B is farther), so $V_B > V_A$: $(V_B - V_A)$ is **positive**.

(b) A small negative charge is attracted toward the positive charge and moves from higher to lower potential energy. So the potential energy difference of the negative charge between Q and P, $U_Q - U_P$, is **positive**. Similarly, $(U)_A > (U)_B$, so $U_A - U_B$ is **positive**.

(c) Moving a positive charge from Q to P (toward the positive charge) needs work by an external agency against the field. So the work done **by the field is negative**.

(d) Moving a negative charge from B to A (toward the negative charge) is against the repulsion, so the work done by the external agency is **positive**.

(e) Because of repulsion on the negative charge, its velocity decreases, so its **kinetic energy decreases** in going from B to A.

Example 2.4

Question: Four charges are arranged at the corners of a square ABCD of side d , as shown in Fig. 2.15. (a) Find the work required to put together this arrangement. (b) A charge q_0 is brought to the centre E of the square, the four charges being held fixed at its corners. How much extra work is needed to do this?

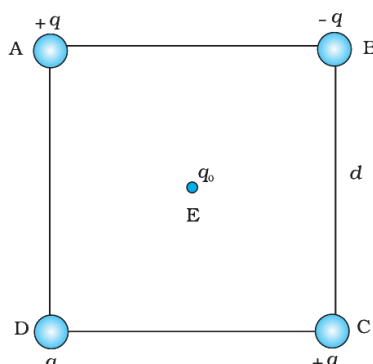


Fig. 2.15

Solution:

(a) The work depends only on the final arrangement. Bring $+q$ to A, then $-q$ to B, $+q$ to C and $-q$ to D:

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(i) $+q$ to A: **zero** (no other charge present).

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(ii) $-q$ to B: $-q \times \frac{q}{4\pi\epsilon_0 d} = -\frac{q^2}{4\pi\epsilon_0 d}$

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(iii) $+q$ to C: $+q \left(\frac{q}{4\pi\epsilon_0 d\sqrt{2}} - \frac{q}{4\pi\epsilon_0 d} \right) = -\frac{q^2}{4\pi\epsilon_0 d} \left(1 - \frac{1}{\sqrt{2}} \right)$

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(iv) $-q$ to D: $-q \left(\frac{q}{4\pi\epsilon_0 d} - \frac{q}{4\pi\epsilon_0 d\sqrt{2}} + \frac{q}{4\pi\epsilon_0 d} \right) = -\frac{q^2}{4\pi\epsilon_0 d} \left(2 - \frac{1}{\sqrt{2}} \right)$

Adding:

$$W = -\frac{q^2}{4\pi\epsilon_0 d} \left\{ 0 + 1 + \left(1 - \frac{1}{\sqrt{2}} \right) + \left(2 - \frac{1}{\sqrt{2}} \right) \right\} = -\frac{q^2}{4\pi\epsilon_0 d} (4 - \sqrt{2})$$

This is the total electrostatic energy of the charges, whatever the order in which they are brought.

(b) Extra work = $q_0 \times$ (potential at E due to the four charges). At E, the potentials due to A and C cancel those due to B and D, so $V_E = 0$.

Answer: (a) $-\frac{q^2}{4\pi\epsilon_0 d} (4 - \sqrt{2})$; (b) **zero** extra work.

Example 2.5

Question: (a) Determine the electrostatic potential energy of a system consisting of two charges $7 \mu\text{C}$ and $-2 \mu\text{C}$ (and with no external field) placed at $(-9 \text{ cm}, 0, 0)$ and $(9 \text{ cm}, 0, 0)$ respectively. (b) How much work is required to separate the two charges infinitely away from each other? (c) Suppose that the same system of charges is now placed in an external electric field $E = A(1/r^2)$; $A = 9 \times 10^5 \text{ N C}^{-1} \text{ m}^2$. What would the electrostatic energy of the configuration be?

Solution: Separation $r = 18 \text{ cm} = 0.18 \text{ m}$.

(a)

$$U = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r} = 9 \times 10^9 \times \frac{7 \times (-2) \times 10^{-12}}{0.18} = -0.7 \text{ J}$$

(b) $W = U_2 - U_1 = 0 - (-0.7) = 0.7 \text{ J}$.

(c) The mutual interaction energy (-0.7 J) is unchanged. In addition, each charge interacts with the external field. The field $E = A/r^2$ corresponds to a potential $V = A/r$, and each charge is 0.09 m from the origin:

$$q_1 V(r_1) + q_2 V(r_2) = A \frac{7 \mu\text{C}}{0.09 \text{ m}} + A \frac{-2 \mu\text{C}}{0.09 \text{ m}} = 70 - 20 = 50 \text{ J}$$

$$U_{\text{total}} = 70 - 20 - 0.7 = 49.3 \text{ J}$$

Answer: (a) -0.7 J ; (b) 0.7 J ; (c) 49.3 J .

Example 2.6

Question: A molecule of a substance has a permanent electric dipole moment of magnitude 10^{-29} C m . A mole of this substance is polarised (at low temperature) by applying a strong electrostatic field of magnitude 10^6 V m^{-1} . The direction of the field is suddenly changed by an angle of 60° . Estimate the heat released by the substance in aligning its dipoles along the new direction of the field. For simplicity, assume 100% polarisation of the sample.

Solution: Total dipole moment of one mole (6×10^{23} molecules):

$$p = 6 \times 10^{23} \times 10^{-29} = 6 \times 10^{-6} \text{ C m}$$

Initial potential energy ($\theta = 0^\circ$): $U_i = -pE \cos 0^\circ = -6 \times 10^{-6} \times 10^6 = -6 \text{ J}$.

Final potential energy ($\theta = 60^\circ$): $U_f = -6 \times 10^{-6} \times 10^6 \times \cos 60^\circ = -3 \text{ J}$.

Change in potential energy $= -3 - (-6) = 3 \text{ J}$. When the dipoles realign with the new field, this energy is lost.

Answer: About **3 J** of heat is released.

Example 2.7

Question: (a) A comb run through one's dry hair attracts small bits of paper. Why? What happens if the hair is wet or if it is a rainy day? (Remember, a paper does not conduct electricity.) (b) Ordinary rubber is an insulator. But special rubber tyres of aircraft are made slightly conducting. Why is this necessary? (c) Vehicles carrying inflammable materials usually have metallic ropes touching the ground during motion. Why? (d) A bird perches on a bare high power line, and nothing happens to the bird. A man standing on the ground touches the same line and gets a fatal shock. Why?

Answer:

(a) The comb gets charged by friction. The molecules of the paper are **polarised** by the charged comb, giving a net force of attraction. If the hair is wet or the day is rainy, friction between hair and comb reduces (and moisture conducts the charge away), so the comb does not get charged and does not attract the paper.

(b) To conduct the static charge produced by friction (during landing) to the ground. Too much accumulated static electricity may cause a **spark** and start a fire.

(c) For the same reason as (b): the ropes conduct the static charge produced by friction to the ground, preventing sparks.

(d) A current flows only when there is a **potential difference**. Both feet of the bird are on the same wire at the same potential, so no current flows. The man touches the line (high potential) while standing on the ground (zero potential), so a large current flows through his body.

Example 2.8

Question: A slab of material of dielectric constant K has the same area as the plates of a parallel-plate capacitor but has a thickness $(3/4)d$, where d is the separation of the plates. How is the capacitance changed when the slab is inserted between the plates?

Solution: Let $E_0 = V_0/d$ be the field without the dielectric. With the slab, the field in the dielectric is $E = E_0/K$, while in the remaining air gap $(d/4)$ it is E_0 :

$$V = E_0 \left(\frac{1}{4} d \right) + \frac{E_0}{K} \left(\frac{3}{4} d \right) = E_0 d \left(\frac{1}{4} + \frac{3}{4K} \right) = V_0 \frac{K+3}{4K}$$

The free charge Q_0 is unchanged, so

$$C = \frac{Q_0}{V} = \frac{4K}{K+3} \frac{Q_0}{V_0} = \frac{4K}{K+3} C_0$$

Answer: The potential difference decreases by the factor $(K+3)/4K$, and the capacitance increases to $\frac{4K}{K+3} C_0$.

Example 2.9

Question: A network of four $10 \mu\text{F}$ capacitors is connected to a 500 V supply, as shown in Fig. 2.29. Determine (a) the equivalent capacitance of the network and (b) the charge on each capacitor. (Note, the charge on a capacitor is the charge on the plate with higher potential, equal and opposite to the charge on the plate with lower potential.)

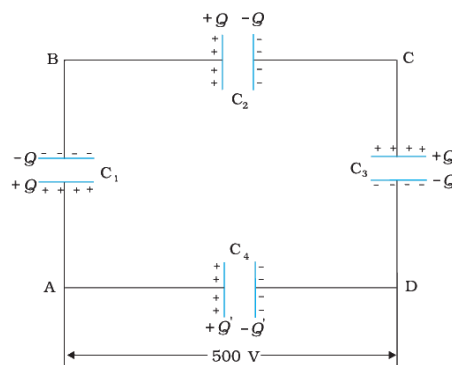


Fig. 2.29

Solution:

(a) C_1 , C_2 and C_3 are in series:

$$\frac{1}{C'} = \frac{1}{10} + \frac{1}{10} + \frac{1}{10} \Rightarrow C' = \frac{10}{3} \mu\text{F}$$

C' is in parallel with C_4 :

$$C = C' + C_4 = \left(\frac{10}{3} + 10 \right) \mu\text{F} = 13.3 \mu\text{F}$$

(b) C_1 , C_2 , C_3 carry the same charge Q . The potential differences across AB, BC and CD add to 500 V :

$$\frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3} = 500 \text{ V} \Rightarrow Q = 500 \times \frac{10}{3} \mu\text{C} = 1.7 \times 10^{-3} \text{ C}$$

For C_4 : $Q'/C_4 = 500 \text{ V}$, so $Q' = 500 \times 10 \mu\text{C} = 5.0 \times 10^{-3} \text{ C}$.

Answer: (a) $13.3 \mu\text{F}$; (b) $1.7 \times 10^{-3} \text{ C}$ on each of C_1 , C_2 , C_3 , and $5.0 \times 10^{-3} \text{ C}$ on C_4 .

Example 2.10

Question: (a) A 900 pF capacitor is charged by a 100 V battery [Fig. 2.31 (a)]. How much electrostatic energy is stored by the capacitor? (b) The capacitor is disconnected from the battery and connected to another 900 pF capacitor [Fig. 2.31 (b)]. What is the electrostatic energy stored by the system?

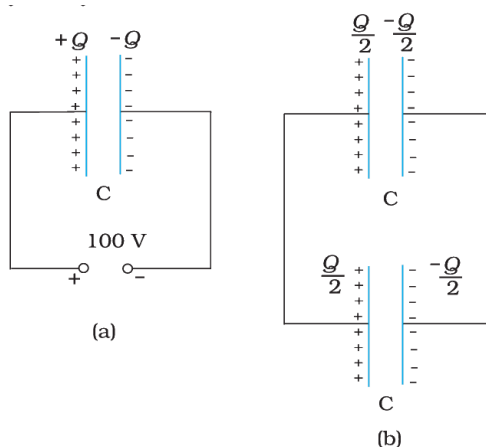


Fig. 2.31

Solution:

(a) $Q = CV = 900 \times 10^{-12} \times 100 = 9 \times 10^{-8} \text{ C}.$

$$U = \frac{1}{2} CV^2 = \frac{1}{2} QV = \frac{1}{2} \times 9 \times 10^{-8} \times 100 = 4.5 \times 10^{-6} \text{ J}$$

(b) In the steady state, both capacitors have their positive plates at the same potential and their negative plates at the same potential. Let the common potential difference be V' . The charge on each is $Q' = CV'$. By charge conservation, $Q' = Q/2$, so $V' = V/2$.

$$U' = 2 \times \frac{1}{2} Q'V' = \frac{1}{4} QV = 2.25 \times 10^{-6} \text{ J}$$

Answer: (a) $4.5 \times 10^{-6} \text{ J}$; (b) $2.25 \times 10^{-6} \text{ J}$. Though no charge is lost, the final energy is **half** the initial energy. The remaining energy is lost as heat and electromagnetic radiation during the transient current that flows from the first capacitor to the second.

Part B — NCERT Exercises

Exercise 2.1

Question: Two charges $5 \times 10^{-8} \text{ C}$ and $-3 \times 10^{-8} \text{ C}$ are located 16 cm apart. At what point(s) on the line joining the two charges is the electric potential zero? Take the potential at infinity to be zero.

Solution: Take the positive charge at the origin and the negative charge at $x = 16 \text{ cm}$. Let $V = 0$ at distance x (in cm) from the positive charge.

Between the charges:

$$\frac{k \times 5 \times 10^{-8}}{x} = \frac{k \times 3 \times 10^{-8}}{16 - x} \Rightarrow 5(16 - x) = 3x \Rightarrow x = 10 \text{ cm}$$

Outside, beyond the negative charge:

$$\frac{5}{x} = \frac{3}{x - 16} \Rightarrow 5x - 80 = 3x \Rightarrow x = 40 \text{ cm}$$

Answer: $V = 0$ at **10 cm** and **40 cm** from the positive charge, on the side of the negative charge.

Exercise 2.2

Question: A regular hexagon of side 10 cm has a charge $5 \mu\text{C}$ at each of its vertices. Calculate the potential at the centre of the hexagon.

Solution: In a regular hexagon, the distance from the centre to each vertex equals the side: $r = 0.1$ m. Potential is a scalar, so the six potentials add:

$$V = 6 \times \frac{kq}{r} = 6 \times \frac{9 \times 10^9 \times 5 \times 10^{-6}}{0.1} = 6 \times 4.5 \times 10^5$$

Answer: $V = 2.7 \times 10^6$ V.

Exercise 2.3

Question: Two charges $2 \mu\text{C}$ and $-2 \mu\text{C}$ are placed at points A and B 6 cm apart. (a) Identify an equipotential surface of the system. (b) What is the direction of the electric field at every point on this surface?

Answer:

(a) The plane **perpendicular to AB through its midpoint** (the equatorial plane of the dipole). Every point on it is equidistant from both charges, so the potential is zero everywhere on this plane.

(b) The field is **normal to the plane**, directed from A to B (from the positive charge toward the negative charge, i.e. opposite to the dipole moment).

Exercise 2.4

Question: A spherical conductor of radius 12 cm has a charge of 1.6×10^{-7} C distributed uniformly on its surface. What is the electric field (a) inside the sphere, (b) just outside the sphere, (c) at a point 18 cm from the centre of the sphere?

Solution:

(a) Inside a conductor, $\mathbf{E} = \mathbf{0}$.

(b) Just outside, the field is as if the charge were at the centre ($r = R = 0.12$ m):

$$E = \frac{kq}{R^2} = \frac{9 \times 10^9 \times 1.6 \times 10^{-7}}{(0.12)^2} = 10^5 \text{ N C}^{-1}$$

(c) At $r = 0.18$ m:

$$E = \frac{9 \times 10^9 \times 1.6 \times 10^{-7}}{(0.18)^2} = 4.4 \times 10^4 \text{ N C}^{-1}$$

Answer: (a) zero; (b) 10^5 N C^{-1} ; (c) $4.4 \times 10^4 \text{ N C}^{-1}$.

Exercise 2.5

Question: A parallel plate capacitor with air between the plates has a capacitance of 8 pF ($1 \text{ pF} = 10^{-12} \text{ F}$). What will be the capacitance if the distance between the plates is reduced by half, and the space between them is filled with a substance of dielectric constant 6?

Solution:

$$C_0 = \frac{\epsilon_0 A}{d} = 8 \text{ pF}, \quad C = \frac{K \epsilon_0 A}{d/2} = 2K C_0 = 2 \times 6 \times 8 \text{ pF}$$

Answer: $C = 96 \text{ pF}$.

Exercise 2.6

Question: Three capacitors each of capacitance 9 pF are connected in series. (a) What is the total capacitance of the combination? (b) What is the potential difference across each capacitor if the combination is connected to a 120 V supply?

Solution:

(a)

$$\frac{1}{C} = \frac{1}{9} + \frac{1}{9} + \frac{1}{9} = \frac{3}{9} \Rightarrow C = 3 \text{ pF}$$

(b) The capacitors are identical and carry the same charge, so the voltage divides equally:

$$V_{\text{each}} = \frac{120}{3} = 40 \text{ V}$$

Answer: (a) 3 pF; (b) 40 V across each.

Exercise 2.7

Question: Three capacitors of capacitances 2 pF, 3 pF and 4 pF are connected in parallel. (a) What is the total capacitance of the combination? (b) Determine the charge on each capacitor if the combination is connected to a 100 V supply.

Solution:

(a) $C = 2 + 3 + 4 = 9 \text{ pF}$.

(b) Each capacitor has $V = 100 \text{ V}$, so $Q = CV$:

- $Q_1 = 2 \times 10^{-12} \times 100 = 2 \times 10^{-10} \text{ C}$
- $Q_2 = 3 \times 10^{-12} \times 100 = 3 \times 10^{-10} \text{ C}$
- $Q_3 = 4 \times 10^{-12} \times 100 = 4 \times 10^{-10} \text{ C}$

Answer: (a) 9 pF; (b) $2 \times 10^{-10} \text{ C}$, $3 \times 10^{-10} \text{ C}$ and $4 \times 10^{-10} \text{ C}$.

Exercise 2.8

Question: In a parallel plate capacitor with air between the plates, each plate has an area of $6 \times 10^{-3} \text{ m}^2$ and the distance between the plates is 3 mm. Calculate the capacitance of the capacitor. If this capacitor is connected to a 100 V supply, what is the charge on each plate of the capacitor?

Solution:

$$C = \frac{\epsilon_0 A}{d} = \frac{8.854 \times 10^{-12} \times 6 \times 10^{-3}}{3 \times 10^{-3}} = 1.77 \times 10^{-11} \text{ F} = 17.7 \text{ pF}$$

$$Q = CV = 1.77 \times 10^{-11} \times 100 = 1.77 \times 10^{-9} \text{ C}$$

Answer: $C = 17.7 \text{ pF}$; charge on each plate = $1.8 \times 10^{-9} \text{ C}$ (+ on one plate, – on the other).

Exercise 2.9

Question: Explain what would happen if in the capacitor given in Exercise 2.8, a 3 mm thick mica sheet (of dielectric constant 6) were inserted between the plates, (a) while the voltage supply remained connected, (b) after the supply was disconnected.

Solution: The mica sheet fills the gap, so $C = KC_0 = 6 \times 17.7 = 106 \text{ pF} = 1.06 \times 10^{-10} \text{ F}$.

(a) **Supply connected:** V stays 100 V.

$$Q = CV = 1.06 \times 10^{-10} \times 100 = 1.06 \times 10^{-8} \text{ C}$$

The charge increases 6 times (the battery supplies the extra charge).

(b) **Supply disconnected:** Q stays $1.77 \times 10^{-9} \text{ C}$.

$$V = \frac{Q}{C} = \frac{1.77 \times 10^{-9}}{1.06 \times 10^{-10}} = 16.7 \text{ V}$$

The potential difference falls to $1/6$ of its value.

Answer: (a) $C = 106 \text{ pF}$, $Q = 1.06 \times 10^{-8} \text{ C}$, $V = 100 \text{ V}$; (b) $C = 106 \text{ pF}$, $Q = 1.77 \times 10^{-9} \text{ C}$, $V = 16.7 \text{ V}$.

Exercise 2.10

Question: A 12 pF capacitor is connected to a 50 V battery. How much electrostatic energy is stored in the capacitor?

Solution:

$$U = \frac{1}{2} CV^2 = \frac{1}{2} \times 12 \times 10^{-12} \times (50)^2 = 1.5 \times 10^{-8} \text{ J}$$

Answer: $1.5 \times 10^{-8} \text{ J}$.

Exercise 2.11

Question: A 600 pF capacitor is charged by a 200 V supply. It is then disconnected from the supply and is connected to another uncharged 600 pF capacitor. How much electrostatic energy is lost in the process?

Solution: Initial energy:

$$U_i = \frac{1}{2} CV^2 = \frac{1}{2} \times 600 \times 10^{-12} \times (200)^2 = 1.2 \times 10^{-5} \text{ J}$$

On connecting, the charge is shared equally (identical capacitors), so the common potential is $V' = 100 \text{ V}$:

$$U_f = 2 \times \frac{1}{2} \times 600 \times 10^{-12} \times (100)^2 = 6 \times 10^{-6} \text{ J}$$

$$\Delta U = U_i - U_f = 1.2 \times 10^{-5} - 6 \times 10^{-6} = 6 \times 10^{-6} \text{ J}$$

Answer: $6 \times 10^{-6} \text{ J}$ is lost (as heat and electromagnetic radiation) — half the initial energy.