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Chapter 4 : Moving Charges and Magnetism (Solved Examples & Exercises)

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Notes

Step-by-step solutions · NCERT Examples 4.1–4.12 and Exercises 4.1–4.13 · Class 12 Physics

Part A — NCERT Solved Examples

Example 4.1

Question: A straight wire of mass 200 g and length 1.5 m carries a current of 2 A. It is suspended in mid-air by a uniform horizontal magnetic field B (Fig. 4.3). What is the magnitude of the magnetic field?

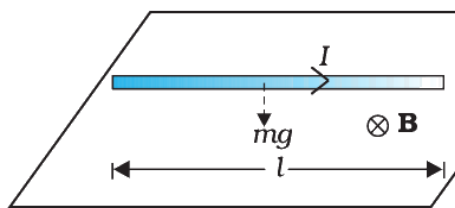


Fig. 4.3

Solution: From $F = IlB$, there is an upward force of magnitude IlB . For mid-air suspension, this must balance the weight:

$$mg = IlB \quad \Rightarrow \quad B = \frac{mg}{Il} = \frac{0.2 \times 9.8}{2 \times 1.5} = 0.65 \text{ T}$$

It would have been enough to specify m/l , the mass per unit length of the wire. The Earth's field (about 4×10^{-5} T) has been ignored.

Answer: $B = 0.65 \text{ T}$

Example 4.2

Question: If the magnetic field is parallel to the positive y-axis and the charged particle is moving along the positive x-axis (Fig. 4.4), which way would the Lorentz force be for (a) an electron (negative charge), (b) a proton (positive charge)?

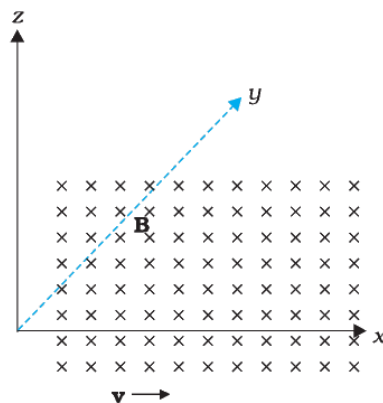


Fig. 4.4

Solution: $\vec{v}$ is along $\hat{i}$ and $\vec{B}$ along $\hat{j}$, so $\vec{v} \times \vec{B}$ is along $\hat{i} \times \hat{j} = \hat{k}$ (+z).

Answer: (a) For an electron, the force is along $-\mathbf{z}$. (b) For a proton, the force is along $+\mathbf{z}$.

Example 4.3

Question: What is the radius of the path of an electron (mass 9×10^{-31} kg and charge 1.6×10^{-19} C) moving at a speed of 3×10^7 m/s in a magnetic field of 6×10^{-4} T perpendicular to it? What is its frequency? Calculate its energy in keV. (1 eV = 1.6×10^{-19} J)

Solution:

$$r = \frac{mv}{qB} = \frac{9 \times 10^{-31} \times 3 \times 10^7}{1.6 \times 10^{-19} \times 6 \times 10^{-4}} = 28 \times 10^{-2} \text{ m} = 28 \text{ cm}$$

$$\nu = \frac{v}{2\pi r} = \frac{3 \times 10^7}{2\pi \times 0.28} = 17 \times 10^6 \text{ Hz} = 17 \text{ MHz}$$

$$E = \frac{1}{2}mv^2 = \frac{1}{2} \times 9 \times 10^{-31} \times 9 \times 10^{14} = 40.5 \times 10^{-17} \text{ J} \approx 4 \times 10^{-16} \text{ J} = 2.5 \text{ keV}$$

Answer: $r = 28 \text{ cm}$, $\nu = 17 \text{ MHz}$, $E \approx 2.5 \text{ keV}$

Example 4.4

Question: An element $\Delta \vec{l} = \Delta x \hat{i}$ is placed at the origin and carries a large current $I = 10 \text{ A}$ (Fig. 4.8). What is the magnetic field on the y-axis at a distance of 0.5 m? $\Delta x = 1 \text{ cm}$.

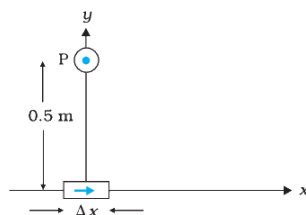


Fig. 4.8

Solution: Using the Biot-Savart law, $\left| d\vec{B} \right| = \frac{\mu_0 I dl \sin \theta}{4\pi r^2}$, with $dl = \Delta x = 10^{-2} \text{ m}$, $I = 10 \text{ A}$, $r = 0.5 \text{ m}$, $\theta = 90^\circ$:

$$\left| d\vec{B} \right| = \frac{10^{-7} \times 10 \times 10^{-2}}{25 \times 10^{-2}} = 4 \times 10^{-8} \text{ T}$$

Direction: $d\vec{l} \times \vec{r} = \Delta x \hat{i} \times y \hat{j} = y \Delta x (\hat{i} \times \hat{j}) = y \Delta x \hat{k}$, i.e. along $+\mathbf{z}$.

(Cyclic property: $\hat{i} \times \hat{j} = \hat{k}$, $\hat{j} \times \hat{k} = \hat{i}$, $\hat{k} \times \hat{i} = \hat{j}$.)

Answer: $4 \times 10^{-8} \text{ T}$ along the $+\mathbf{z}$ direction (a small field).

Example 4.5

Question: A straight wire carrying a current of 12 A is bent into a semi-circular arc of radius 2.0 cm as shown in Fig. 4.11(a). Consider the magnetic field B at the centre of the arc. (a) What is the magnetic field due to the straight segments? (b) In what way does the contribution to B from the semicircle differ from that of a circular loop and in what way does it resemble it? (c) Would your answer be different if the wire were bent into a semi-circular arc of the same radius but in the opposite way, as shown in Fig. 4.11(b)?

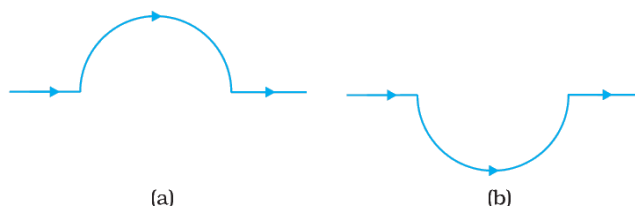


Fig. 4.11

Solution:

- (a) For each element of the straight segments, $d\vec{l}$ and $\vec{r}$ are parallel, so $d\vec{l} \times \vec{r} = 0$. **The straight segments do not contribute to B .**
- (b) For all elements of the semicircular arc, $d\vec{l} \times \vec{r}$ are parallel to each other (into the page), so all contributions add. The direction is given by the right-hand rule, as for a circular loop, but the **magnitude is half** that of a full loop:

$$B = \frac{\mu_0 I}{4R} = \frac{4\pi \times 10^{-7} \times 12}{4 \times 0.02} = 1.9 \times 10^{-4} \text{ T, normal to the paper, into it}$$
- (c) Same magnitude of B , but **opposite in direction** to that in (b).

Example 4.6

Question: Consider a tightly wound 100-turn coil of radius 10 cm, carrying a current of 1 A. What is the magnitude of the magnetic field at the centre of the coil?

Solution: Since the coil is tightly wound, each circular element has the same radius $R = 0.1$ m. $N = 100$.

$$B = \frac{\mu_0 NI}{2R} = \frac{4\pi \times 10^{-7} \times 10^2 \times 1}{2 \times 10^{-1}} = 2\pi \times 10^{-4} = 6.28 \times 10^{-4} \text{ T}$$

Answer: $6.28 \times 10^{-4} \text{ T}$

Example 4.7

Question: Figure 4.13 shows a long straight wire of circular cross-section (radius a) carrying steady current I . The current I is uniformly distributed across the cross-section. Calculate the magnetic field in the regions $r < a$ and $r > a$.

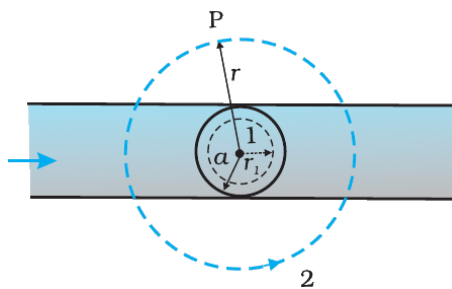


Fig. 4.13

Solution:

(a) $r > a$: The Amperian loop 2 is a circle concentric with the cross-section; $L = 2\pi r$ and $I_e = I$:

$$B(2\pi r) = \mu_0 I \Rightarrow B = \frac{\mu_0 I}{2\pi r}, \quad B \propto \frac{1}{r}$$

(b) $r < a$: The enclosed current is less than I . Since the distribution is uniform,

$$I_e = I \left(\frac{\pi r^2}{\pi a^2} \right) = \frac{I r^2}{a^2}$$

$$B(2\pi r) = \mu_0 \frac{I r^2}{a^2} \Rightarrow B = \left(\frac{\mu_0 I}{2\pi a^2} \right) r, \quad B \propto r$$

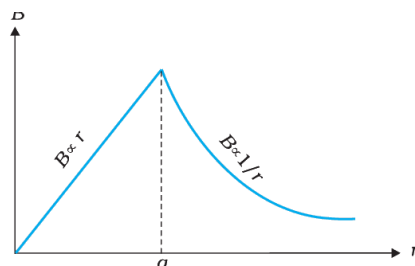


Fig. 4.14: Magnitude of B against distance r from the centre of the wire

The field is tangential to the respective circular loop and is given by the right-hand rule. This example has the symmetry needed for Ampere's law to be applied easily.

Example 4.8

Question: A solenoid of length 0.5 m has a radius of 1 cm and is made up of 500 turns. It carries a current of 5 A. What is the magnitude of the magnetic field inside the solenoid?

Solution: Number of turns per unit length:

$$n = \frac{500}{0.5} = 1000 \text{ turns/m}$$

Since $l/a = 0.5/0.01 = 50$, i.e. $l \gg a$, the long-solenoid formula applies:

$$B = \mu_0 n I = 4\pi \times 10^{-7} \times 10^3 \times 5 = 6.28 \times 10^{-3} \text{ T}$$

Answer: $6.28 \times 10^{-3} \text{ T}$

Example 4.9

Question: The horizontal component of the Earth's magnetic field at a certain place is $3.0 \times 10^{-5} \text{ T}$ and the direction of the field is from the geographic south to the geographic north. A very long straight conductor is carrying a steady current of 1 A. What is the force per unit length on it when it is placed on a horizontal table and the direction of the current is (a) east to west; (b) south to north?

Solution: $\vec{F} = I \vec{l} \times \vec{B} \Rightarrow F = IlB \sin \theta$, and the force per unit length is $f = IB \sin \theta$.

(a) Current east to west: $\theta = 90^\circ$.

$$f = IB = 1 \times 3 \times 10^{-5} = 3 \times 10^{-5} \text{ N m}^{-1}$$

This is larger than the value $2 \times 10^{-7} \text{ N m}^{-1}$ used in the definition of the ampere, so it is important to eliminate the Earth's field and other stray fields while standardising the ampere. The force is **downwards** (from the cross product).

(b) Current south to north: $\theta = 0^\circ$, so $f = 0$. **There is no force** on the conductor.

Example 4.10

Question: A 100-turn closely wound circular coil of radius 10 cm carries a current of 3.2 A. (a) What is the field at the centre of the coil? (b) What is the magnetic moment of this coil? The coil is placed in a vertical plane and is free to rotate about a horizontal axis which coincides with its diameter. A uniform magnetic field of 2 T in the horizontal direction exists such that initially the axis of the coil is in the direction of the field. The coil rotates through an angle of 90° under the influence of the magnetic field. (c) What are the magnitudes of the torques on the coil in the initial and final position? (d) What is the angular speed acquired by the coil when it has rotated by 90° ? The moment of inertia of the coil is 0.1 kg m^2 .

Solution:

(a) $N = 100$, $I = 3.2 \text{ A}$, $R = 0.1 \text{ m}$ (using $\pi \times 3.2 \approx 10$):

$$B = \frac{\mu_0 NI}{2R} = \frac{4\pi \times 10^{-7} \times 100 \times 3.2}{2 \times 10^{-1}} = \frac{4 \times 10^{-5} \times 10}{2 \times 10^{-1}} = 2 \times 10^{-3} \text{ T}$$

The direction is given by the right-hand thumb rule.

(b)

$$m = NIA = NI\pi r^2 = 100 \times 3.2 \times 3.14 \times 10^{-2} = 10 \text{ A m}^2$$

$$(c) \tau = \left| \vec{m} \times \vec{B} \right| = mB \sin \theta.$$

- Initially $\theta = 0$, so $\tau_i = 0$.
- Finally $\theta = 90^\circ$, so $\tau_f = mB = 10 \times 2 = 20 \text{ N m}$.

(d) From Newton's second law for rotation, $J \frac{d\omega}{dt} = mB \sin \theta$. Using $\frac{d\omega}{dt} = \frac{d\omega}{d\theta} \frac{d\theta}{dt} = \omega \frac{d\omega}{d\theta}$:

$$J \omega d\omega = mB \sin \theta d\theta$$

Integrating from $\theta = 0$ to $\pi/2$:

$$J \frac{\omega_f^2}{2} = -mB \cos \theta \Big|_0^{\pi/2} = mB$$

$$\omega_f = \left(\frac{2mB}{J} \right)^{1/2} = \left(\frac{2 \times 20}{10^{-1}} \right)^{1/2} = 20 \text{ s}^{-1}$$

Answer: (a) $2 \times 10^{-3} \text{ T}$ (b) 10 A m^2 (c) 0 and 20 N m (d) 20 s^{-1}

Example 4.11

Question: (a) A current-carrying circular loop lies on a smooth horizontal plane. Can a uniform magnetic field be set up in such a manner that the loop turns around itself (i.e., turns about the vertical axis)? (b) A current-carrying circular loop is located in a uniform external magnetic field. If the loop is free to turn, what is its orientation of stable equilibrium? Show that in this orientation, the flux of the total field (external field + field produced by the loop) is maximum. (c) A loop of irregular shape carrying current is located in an external magnetic field. If the wire is flexible, why does it change to a circular shape?

Answer:

- (a) No. That would require $\vec{\tau}$ to be in the vertical direction. But $\vec{\tau} = I\vec{A} \times \vec{B}$, and since $\vec{A}$ of the horizontal loop is vertical, $\vec{\tau}$ lies in the plane of the loop for any $\vec{B}$.
- (b) Stable equilibrium is when the area vector $\vec{A}$ of the loop is **along the external field**. In this orientation, the field produced by the loop is in the same direction as the external field, both normal to the plane of the loop, giving the maximum flux of the total field.
- (c) It takes a circular shape with its plane normal to the field to **maximise the flux**, since for a given perimeter a circle encloses a greater area than any other shape.

Example 4.12

Question: In the circuit (Fig. 4.23) the current is to be measured. What is the value of the current if the ammeter shown (a) is a galvanometer with a resistance $R_G = 60.00 \, \Omega$; (b) is a galvanometer described in (a) but converted to an ammeter by a shunt resistance $r_s = 0.02 \, \Omega$; (c) is an ideal ammeter with zero resistance?

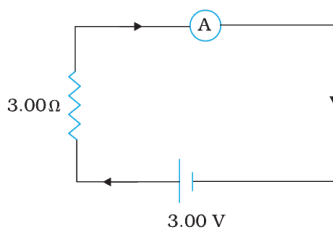


Fig. 4.23

Solution:

(a) Total resistance $= R_G + 3 = 63 \, \Omega$. Hence $I = \frac{3}{63} = 0.048 \, \text{A}$.

(b) Resistance of the galvanometer converted to an ammeter:

$$\frac{R_G r_s}{R_G + r_s} = \frac{60 \times 0.02}{60 + 0.02} \approx 0.02 \, \Omega$$

Total resistance $= 0.02 + 3 = 3.02 \, \Omega$. Hence $I = \frac{3}{3.02} = 0.99 \, \text{A}$.

(c) For an ideal ammeter, $I = \frac{3}{3} = 1.00 \, \text{A}$.

Part B — NCERT Exercises

Exercise 4.1

Question: A circular coil of wire consisting of 100 turns, each of radius 8.0 cm, carries a current of 0.40 A. What is the magnitude of the magnetic field B at the centre of the coil?

Solution: $N = 100$, $R = 0.08 \, \text{m}$, $I = 0.40 \, \text{A}$.

$$B = \frac{\mu_0 NI}{2R} = \frac{4\pi \times 10^{-7} \times 100 \times 0.40}{2 \times 0.08} = 3.14 \times 10^{-4} \, \text{T}$$

Answer: $B = 3.1 \times 10^{-4} \, \text{T}$

Exercise 4.2

Question: A long straight wire carries a current of 35 A. What is the magnitude of the field B at a point 20 cm from the wire?

Solution:

$$B = \frac{\mu_0 I}{2\pi r} = \frac{2 \times 10^{-7} \times 35}{0.20} = 3.5 \times 10^{-5} \, \text{T}$$

Answer: $B = 3.5 \times 10^{-5} \, \text{T}$

Exercise 4.3

Question: A long straight wire in the horizontal plane carries a current of 50 A in north to south direction. Give the magnitude and direction of B at a point 2.5 m east of the wire.

Solution:

$$B = \frac{\mu_0 I}{2\pi r} = \frac{2 \times 10^{-7} \times 50}{2.5} = 4 \times 10^{-6} \text{ T}$$

Direction: Point the right thumb towards the south (the current); at a point to the east of the wire, the fingers curl **vertically upwards**. (Check: $(-\hat{j}) \times \hat{i} = +\hat{k}$, with x east, y north, z up.)

Answer: 4×10^{-6} T, vertically upward

Exercise 4.4

Question: A horizontal overhead power line carries a current of 90 A in east to west direction. What is the magnitude and direction of the magnetic field due to the current 1.5 m below the line?

Solution:

$$B = \frac{\mu_0 I}{2\pi r} = \frac{2 \times 10^{-7} \times 90}{1.5} = 1.2 \times 10^{-5} \text{ T}$$

Direction: Current along $-\hat{i}$ (west), point below the line ($-\hat{k}$): $(-\hat{i}) \times (-\hat{k}) = \hat{i} \times \hat{k} = -\hat{j}$, i.e. towards the south.

Answer: 1.2×10^{-5} T, towards the south

Exercise 4.5

Question: What is the magnitude of magnetic force per unit length on a wire carrying a current of 8 A and making an angle of 30° with the direction of a uniform magnetic field of 0.15 T?

Solution:

$$\frac{F}{l} = IB \sin \theta = 8 \times 0.15 \times \sin 30^\circ = 0.6 \text{ N m}^{-1}$$

Answer: 0.6 N m^{-1}

Exercise 4.6

Question: A 3.0 cm wire carrying a current of 10 A is placed inside a solenoid perpendicular to its axis. The magnetic field inside the solenoid is given to be 0.27 T. What is the magnetic force on the wire?

Solution: The field inside the solenoid is along its axis, so $\theta = 90^\circ$:

$$F = IlB \sin \theta = 10 \times 0.03 \times 0.27 \times 1 = 8.1 \times 10^{-2} \text{ N}$$

The force is perpendicular to both the wire and the axis of the solenoid (by Fleming's left-hand rule).

Answer: 8.1×10^{-2} N

Exercise 4.7

Question: Two long and parallel straight wires A and B carrying currents of 8.0 A and 5.0 A in the same direction are separated by a distance of 4.0 cm. Estimate the force on a 10 cm section of wire A.

Solution:

$$F = \frac{\mu_0 I_A I_B}{2\pi d} L = \frac{2 \times 10^{-7} \times 8 \times 5}{0.04} \times 0.10 = 2 \times 10^{-5} \text{ N}$$

The currents are parallel, so the force is **attractive** — perpendicular to A, directed towards B.

Answer: 2×10^{-5} N, attractive

Exercise 4.8

Question: A closely wound solenoid 80 cm long has 5 layers of windings of 400 turns each. The diameter of the solenoid is 1.8 cm. If the current carried is 8.0 A, estimate the magnitude of B inside the solenoid near its centre.

Solution: Total turns $N = 5 \times 400 = 2000$; length $l = 0.8$ m.

$$n = \frac{N}{l} = \frac{2000}{0.8} = 2500 \text{ m}^{-1}$$

$$B = \mu_0 n I = 4\pi \times 10^{-7} \times 2500 \times 8 = 2.5 \times 10^{-2} \text{ T}$$

The diameter is not needed, since $l \gg$ diameter (long solenoid).

Answer: $B \approx 2.5 \times 10^{-2} \text{ T}$

Exercise 4.9

Question: A square coil of side 10 cm consists of 20 turns and carries a current of 12 A. The coil is suspended vertically and the normal to the plane of the coil makes an angle of 30° with the direction of a uniform horizontal magnetic field of magnitude 0.80 T. What is the magnitude of torque experienced by the coil?

Solution: $A = (0.1)^2 = 0.01 \text{ m}^2$.

$$\tau = NIAB \sin \theta = 20 \times 12 \times 0.01 \times 0.80 \times \sin 30^\circ = 0.96 \text{ N m}$$

Answer: $\tau = 0.96 \text{ N m}$

Exercise 4.10

Question: Two moving coil meters, M_1 and M_2 , have the following particulars: $R_1 = 10 \Omega$, $N_1 = 30$, $A_1 = 3.6 \times 10^{-3} \text{ m}^2$, $B_1 = 0.25 \text{ T}$; $R_2 = 14 \Omega$, $N_2 = 42$, $A_2 = 1.8 \times 10^{-3} \text{ m}^2$, $B_2 = 0.50 \text{ T}$. (The spring constants are identical for the two meters.) Determine the ratio of (a) current sensitivity and (b) voltage sensitivity of M_2 and M_1 .

Solution:

(a) Current sensitivity $= \frac{NAB}{k}$:

$$\frac{(CS)_2}{(CS)_1} = \frac{N_2 A_2 B_2}{N_1 A_1 B_1} = \frac{42 \times 1.8 \times 10^{-3} \times 0.50}{30 \times 3.6 \times 10^{-3} \times 0.25} = \frac{0.0378}{0.027} = 1.4$$

(b) Voltage sensitivity $= \frac{NAB}{kR}$:

$$\frac{(VS)_2}{(VS)_1} = 1.4 \times \frac{R_1}{R_2} = 1.4 \times \frac{10}{14} = 1$$

Answer: (a) 1.4 (b) 1

Exercise 4.11

Question: In a chamber, a uniform magnetic field of 6.5 G ($1 \text{ G} = 10^{-4} \text{ T}$) is maintained. An electron is shot into the field with a speed of $4.8 \times 10^6 \text{ m s}^{-1}$ normal to the field. Explain why the path of the electron is a circle. Determine the radius of the circular orbit. ($e = 1.6 \times 10^{-19} \text{ C}$, $m_e = 9.1 \times 10^{-31} \text{ kg}$)

Solution: The magnetic force $-e\vec{v} \times \vec{B}$ is always perpendicular to the velocity and has constant magnitude evB . It does no work, so the speed stays constant and the force acts as a **centripetal force**. Hence the path is a **circle**.

$$r = \frac{mv}{eB} = \frac{9.1 \times 10^{-31} \times 4.8 \times 10^6}{1.6 \times 10^{-19} \times 6.5 \times 10^{-4}} = 4.48 \times 10^{-2} \text{ m} \approx 4.5 \text{ cm}$$

(With the standard value $e = 1.6 \times 10^{-19} \text{ C}$, $r = 4.2 \text{ cm}$.)

Answer: $r \approx 4.5 \text{ cm}$ (4.2 cm with $e = 1.6 \times 10^{-19} \text{ C}$)

Exercise 4.12

Question: In Exercise 4.11 obtain the frequency of revolution of the electron in its circular orbit. Does the answer depend on the speed of the electron? Explain.

Solution:

$$\nu = \frac{eB}{2\pi m} = \frac{1.5 \times 10^{-19} \times 6.5 \times 10^{-4}}{2\pi \times 9.1 \times 10^{-31}} = 1.7 \times 10^7 \text{ Hz} = 17 \text{ MHz}$$

(With $e = 1.6 \times 10^{-19} \text{ C}$, $\nu = 18 \text{ MHz}$.)

The frequency **does not depend on the speed**. A faster electron moves in a proportionally larger circle ($r \propto v$), so the time for one revolution, $T = 2\pi r/v = 2\pi m/eB$, stays the same.

Answer: $\nu \approx 17 \text{ MHz}$ (18 MHz with $e = 1.6 \times 10^{-19} \text{ C}$); independent of speed.

Exercise 4.13

Question: (a) A circular coil of 30 turns and radius 8.0 cm carrying a current of 6.0 A is suspended vertically in a uniform horizontal magnetic field of magnitude 1.0 T. The field lines make an angle of 60° with the normal of the coil. Calculate the magnitude of the counter torque that must be applied to prevent the coil from turning. (b) Would your answer change, if the circular coil in (a) were replaced by a planar coil of some irregular shape that encloses the same area? (All other particulars are also unaltered.)

Solution:

(a) $A = \pi r^2 = \pi (0.08)^2 = 2.01 \times 10^{-2} \text{ m}^2$.

$$\tau = NIAB \sin \theta = 30 \times 6.0 \times 2.01 \times 10^{-2} \times 1.0 \times \sin 60^\circ = 3.13 \text{ N m}$$

The counter torque must be equal and opposite: **3.13 N m**.

(b) **No**. The torque $\tau = NIAB \sin \theta$ depends only on the area of the coil, not on its shape (for a planar loop).

Answer: (a) 3.13 N m (b) No change