



Chapter 7 : Alternating Current (Solved Examples & Exercises)

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Notes

Step-by-step solutions · NCERT Examples 7.1–7.10 and Exercises 7.1–7.8 · Class 12 Physics

Part A — NCERT Solved Examples

Example 7.1

Question: A light bulb is rated at 100 W for a 220 V supply. Find (a) the resistance of the bulb; (b) the peak voltage of the source; and (c) the rms current through the bulb.

Solution:

(a) $P = 100 \text{ W}$ and $V = 220 \text{ V}$:

$$R = \frac{V^2}{P} = \frac{(220)^2}{100} = 484 \, \Omega$$

(b) The peak voltage of the source:

$$v_m = \sqrt{2} V = 311 \text{ V}$$

(c) Since $P = IV$:

$$I = \frac{P}{V} = \frac{100}{220} = 0.454 \text{ A}$$

Example 7.2

Question: A pure inductor of 25.0 mH is connected to a source of 220 V. Find the inductive reactance and rms current in the circuit if the frequency of the source is 50 Hz.

Solution:

$$X_L = 2\pi\nu L = 2 \times 3.14 \times 50 \times 25 \times 10^{-3} = 7.85 \, \Omega$$

$$I = \frac{V}{X_L} = \frac{220}{7.85} = 28 \text{ A}$$

Example 7.3

Question: A lamp is connected in series with a capacitor. Predict your observations for dc and ac connections. What happens in each case if the capacitance of the capacitor is reduced?

Solution:

- **dc source:** The capacitor gets charged, and after charging **no current flows** in the circuit, so the lamp **will not glow**. There will be no change even if C is reduced.

- **ac source:** The capacitor offers capacitive reactance $1/\omega C$ and current flows in the circuit. Consequently, the **lamp will shine**. Reducing C will **increase the reactance**, and the lamp will shine **less brightly** than before.

Example 7.4

Question: A $15.0\ \mu\text{F}$ capacitor is connected to a $220\ \text{V}$, $50\ \text{Hz}$ source. Find the capacitive reactance and the current (rms and peak) in the circuit. If the frequency is doubled, what happens to the capacitive reactance and the current?

Solution:

$$X_C = \frac{1}{2\pi\nu C} = \frac{1}{2\pi \times 50 \times 15.0 \times 10^{-6}} = 212\ \Omega$$

$$I = \frac{V}{X_C} = \frac{220}{212} = 1.04\ \text{A}, \quad i_m = \sqrt{2} I = 1.41 \times 1.04 = 1.47\ \text{A}$$

The current oscillates between $+1.47\ \text{A}$ and $-1.47\ \text{A}$, and is ahead of the voltage by $\pi/2$.

If the frequency is doubled, the capacitive reactance is **halved** and consequently the current is **doubled**.

Example 7.5

Question: A light bulb and an open coil inductor are connected to an ac source through a key as shown in Fig. 7.9. The switch is closed and after sometime, an iron rod is inserted into the interior of the inductor. The glow of the light bulb (a) increases; (b) decreases; (c) is unchanged, as the iron rod is inserted. Give your answer with reasons.

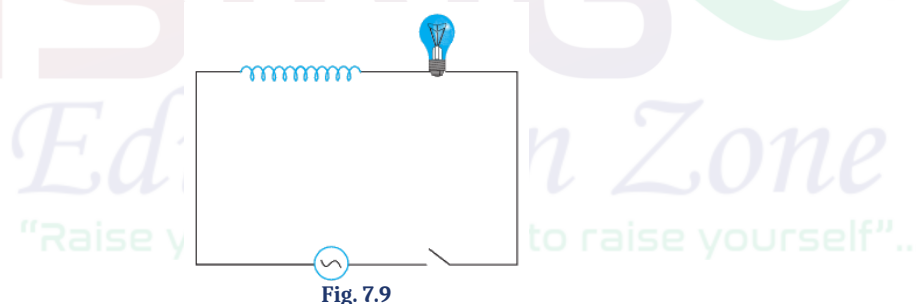


Fig. 7.9

Solution: As the iron rod is inserted, the magnetic field inside the coil magnetises the iron, increasing the magnetic field inside it. Hence the **inductance of the coil increases**, and so does its inductive reactance. As a result, a larger fraction of the applied ac voltage appears across the inductor, leaving less voltage across the bulb.

Answer: (b) The glow of the light bulb **decreases**.

Example 7.6

Question: A resistor of $200\ \Omega$ and a capacitor of $15.0\ \mu\text{F}$ are connected in series to a $220\ \text{V}$, $50\ \text{Hz}$ ac source. (a) Calculate the current in the circuit; (b) Calculate the voltage (rms) across the resistor and the capacitor. Is the algebraic sum of these voltages more than the source voltage? If yes, resolve the paradox.

Solution: $R = 200\ \Omega$, $C = 15.0 \times 10^{-6}\ \text{F}$, $V = 220\ \text{V}$, $\nu = 50\ \text{Hz}$.

(a)

$$Z = \sqrt{R^2 + X_C^2} = \sqrt{(200)^2 + (2 \times 3.14 \times 50 \times 15.0 \times 10^{-6})^{-2}} = \sqrt{(200)^2 + (212.3)^2} = 291.67\ \Omega$$

$$I = \frac{V}{Z} = \frac{220}{291.5} = 0.755\ \text{A}$$

(b) The current is the same throughout the circuit:

$$V_R = IR = 0.755 \times 200 = 151 \text{ V}, \quad V_C = IX_C = 0.755 \times 212.3 = 160.3 \text{ V}$$

The algebraic sum $V_R + V_C = 311.3 \text{ V}$ is **more** than the source voltage of 220 V.

Resolving the paradox: The two voltages are **not in the same phase** — they are 90° out of phase — so they cannot be added like ordinary numbers. They must be added using the Pythagorean theorem:

$$V_{R+C} = \sqrt{V_R^2 + V_C^2} = 220 \text{ V}$$

which equals the source voltage.

Example 7.7

Question: (a) For circuits used for transporting electric power, a low power factor implies large power loss in transmission. Explain. (b) Power factor can often be improved by the use of a capacitor of appropriate capacitance in the circuit. Explain.

Solution:

(a) $P = IV \cos \phi$, where $\cos \phi$ is the power factor. To supply a given power at a given voltage, if $\cos \phi$ is small, the current must be increased accordingly. This leads to a **large power loss** ($I^2 R$) in transmission.

(b) Suppose the current I lags the voltage by ϕ ; the power factor is $\cos \phi = R/Z$. We improve it (towards 1) by making Z tend to R .

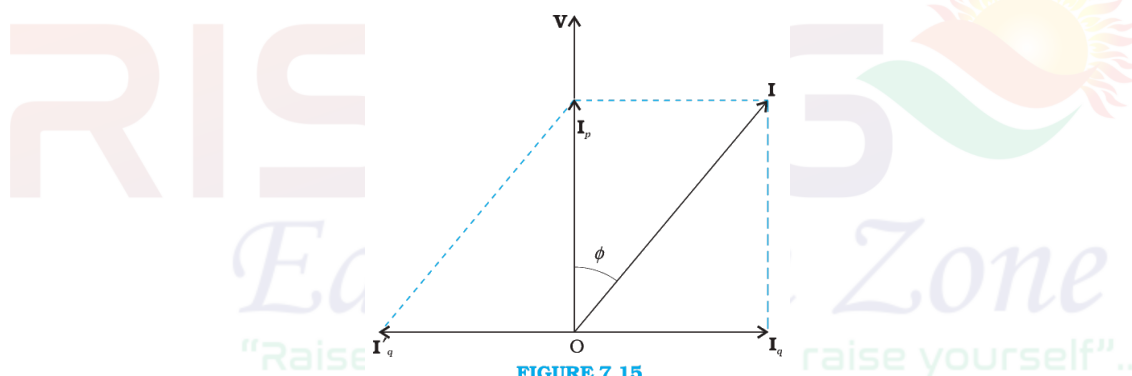


FIGURE 7.15
Fig. 7.15

Resolve I into two components: I_p along V (the **power component**, which corresponds to power loss) and I_q perpendicular to V (the **wattless component**, no power loss). To improve the power factor, the lagging wattless current I_q is neutralised by an equal **leading** wattless current I'_q . This is done by connecting a **capacitor of appropriate value in parallel**, so that I_q and I'_q cancel and P is effectively $I_p V$.

Example 7.8

Question: A sinusoidal voltage of peak value 283 V and frequency 50 Hz is applied to a series LCR circuit in which $R = 3 \Omega$, $L = 25.48 \text{ mH}$, and $C = 796 \mu\text{F}$. Find (a) the impedance of the circuit; (b) the phase difference between the voltage across the source and the current; (c) the power dissipated in the circuit; and (d) the power factor.

Solution:

(a)

$$X_L = 2\pi\nu L = 2 \times 3.14 \times 50 \times 25.48 \times 10^{-3} = 8 \Omega$$

$$X_C = \frac{1}{2\pi\nu C} = \frac{1}{2 \times 3.14 \times 50 \times 796 \times 10^{-6}} = 4 \Omega$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{3^2 + (8 - 4)^2} = 5 \, \Omega$$

(b)

$$\phi = \tan^{-1} \frac{X_C - X_L}{R} = \tan^{-1} \left(\frac{4 - 8}{3} \right) = -53.1^\circ$$

Since ϕ is negative, the current in the circuit **lags** the voltage across the source.

(c)

$$I = \frac{i_m}{\sqrt{2}} = \frac{1}{\sqrt{2}} \left(\frac{283}{5} \right) = 40 \, \text{A}, \quad P = I^2 R = (40)^2 \times 3 = 4800 \, \text{W}$$

(d) Power factor = $\cos \phi = \cos(-53.1^\circ) = 0.6$

Example 7.9

Question: Suppose the frequency of the source in the previous example can be varied. (a) What is the frequency of the source at which resonance occurs? (b) Calculate the impedance, the current, and the power dissipated at the resonant condition.

Solution:

(a)

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{25.48 \times 10^{-3} \times 796 \times 10^{-6}}} = 222.1 \, \text{rad/s}$$

$$\nu_r = \frac{\omega_0}{2\pi} = \frac{222.1}{2 \times 3.14} = 35.4 \, \text{Hz}$$

(b) At resonance, the impedance equals the resistance: $Z = R = 3 \, \Omega$.

$$I = \frac{V}{Z} = \frac{V}{R} = \left(\frac{283}{\sqrt{2}} \right) \frac{1}{3} = 66.7 \, \text{A}$$

$$P = I^2 R = (66.7)^2 \times 3 = 13.35 \, \text{kW}$$

The power dissipated at resonance is **more** than in Example 7.8.

Example 7.10

Question: At an airport, a person is made to walk through the doorway of a metal detector, for security reasons. If she/he is carrying anything made of metal, the metal detector emits a sound. On what principle does this detector work?

Solution: The metal detector works on the principle of **resonance in ac circuits**. When you walk through it, you are in fact walking through a coil of many turns. The coil is connected to a capacitor tuned so that the circuit is in resonance. When you walk through with metal in your pocket, the **impedance of the circuit changes**, resulting in a significant change in the current. This change in current is detected, and the electronic circuitry causes a sound to be emitted as an alarm.

Part B — NCERT Exercises

Exercise 7.1

Question: A $100 \, \Omega$ resistor is connected to a 220 V, 50 Hz ac supply. (a) What is the rms value of current in the circuit? (b) What is the net power consumed over a full cycle?

Solution:

$$(a) I = \frac{V}{R} = \frac{220}{100} = 2.2 \text{ A}$$

$$(b) P = VI = 220 \times 2.2 = 484 \text{ W}$$

Answer: (a) 2.2 A (b) 484 W

Exercise 7.2

Question: (a) The peak voltage of an ac supply is 300 V. What is the rms voltage? (b) The rms value of current in an ac circuit is 10 A. What is the peak current?

Solution:

$$(a) V = \frac{v_m}{\sqrt{2}} = \frac{300}{1.414} = 212.1 \text{ V}$$

$$(b) i_m = \sqrt{2} I = 1.414 \times 10 = 14.1 \text{ A}$$

Exercise 7.3

Question: A 44 mH inductor is connected to 220 V, 50 Hz ac supply. Determine the rms value of the current in the circuit.

Solution:

$$X_L = 2\pi\nu L = 2\pi \times 50 \times 44 \times 10^{-3} = 13.8 \Omega$$

$$I = \frac{V}{X_L} = \frac{220}{13.8} = 15.9 \text{ A}$$

Answer: $\approx 15.9 \text{ A}$

Exercise 7.4

Question: A 60 μF capacitor is connected to a 110 V, 60 Hz ac supply. Determine the rms value of the current in the circuit.

Solution:

$$X_C = \frac{1}{2\pi\nu C} = \frac{1}{2\pi \times 60 \times 60 \times 10^{-6}} = 44.2 \Omega$$

$$I = \frac{V}{X_C} = \frac{110}{44.2} = 2.49 \text{ A}$$

Answer: $\approx 2.49 \text{ A}$

Exercise 7.5

Question: In Exercises 7.3 and 7.4, what is the net power absorbed by each circuit over a complete cycle? Explain your answer.

Answer: **Zero** in both cases. In a purely inductive circuit the current lags the voltage by $\pi/2$, and in a purely capacitive circuit it leads by $\pi/2$. So $\cos \phi = \cos 90^\circ = 0$, and

$$P = VI \cos \phi = 0$$

The energy stored in the inductor (or capacitor) in one quarter cycle is returned to the source in the next quarter cycle.

Exercise 7.6

Question: A charged 30 μF capacitor is connected to a 27 mH inductor. What is the angular frequency of free oscillations of the circuit?

Solution:

$$\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{27 \times 10^{-3} \times 30 \times 10^{-6}}} = \frac{1}{9 \times 10^{-4}} = 1.11 \times 10^3 \text{ rad s}^{-1}$$

Answer: $\omega \approx 1.1 \times 10^3 \text{ rad s}^{-1}$

Exercise 7.7

Question: A series LCR circuit with $R = 20 \Omega$, $L = 1.5 \text{ H}$ and $C = 35 \mu\text{F}$ is connected to a variable-frequency 200 V ac supply. When the frequency of the supply equals the natural frequency of the circuit, what is the average power transferred to the circuit in one complete cycle?

Solution: At resonance, $Z = R = 20 \Omega$, so

$$I = \frac{V}{R} = \frac{200}{20} = 10 \text{ A}, \quad P = I^2 R = 100 \times 20 = 2000 \text{ W}$$

Answer: 2000 W (2 kW)

Exercise 7.8

Question: Figure 7.17 shows a series LCR circuit connected to a variable frequency 230 V source. $L = 5.0 \text{ H}$, $C = 80 \mu\text{F}$, $R = 40 \Omega$. (a) Determine the source frequency which drives the circuit in resonance. (b) Obtain the impedance of the circuit and the amplitude of current at the resonating frequency. (c) Determine the rms potential drops across the three elements of the circuit. Show that the potential drop across the LC combination is zero at the resonating frequency.

Solution:

(a)

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{5.0 \times 80 \times 10^{-6}}} = 50 \text{ rad s}^{-1}, \quad \nu_0 = \frac{50}{2\pi} = 7.96 \text{ Hz}$$

(b) At resonance, $Z = R = 40 \Omega$.

$$I = \frac{230}{40} = 5.75 \text{ A}, \quad i_m = \sqrt{2} \times 5.75 = 8.13 \text{ A}$$

(c) rms potential drops:

$$\begin{aligned} V_R &= IR = 5.75 \times 40 = 230 \text{ V} \\ V_L &= I\omega_0 L = 5.75 \times 50 \times 5 = 1437.5 \text{ V} \\ V_C &= \frac{I}{\omega_0 C} = \frac{5.75}{50 \times 80 \times 10^{-6}} = 1437.5 \text{ V} \end{aligned}$$

V_L and V_C are equal in magnitude but **180° out of phase**, so the potential drop across the LC combination is

$$V_{LC} = |V_L - V_C| = 0$$

All of the source voltage (230 V) appears across R.