



Chapter 6 : Electromagnetic Induction (Solved Examples & Exercises)

by - Tomar Sir

Notes

Step-by-step solutions · NCERT Examples 6.1–6.10 and Exercises 6.1–6.8 · Class 12 Physics

Part A — NCERT Solved Examples

Example 6.1

Question: Consider Experiment 6.2. (a) What would you do to obtain a large deflection of the galvanometer? (b) How would you demonstrate the presence of an induced current in the absence of a galvanometer?

Solution:

- (a) To obtain a large deflection, one or more of the following steps can be taken: (i) use a rod made of **soft iron** inside the coil C_2 ; (ii) connect the coil to a **powerful battery**; (iii) move the arrangement **rapidly** towards the test coil C_1 .
- (b) Replace the galvanometer by a **small bulb**, of the kind found in a small torch. The relative motion between the two coils will cause the bulb to glow and thus demonstrate the presence of an induced current.

In experimental physics one must learn to innovate. Michael Faraday is ranked as one of the best experimentalists ever.

Example 6.2

Question: A square loop of side 10 cm and resistance 0.5Ω is placed vertically in the east-west plane. A uniform magnetic field of 0.10 T is set up across the plane in the north-east direction. The magnetic field is decreased to zero in 0.70 s at a steady rate. Determine the magnitudes of induced emf and current during this time-interval.

Solution: The area vector of the loop points north-south, and the field is north-east, so $\theta = 45^\circ$. The initial flux is

$$\Phi = BA \cos \theta = \frac{0.1 \times 10^{-2}}{\sqrt{2}} \text{ Wb}$$

Final flux = 0. The change takes 0.70 s, so

$$\varepsilon = \frac{|\Delta\Phi|}{t} = \frac{10^{-3}}{\sqrt{2} \times 0.7} = 1.0 \text{ mV}$$

$$I = \frac{\varepsilon}{R} = \frac{10^{-3}}{0.5} = 2 \text{ mA}$$

The earth's magnetic field also produces a flux through the loop, but it is steady and induces no emf.

Answer: $\varepsilon = 1.0 \text{ mV}$; $I = 2 \text{ mA}$

Example 6.3

Question: A circular coil of radius 10 cm, 500 turns and resistance $2\ \Omega$ is placed with its plane perpendicular to the horizontal component of the earth's magnetic field. It is rotated about its vertical diameter through 180° in 0.25 s. Estimate the magnitudes of the emf and current induced in the coil. Horizontal component of the earth's magnetic field at the place is $3.0 \times 10^{-5}\ \text{T}$.

Solution:

$$\Phi_{\text{initial}} = BA \cos 0^\circ = 3.0 \times 10^{-5} \times (\pi \times 10^{-2}) = 3\pi \times 10^{-7}\ \text{Wb}$$

$$\Phi_{\text{final}} = BA \cos 180^\circ = -3\pi \times 10^{-7}\ \text{Wb}$$

$$\varepsilon = N \frac{\Delta\Phi}{t} = \frac{500 \times 6\pi \times 10^{-7}}{0.25} = 3.8 \times 10^{-3}\ \text{V}$$

$$I = \frac{\varepsilon}{R} = 1.9 \times 10^{-3}\ \text{A}$$

These are **estimated (average)** values; the instantaneous values depend on the speed of rotation at that instant.

Example 6.4

Question: Figure 6.7 shows planar loops of different shapes moving out of or into a region of a magnetic field which is directed normal to the plane of the loop away from the reader. Determine the direction of induced current in each loop using Lenz's law.

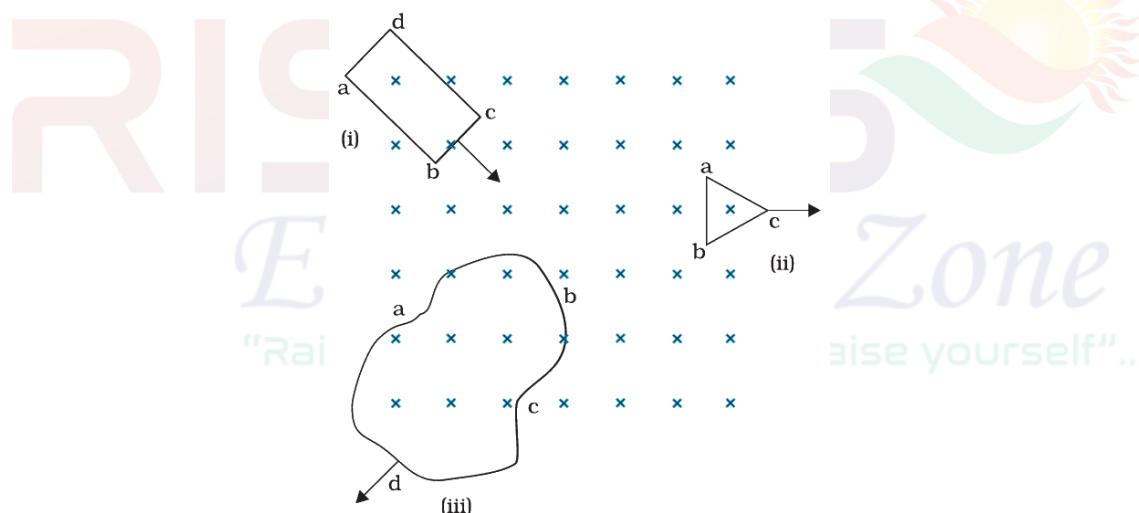


Fig. 6.7

Solution:

- (i) The flux through the rectangular loop abcd **increases** as it moves into the field. The induced current flows along **bcdab** to oppose the increasing flux.
- (ii) The flux through the triangular loop abc **decreases** as it moves out. The induced current flows along **back** to oppose the change.
- (iii) The flux through the irregular loop abcd **decreases** as it moves out. The induced current flows along **cdabc**.

There is no induced current while a loop is completely inside or completely outside the field.

Example 6.5

Question: (a) A closed loop is held stationary in the magnetic field between the north and south poles of two permanent magnets held fixed. Can we hope to generate current in the loop by using very strong magnets? (b) A

closed loop moves normal to the constant electric field between the plates of a large capacitor. Is a current induced in the loop (i) when it is wholly inside the region between the capacitor plates (ii) when it is partially outside the plates of the capacitor? The electric field is normal to the plane of the loop. (c) A rectangular loop and a circular loop are moving out of a uniform magnetic field region (Fig. 6.8) to a field-free region with a constant velocity v . In which loop do you expect the induced emf to be constant during the passage out of the field region? The field is normal to the loops. (d) Predict the polarity of the capacitor in the situation described by Fig. 6.9.



Fig. 6.8

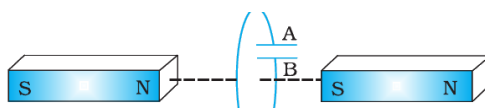


Fig. 6.9: Two magnets moving towards a coil connected to a capacitor with plates A and B

Solution:

- (a) **No**. However strong the magnet, a current can be induced only by **changing** the magnetic flux through the loop.
- (b) **No** current is induced in either case. A current cannot be induced by changing the **electric** flux.
- (c) The induced emf is constant only for the **rectangular loop**. For the circular loop, the rate of change of area inside the field is not constant as it leaves, so the emf varies.
- (d) Plate A will be **positive** with respect to plate B.

Example 6.6

Question: A metallic rod of 1 m length is rotated with a frequency of 50 rev/s, with one end hinged at the centre and the other end at the circumference of a circular metallic ring of radius 1 m, about an axis passing through the centre and perpendicular to the plane of the ring (Fig. 6.11). A constant and uniform magnetic field of 1 T parallel to the axis is present everywhere. What is the emf between the centre and the metallic ring?

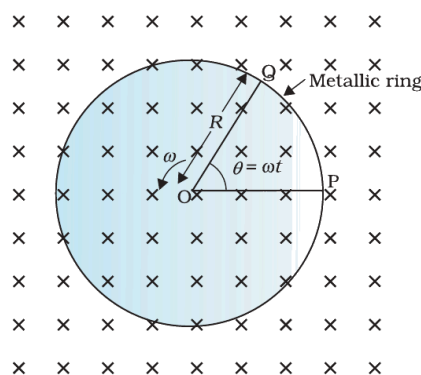


Fig. 6.11

Solution:

Method I: Free electrons in the rotating rod move to the outer end due to the Lorentz force until a steady emf builds up. An element dr moving with speed $v = \omega r$ has emf $d\varepsilon = Bv dr$:

$$\varepsilon = \int_0^R Bv dr = \int_0^R B\omega r dr = \frac{B\omega R^2}{2}$$

$$\varepsilon = \frac{1}{2} \times 1.0 \times 2\pi \times 50 \times 1^2 = 157 \text{ V}$$

Method II: Imagine a closed loop OPQ. The area of sector OPQ is $\pi R^2 \frac{\theta}{2\pi} = \frac{1}{2} R^2 \theta$:

$$\varepsilon = B \frac{d}{dt} \left(\frac{1}{2} R^2 \theta \right) = \frac{1}{2} B R^2 \frac{d\theta}{dt} = \frac{B \omega R^2}{2}$$

with $\frac{d\theta}{dt} = \omega = 2\pi\nu$. Both methods give the same result.

Answer: $\varepsilon = 157 \text{ V}$

Example 6.7

Question: A wheel with 10 metallic spokes each 0.5 m long is rotated with a speed of 120 rev/min in a plane normal to the horizontal component of earth's magnetic field H_E at a place. If $H_E = 0.4 \text{ G}$ at the place, what is the induced emf between the axle and the rim of the wheel? Note that $1 \text{ G} = 10^{-4} \text{ T}$.

Solution: $\omega = 120 \text{ rev/min} = 2 \text{ rev/s} = 4\pi \text{ rad/s}$.

$$\varepsilon = \frac{1}{2} \omega B R^2 = \frac{1}{2} \times 4\pi \times 0.4 \times 10^{-4} \times (0.5)^2 = 6.28 \times 10^{-5} \text{ V}$$

The number of spokes is immaterial because the emfs across the spokes are **in parallel**.

Answer: $6.28 \times 10^{-5} \text{ V}$

Example 6.8

Question: Two concentric circular coils, one of small radius r_1 and the other of large radius r_2 , such that $r_1 \ll r_2$, are placed co-axially with centres coinciding. Obtain the mutual inductance of the arrangement.

Solution: Let a current I_2 flow through the outer coil. The field at its centre is $B_2 = \frac{\mu_0 I_2}{2r_2}$. Since the inner coil is very small, B_2 may be taken as constant over its area:

$$\Phi_1 = \pi r_1^2 B_2 = \frac{\mu_0 \pi r_1^2}{2r_2} I_2 = M_{12} I_2$$

$$M_{12} = M_{21} = \frac{\mu_0 \pi r_1^2}{2r_2}$$

This uses an approximate value of Φ_1 (B assumed uniform over πr_1^2), which is acceptable because $r_1 \ll r_2$.

Example 6.9

Question: (a) Obtain the expression for the magnetic energy stored in a solenoid in terms of magnetic field B , area A and length l of the solenoid. (b) How does this magnetic energy compare with the electrostatic energy stored in a capacitor?

Solution:

(a) Using $U_B = \frac{1}{2} L I^2$, $B = \mu_0 n I$ and $L = \mu_0 n^2 A l$:

$$U_B = \frac{1}{2} L \left(\frac{B}{\mu_0 n} \right)^2 = \frac{1}{2} (\mu_0 n^2 A l) \left(\frac{B}{\mu_0 n} \right)^2 = \frac{1}{2\mu_0} B^2 A l$$

(b) The magnetic energy per unit volume is

$$u_B = \frac{U_B}{V} = \frac{U_B}{A l} = \frac{B^2}{2\mu_0}$$

The electrostatic energy per unit volume in a parallel plate capacitor is $u_E = \frac{1}{2} \varepsilon_0 E^2$.

In both cases, the **energy is proportional to the square of the field strength**. Though derived for special cases, these expressions are general — valid for any region where a magnetic and/or electric field exists.

Example 6.10

Question: Kamla peddles a stationary bicycle. The pedals of the bicycle are attached to a 100 turn coil of area 0.10 m^2 . The coil rotates at half a revolution per second and it is placed in a uniform magnetic field of 0.01 T perpendicular to the axis of rotation of the coil. What is the maximum voltage generated in the coil?

Solution: $\nu = 0.5 \text{ Hz}$, $N = 100$, $A = 0.1 \text{ m}^2$, $B = 0.01 \text{ T}$.

$$\varepsilon_0 = NBA (2\pi\nu) = 100 \times 0.01 \times 0.1 \times 2 \times 3.14 \times 0.5 = 0.314 \text{ V}$$

Answer: The maximum voltage is 0.314 V .

Part B — NCERT Exercises

Exercise 6.1

Question: Predict the direction of induced current in the situations described by the following Figs. 6.15(a) to (f).

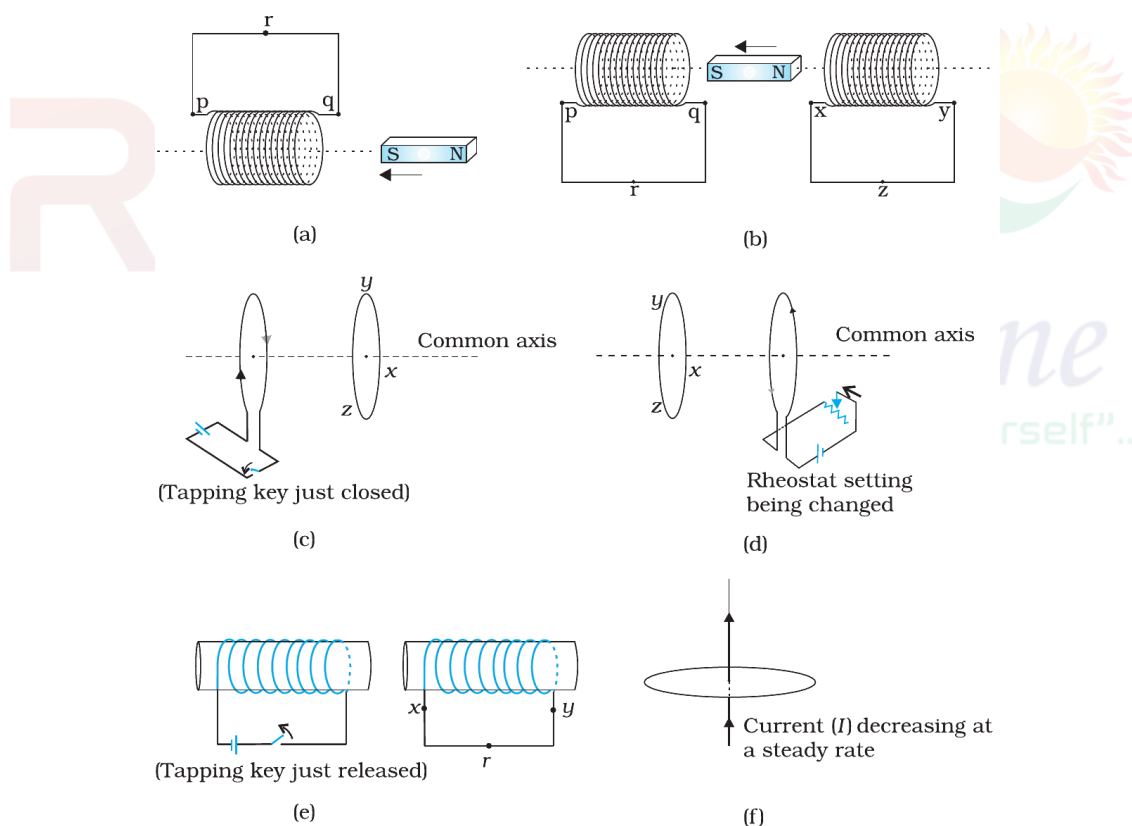


Fig. 6.15

Answer: Using Lenz's law (the induced current opposes the change in flux):

- (a) Along **qrpq**.
- (b) Along **prq** in the left coil and along **yzx** in the right coil.
- (c) Along **yzxy**.
- (d) Along **zyxz**.
- (e) Along **xry**.

- **(f) No current** is induced, since the field lines of the straight wire lie in the plane of the loop — the flux through it is zero and does not change.

Exercise 6.2

Question: Use Lenz's law to determine the direction of induced current in the situations described by Fig. 6.16:

(a) A wire of irregular shape turning into a circular shape; (b) A circular loop being deformed into a narrow straight wire.

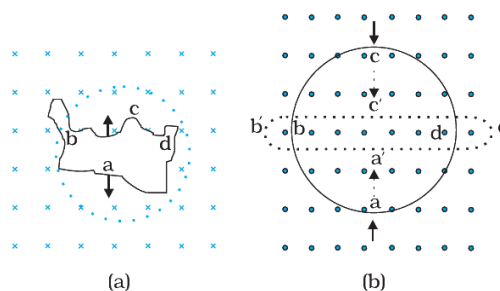


Fig. 6.16

Answer:

- **(a)** As the irregular wire turns into a circle, its area **increases**, so the flux (field into the page) increases. The induced current flows to oppose the increase: along **adcb**.
- **(b)** As the circle is deformed into a narrow wire, its area **decreases**, so the flux (field out of the page) decreases. The induced current flows to maintain the flux: along **a'd'c'b'**.

Exercise 6.3

Question: A long solenoid with 15 turns per cm has a small loop of area 2.0 cm^2 placed inside the solenoid normal to its axis. If the current carried by the solenoid changes steadily from 2.0 A to 4.0 A in 0.1 s, what is the induced emf in the loop while the current is changing?

Solution: $n = 15 \text{ turns/cm} = 1500 \text{ m}^{-1}$; $A = 2.0 \times 10^{-4} \text{ m}^2$; $dI/dt = 2.0/0.1 = 20 \text{ A/s}$.

$$\varepsilon = A \frac{dB}{dt} = \mu_0 n A \frac{dI}{dt} = 4\pi \times 10^{-7} \times 1500 \times 2 \times 10^{-4} \times 20 = 7.5 \times 10^{-6} \text{ V}$$

Answer: $\varepsilon \approx 7.5 \times 10^{-6} \text{ V}$

Exercise 6.4

Question: A rectangular wire loop of sides 8 cm and 2 cm with a small cut is moving out of a region of uniform magnetic field of magnitude 0.3 T directed normal to the loop. What is the emf developed across the cut if the velocity of the loop is 1 cm s^{-1} in a direction normal to the (a) longer side, (b) shorter side of the loop? For how long does the induced voltage last in each case?

Solution: $\varepsilon = Blv$, where l is the side perpendicular to the velocity (the side cutting the field boundary), and $v = 0.01 \text{ m/s}$.

(a) Velocity normal to the longer side ($l = 0.08 \text{ m}$):

$$\varepsilon = 0.3 \times 0.08 \times 0.01 = 2.4 \times 10^{-4} \text{ V}$$

The loop must move 2 cm to leave the field: time = $2/1 = 2 \text{ s}$.

(b) Velocity normal to the shorter side ($l = 0.02 \text{ m}$):

$$\varepsilon = 0.3 \times 0.02 \times 0.01 = 0.6 \times 10^{-4} \text{ V}$$

The loop must move 8 cm: time = $8/1 = 8 \text{ s}$.

Answer: (a) 2.4×10^{-4} V for 2 s (b) 0.6×10^{-4} V for 8 s

Exercise 6.5

Question: A 1.0 m long metallic rod is rotated with an angular frequency of 400 rad s^{-1} about an axis normal to the rod passing through its one end. The other end of the rod is in contact with a circular metallic ring. A constant and uniform magnetic field of 0.5 T parallel to the axis exists everywhere. Calculate the emf developed between the centre and the ring.

Solution:

$$\varepsilon = \frac{1}{2} B \omega l^2 = \frac{1}{2} \times 0.5 \times 400 \times (1.0)^2 = 100 \text{ V}$$

Answer: 100 V

Exercise 6.6

Question: A horizontal straight wire 10 m long extending from east to west is falling with a speed of 5.0 m s^{-1} , at right angles to the horizontal component of the earth's magnetic field, $0.30 \times 10^{-4} \text{ Wb m}^{-2}$. (a) What is the instantaneous value of the emf induced in the wire? (b) What is the direction of the emf? (c) Which end of the wire is at the higher electrical potential?

Solution:

(a)

$$\varepsilon = Blv = 0.30 \times 10^{-4} \times 10 \times 5.0 = 1.5 \times 10^{-3} \text{ V}$$

(b) Take x east, y north, z up. $\vec{v} = -5\hat{k}$ (falling), $\vec{B} = B\hat{j}$ (towards north). The force on a positive charge is along $\vec{v} \times \vec{B} \propto (-\hat{k}) \times \hat{j} = +\hat{i}$, i.e. towards the **east**. The emf is directed from **west to east**.

(c) Positive charges collect at the **eastern end**, which is at the higher potential.

Answer: (a) 1.5 mV (b) west to east (c) the eastern end

Exercise 6.7

Question: Current in a circuit falls from 5.0 A to 0.0 A in 0.1 s. If an average emf of 200 V is induced, give an estimate of the self-inductance of the circuit.

Solution:

$$|\varepsilon| = L \frac{\Delta I}{\Delta t} \Rightarrow L = \frac{\varepsilon \Delta t}{\Delta I} = \frac{200 \times 0.1}{5.0} = 4 \text{ H}$$

Answer: $L = 4 \text{ H}$

Exercise 6.8

Question: A pair of adjacent coils has a mutual inductance of 1.5 H. If the current in one coil changes from 0 to 20 A in 0.5 s, what is the change of flux linkage with the other coil?

Solution:

$$\Delta(N\Phi) = M \Delta I = 1.5 \times (20 - 0) = 30 \text{ Wb}$$

(The time 0.5 s is not needed for the flux linkage; it gives the induced emf, $1.5 \times 20/0.5 = 60 \text{ V}$.)

Answer: 30 Wb