

Chapter 7 : Alternating Current (Notes)

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Notes

GSEB and CBSE Board Exam preparation subjective notes · Class 12 Physics · NCERT Chapter 7

Q.1 An ac voltage is applied to a pure resistor. Show that current and voltage are in phase and derive the average power. (3 Marks)

Let the source voltage be

$$v = v_m \sin \omega t$$

where v_m is the amplitude and ω the angular frequency (Fig. 7.1).



Fig. 7.1: AC voltage applied to a resistor

By Kirchhoff's loop rule, $v_m \sin \omega t = iR$, so

$$i = \frac{v_m}{R} \sin \omega t = i_m \sin \omega t, \quad i_m = \frac{v_m}{R}$$

Both v and i reach zero, minimum and maximum values at the same time — **the voltage and current are in phase** (Fig. 7.2). Ohm's law holds for ac as well as dc.

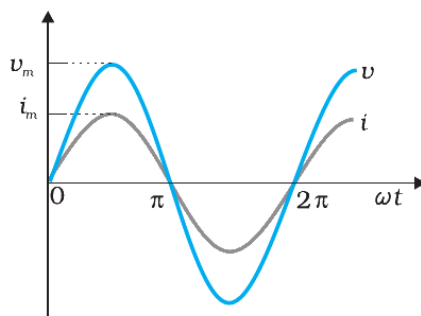


Fig. 7.2: In a pure resistor, the voltage and current are in phase

Average current over a cycle is zero, but there is still heating, since Joule heating depends on i^2 (always positive).

Power: The instantaneous power is $p = i^2 R = i_m^2 R \sin^2 \omega t$. Its average over a cycle:

$$\bar{p} = i_m^2 R \langle \sin^2 \omega t \rangle$$

Since $\sin^2 \omega t = \frac{1}{2} (1 - \cos 2\omega t)$ and $\langle \cos 2\omega t \rangle = 0$, we get $\langle \sin^2 \omega t \rangle = \frac{1}{2}$:

$$p = \frac{1}{2} i_m^2 R$$

Key Examination Points for Full Marks:

- The symbol for an ac source in a circuit diagram is $\ominus$ (a circle with a sine wave).
- The average of a function over a period T : $\langle F(t) \rangle = \frac{1}{T} \int_0^T F(t) dt$.

Q.2 Define rms (effective) current and voltage. Derive $I = i_m/\sqrt{2}$. (2 Marks)

To express ac power in the same form as dc power ($P = I^2 R$), a special value of current is used — the **root mean square (rms) or effective current**:

$$I = \sqrt{\bar{i^2}} = \sqrt{\frac{1}{2} i_m^2} = \frac{i_m}{\sqrt{2}} \Rightarrow I = 0.707 i_m$$

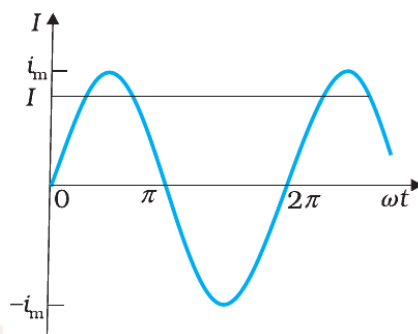


Fig. 7.3: The rms current I is related to the peak current by $I = i_m/\sqrt{2} = 0.707 i_m$

In terms of I , the average power is

$$P = \bar{p} = \frac{1}{2} i_m^2 R = I^2 R$$

Similarly, the **rms voltage**:

$$V = \frac{v_m}{\sqrt{2}} = 0.707 v_m$$

From $v_m = i_m R$: $V = IR$. Also $P = \frac{V^2}{R} = IV$.

Definition: The rms current is the value of the **steady (dc) current that would produce the same average power loss (heat) in a resistor as the alternating current.**

Key Examination Points for Full Marks:

- Ac values are normally quoted as rms values. Household supply 220 V (rms) has a peak value $v_m = \sqrt{2} \times 220 = 311$ V.
- Example 7.1:** A 100 W, 220 V bulb: $R = V^2/P = 484 \Omega$; $v_m = 311$ V; $I = P/V = 0.454$ A.

Q.3 What is a phasor? Explain the phasor diagram for a resistive ac circuit. (2 Marks)

Phasor: A vector which **rotates about the origin** with angular speed ω . The **vertical components** (projections) of the voltage and current phasors represent the sinusoidally varying quantities v and i . The **magnitudes** of the phasors V and I represent the **amplitudes** (peak values) v_m and i_m .

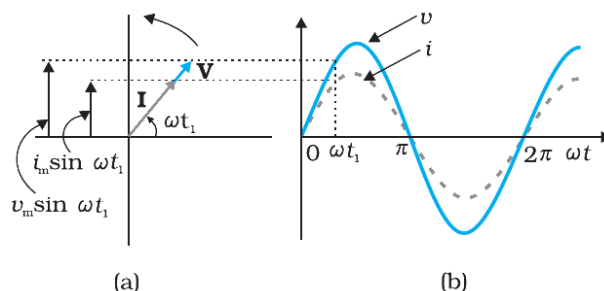


Fig. 7.4: (a) A phasor diagram for the resistive circuit of Fig. 7.1. (b) Graph of v and i versus ωt

At time t_1 , the projections of the phasors are $v_m \sin \omega t_1$ and $i_m \sin \omega t_1$. As they rotate with frequency ω , they generate the curves of Fig. 7.4(b).

For a **resistor**, phasors V and I point in the **same direction** at all times — the phase angle between voltage and current is **zero**.

Key Examination Points for Full Marks:

- Voltage and current are not really vectors; they are **scalar** quantities. Phasors are only a convenient way to add harmonically varying quantities with their phases taken into account.
- The angle between two phasors equals the phase difference between the quantities.

Q.4 An ac voltage is applied to a pure inductor. Show that the current lags the voltage by $\pi/2$ and the average power is zero. (3 Marks)

Let $v = v_m \sin \omega t$ be applied to an inductor L of negligible resistance (Fig. 7.5). By Kirchhoff's loop rule:

$$v - L \frac{di}{dt} = 0 \Rightarrow \frac{di}{dt} = \frac{v_m}{L} \sin \omega t$$

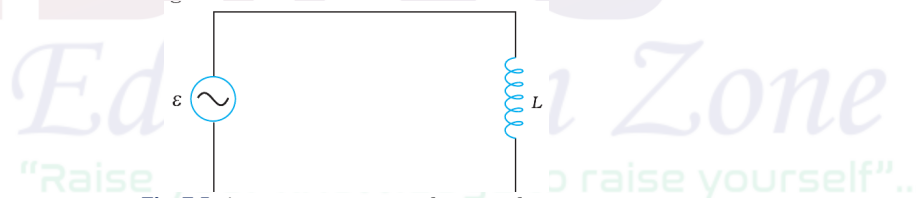


Fig. 7.5: An ac source connected to an inductor

Integrating (the constant is zero, since the current oscillates symmetrically about zero):

$$i = -\frac{v_m}{\omega L} \cos \omega t = i_m \sin \left(\omega t - \frac{\pi}{2} \right)$$

$$i_m = \frac{v_m}{X_L}, \quad X_L = \omega L$$

X_L is the **inductive reactance** (unit: ohm). It limits the current like a resistance, and is directly proportional to L and to the frequency.

Phase: The current **lags** the voltage by $\pi/2$ (one-quarter cycle) (Fig. 7.6).

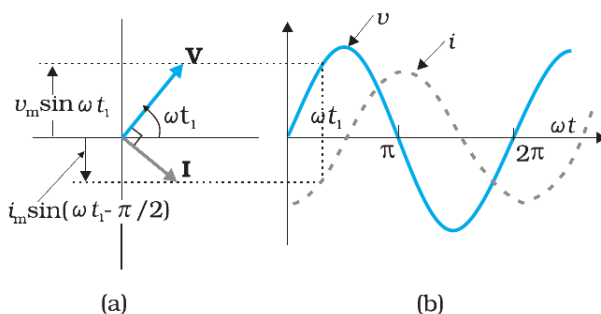


Fig. 7.6: (a) Phasor diagram for the circuit of Fig. 7.5. (b) Graph of v and i versus ωt

Power:

$$p_L = iv = i_m \sin \left(\omega t - \frac{\pi}{2} \right) v_m \sin \omega t = -\frac{i_m v_m}{2} \sin 2\omega t$$

$$P_L = \left\langle -\frac{i_m v_m}{2} \sin 2\omega t \right\rangle = 0$$

The average power supplied to an inductor over a complete cycle is zero.

Key Examination Points for Full Marks:

- For dc ($\omega = 0$), $X_L = 0$ — an inductor offers no opposition to steady current.
- Example 7.2:** $L = 25.0 \text{ mH}$, 220 V , 50 Hz : $X_L = 7.85 \Omega$, $I = 28 \text{ A}$.

Q.5 An ac voltage is applied to a capacitor. Show that the current leads the voltage by $\pi/2$ and the average power is zero. (3 Marks)

Let $v = v_m \sin \omega t$ be applied to a capacitor C (Fig. 7.7). With charge q at time t : $v = \frac{q}{C}$ so $q = Cv_m \sin \omega t$.



Fig. 7.7: An ac source connected to a capacitor

$$i = \frac{dq}{dt} = \omega C v_m \cos \omega t = i_m \sin \left(\omega t + \frac{\pi}{2} \right)$$

$$i_m = \omega C v_m = \frac{v_m}{1/\omega C} = \frac{v_m}{X_C}, \quad X_C = \frac{1}{\omega C}$$

X_C is the **capacitive reactance** (unit: ohm). It is inversely proportional to the frequency and the capacitance.

Phase: The current **leads** the voltage by $\pi/2$ — it reaches its maximum a quarter period earlier (Fig. 7.8).

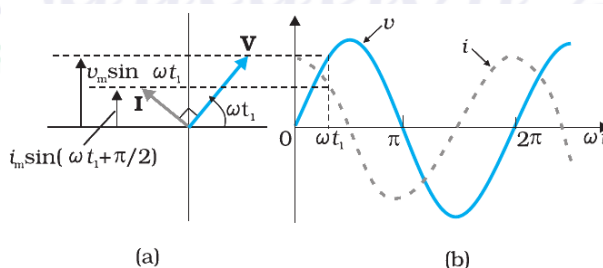


Fig. 7.8: (a) Phasor diagram for the circuit of Fig. 7.7. (b) Graph of v and i versus ωt

Power:

$$p_C = iv = i_m v_m \cos \omega t \sin \omega t = \frac{i_m v_m}{2} \sin 2\omega t \Rightarrow P_C = 0$$

Key Examination Points for Full Marks:

- In dc, a capacitor blocks the current once charged ($X_C = \infty$ for $\omega = 0$). In ac, it is charged and discharged alternately, so current flows.
- Example 7.3:** A lamp in series with a capacitor glows on ac but not on dc; reducing C increases X_C and the lamp becomes dimmer.
- Example 7.4:** $15.0 \mu\text{F}$, 220 V , 50 Hz : $X_C = 212 \Omega$, $I = 1.04 \text{ A}$, $i_m = 1.47 \text{ A}$. If the frequency is doubled, X_C halves and the current doubles.

Q.6 Derive the impedance and phase angle of a series LCR circuit using a phasor diagram. (3 Marks)

A series LCR circuit is connected to an ac source $v = v_m \sin \omega t$ (Fig. 7.10). The same current $i = i_m \sin (\omega t + \phi)$ flows through each element.

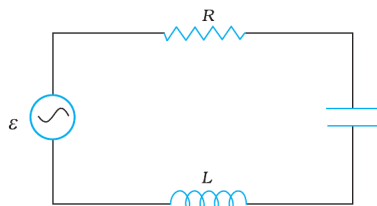


Fig. 7.10: A series LCR circuit connected to an ac source

Let $\vec{V}_L$, $\vec{V}_R$, $\vec{V}_C$ be the voltage phasors. $\vec{V}_R$ is parallel to $\vec{I}$; $\vec{V}_C$ is $\pi/2$ behind $\vec{I}$; $\vec{V}_L$ is $\pi/2$ ahead of $\vec{I}$ (Fig. 7.11). Their amplitudes are

$$v_{Rm} = i_m R, \quad v_{Cm} = i_m X_C, \quad v_{Lm} = i_m X_L$$

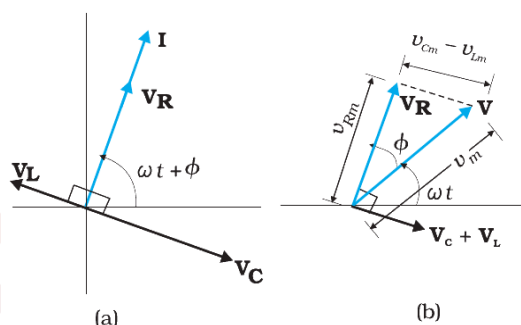


Fig. 7.11: (a) Phasors V_L , V_R , V_C and I . (b) V_L , V_R and $(V_L + V_C)$

Since $\vec{V}_C$ and $\vec{V}_L$ are opposite, they combine into one phasor of magnitude $|v_{Cm} - v_{Lm}|$. $\vec{V}$ is the hypotenuse of a right triangle with sides $\vec{V}_R$ and $(\vec{V}_C + \vec{V}_L)$:

$$v_m^2 = v_{Rm}^2 + (v_{Cm} - v_{Lm})^2 = i_m^2 [R^2 + (X_C - X_L)^2]$$

$$i_m = \frac{v_m}{\sqrt{R^2 + (X_C - X_L)^2}} = \frac{v_m}{Z}$$

Impedance:

$$Z = \sqrt{R^2 + (X_C - X_L)^2}$$

Phase angle (between $\vec{V}_R$ and $\vec{V}$):

$$\tan \phi = \frac{X_C - X_L}{R}$$

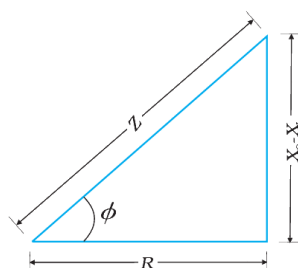


Fig. 7.12: Impedance diagram

- If $X_C > X_L$: ϕ positive; the circuit is **predominantly capacitive** and the current **leads** the voltage.
- If $X_C < X_L$: ϕ negative; the circuit is **predominantly inductive** and the current **lags** the voltage.

Key Examination Points for Full Marks:

- The **impedance diagram** is a right triangle with Z as the hypotenuse, R as the base and $(X_C - X_L)$ as the height.
- The phasor method gives only the **steady-state** solution; the transient solution dies out with time.
- Example 7.6:** $R = 200\ \Omega$, $C = 15.0\ \mu\text{F}$, $220\ \text{V}$, $50\ \text{Hz}$: $Z = 291.5\ \Omega$, $I = 0.755\ \text{A}$, $V_R = 151\ \text{V}$, $V_C = 160.3\ \text{V}$. $V_R + V_C = 311\ \text{V} > 220\ \text{V}$, but $\sqrt{V_R^2 + V_C^2} = 220\ \text{V}$ — voltages must be added as phasors.

Q.7 What is resonance in a series LCR circuit? Derive the resonant frequency. (3 Marks)

Resonance: The phenomenon in which the amplitude of the current is **maximum** when the frequency of the applied voltage equals the natural frequency of the circuit.

The current amplitude is

$$i_m = \frac{v_m}{Z} = \frac{v_m}{\sqrt{R^2 + (X_C - X_L)^2}}$$

As ω varies, at a particular frequency ω_0 , $X_C = X_L$ and the impedance is **minimum**, $Z = R$:

$$\frac{1}{\omega_0 C} = \omega_0 L \Rightarrow \omega_0 = \frac{1}{\sqrt{LC}}, \quad \nu_0 = \frac{1}{2\pi\sqrt{LC}}$$

At resonance, $i_m = \frac{v_m}{R}$ (maximum), and the current is in phase with the voltage.

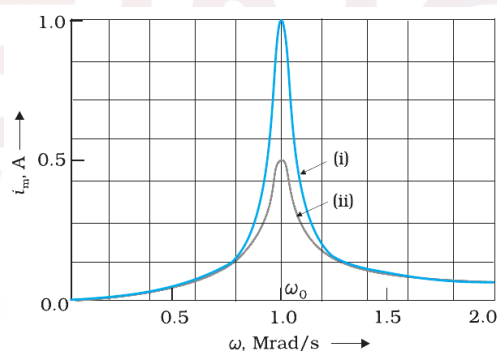


Fig. 7.14: Variation of i_m with ω for (i) $R = 100\ \Omega$ and (ii) $R = 200\ \Omega$, with $L = 1.00\ \text{mH}$, $C = 1.00\ \text{nF}$

For $L = 1.00\ \text{mH}$, $C = 1.00\ \text{nF}$ and $v_m = 100\ \text{V}$, $\omega_0 = 1.00 \times 10^6\ \text{rad/s}$. The peak for $R = 100\ \Omega$ is twice that for $R = 200\ \Omega$ — a smaller R gives a **sharper** resonance.

Applications: Tuning of a radio or TV — the capacitance in the tuning circuit is varied until the resonant frequency equals the frequency of the desired station, giving maximum current for that signal. Metal detectors at airports also work on resonance (Example 7.10).

Key Examination Points for Full Marks:

- Resonance is possible only if both L and C are present.** Only then can V_L and V_C cancel. There is no resonance in an RL or RC circuit.
- Quality factor:** $Q = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 C R}$ — a measure of the sharpness of resonance. $Q = \frac{\omega_0}{\Delta\omega}$ (bandwidth).

Q.8 Derive the expression for average power in an ac circuit. Define power factor and discuss special cases. (3 Marks)

For $v = v_m \sin \omega t$ and $i = i_m \sin (\omega t + \phi)$, the instantaneous power is

$$p = vi = v_m i_m \sin \omega t \sin (\omega t + \phi) = \frac{v_m i_m}{2} [\cos \phi - \cos (2\omega t + \phi)]$$

The second term averages to zero over a cycle. Hence

$$P = \frac{v_m i_m}{2} \cos \phi = \frac{v_m}{\sqrt{2}} \frac{i_m}{\sqrt{2}} \cos \phi \Rightarrow P = VI \cos \phi = I^2 Z \cos \phi$$

The quantity **$\cos \phi$ is the power factor.**

Cases:

- **(i) Resistive circuit:** $\phi = 0$, $\cos \phi = 1$ — **maximum power** dissipation.
- **(ii) Purely inductive or capacitive circuit:** $\phi = \pi/2$, $\cos \phi = 0$ — **no power** is dissipated even though current flows. Such a current is called **wattless current**.
- **(iii) LCR series circuit:** $\phi = \tan^{-1} \frac{X_C - X_L}{R}$; power is dissipated **only in the resistor**.
- **(iv) At resonance:** $X_C = X_L$, $\phi = 0$, $\cos \phi = 1$ and $P = I^2 Z = I^2 R$ — **maximum power**.

Key Examination Points for Full Marks:

- **Example 7.7:** A low power factor means a large current for the same power, so large $I^2 R$ losses in transmission. Power factor is improved by connecting a capacitor in parallel, which cancels the lagging wattless component I_q of the current.

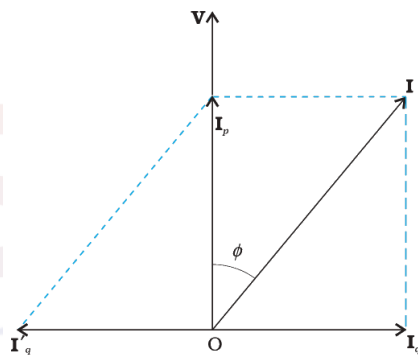


FIGURE 7.15

Fig. 7.15: Resolving the current I into the power component I_p (along V) and the wattless component I_q

- **Example 7.8:** $R = 3 \Omega$, $X_L = 8 \Omega$, $X_C = 4 \Omega$; $Z = 5 \Omega$, $\phi = -53.1^\circ$, $I = 40 \text{ A}$, $P = 4800 \text{ W}$, power factor 0.6.

Q.9 Explain the principle, construction and working of a transformer. (3 Marks)

Derive the voltage and current ratios.

Principle: Mutual induction — an alternating current in one coil induces an emf in a neighbouring coil.

Construction: Two coils insulated from each other, wound on a **soft-iron core** — either one on top of the other (Fig. 7.16(a)) or on separate limbs of the core (Fig. 7.16(b)). The **primary** (input) coil has N_p turns and the **secondary** (output) coil has N_s turns.

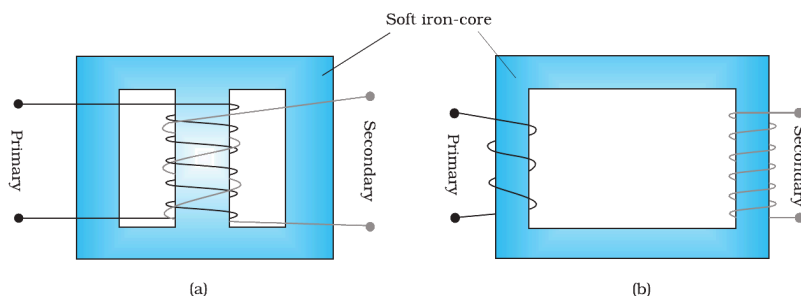


Fig. 7.16: Two arrangements for winding the primary and secondary coils: (a) one on top of the other, (b) on separate limbs of the core

Working: An alternating voltage on the primary produces an alternating flux ϕ in the core, which links the secondary. For an ideal transformer (no resistance, no flux leakage):

$$\varepsilon_s = -N_s \frac{d\phi}{dt}, \quad \varepsilon_p = -N_p \frac{d\phi}{dt}$$

With $\varepsilon_p = v_p$ and $\varepsilon_s = v_s$ (small secondary current):

$$\frac{v_s}{v_p} = \frac{N_s}{N_p}$$

If the transformer is 100% efficient: $i_p v_p = i_s v_s$, so

$$\frac{i_p}{i_s} = \frac{v_s}{v_p} = \frac{N_s}{N_p}$$

- **Step-up transformer** ($N_s > N_p$): $V_s > V_p$ and $I_s < I_p$.
- **Step-down transformer** ($N_s < N_p$): $V_s < V_p$ and $I_s > I_p$.

Energy losses in real transformers:

- i. **Flux leakage** — reduced by winding the coils one over the other.
- ii. **Resistance of the windings** ($I^2 R$) — reduced by using thick wire.
- iii. **Eddy currents** in the core — reduced by using a **laminated core**.
- iv. **Hysteresis** — reduced by using a core material with low hysteresis loss.

Key Examination Points for Full Marks:

- A transformer works only on **ac** (dc gives no changing flux).
- In power transmission, voltage is **stepped up** at the generating station so that the current, and hence the $I^2 R$ loss, is reduced; it is stepped down near the consumers (to 240 V).
- A step-up transformer does not violate conservation of energy — the current is reduced in the same proportion.

Q.10 Explain why a series LCR circuit can be used for tuning, and why an inductor in series with a bulb dims it on inserting an iron rod. (2 Marks)

Tuning: The antenna picks up signals of many frequencies. The capacitance in the tuning circuit is varied so that the resonant frequency $\nu_0 = \frac{1}{2\pi\sqrt{LC}}$ equals the frequency of the desired station. The current for that signal becomes maximum, while signals of other frequencies give much smaller currents.

Iron rod in an inductor (Example 7.5): A bulb and an open-coil inductor are connected in series to an ac source (Fig. 7.9). When an iron rod is inserted, the iron gets magnetised and increases the magnetic field inside the coil, so the **inductance increases**. The inductive reactance $X_L = \omega L$ increases, a larger part of the applied voltage appears across the inductor, and less voltage is left across the bulb. Hence **the glow of the bulb decreases**.

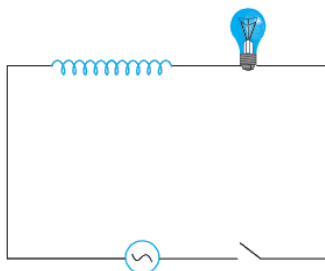


Fig. 7.9: A light bulb and an open-coil inductor connected to an ac source through a key

Key Examination Points for Full Marks:

- A choke coil (large L , small R) controls ac current with almost no power loss, unlike a resistor.

Formula Sheet — Alternating Current

Quantity	Formula	SI Unit
AC voltage and current	$v = v_m \sin \omega t; i = i_m \sin (\omega t + \phi)$	V, A
rms values	$I = \frac{i_m}{\sqrt{2}} = 0.707 i_m; V = \frac{v_m}{\sqrt{2}}$	A, V
Average current (half cycle)	$\bar{i} = \frac{2i_m}{\pi} = 0.637 i_m$	A
Inductive reactance	$X_L = \omega L = 2\pi\nu L$ (current lags by $\pi/2$)	Ω
Capacitive reactance	$X_C = \frac{1}{\omega C}$ (current leads by $\pi/2$)	Ω
Impedance (series LCR)	$Z = \sqrt{R^2 + (X_C - X_L)^2}$	Ω
Phase angle	$\tan \phi = \frac{X_C - X_L}{R}$	—
Resonant frequency	$\omega_0 = \frac{1}{\sqrt{LC}}; \nu_0 = \frac{1}{2\pi\sqrt{LC}}$	$\text{rad s}^{-1}, \text{Hz}$
Quality factor	$Q = \frac{\omega_0 L}{R} = \frac{1}{R} \sqrt{\frac{L}{C}} = \frac{\omega_0}{\Delta\omega}$	—
Average power	$P = VI \cos \phi = I^2 R$	W
Power factor	$\cos \phi = \frac{R}{Z}$	—
LC oscillation	$\omega = \frac{1}{\sqrt{LC}}; q = q_0 \cos \omega t$	rad s^{-1}
Transformer	$\frac{V_s}{V_p} = \frac{N_s}{N_p} = \frac{I_p}{I_s}$	—
Efficiency	$\eta = \frac{P_{out}}{P_{in}} \times 100\%$	—

Dimensions: reactance and impedance $[\text{ML}^2 \text{T}^{-3} \text{A}^{-2}]$; resonant frequency $[\text{T}^{-1}]$; Q and power factor dimensionless. $\sqrt{LC}$ and L/R and CR all have dimensions of **time**.

NEET / JEE Quick Points and Tricks

- 1. Phase rule:** “CIVIL” — in a Capacitor, **I** leads **V**; in an inductor (**L**), **V** leads **I**.
- 2. Ac with dc component:** for $i = I_0 + i_m \sin \omega t$, $I_{rms} = \sqrt{I_0^2 + \frac{i_m^2}{2}}$. For $i = I_1 \sin \omega t + I_2 \cos \omega t$,

$$I_{rms} = \sqrt{\frac{I_1^2 + I_2^2}{2}}.$$
- 3. Time from zero to peak** = $T/4$; from peak to rms value = $T/8$.
- 4. Voltages in series LCR** add as phasors: $V = \sqrt{V_R^2 + (V_L - V_C)^2}$. At resonance V_L and V_C can each be much larger than the source voltage (voltage magnification = Q).
- 5. At resonance:** $Z = R$ (minimum), I maximum, $\phi = 0$, power factor 1, $V_L = V_C$, $V_{LC} = 0$.
- 6. Below resonance** the circuit is capacitive (current leads); **above resonance** it is inductive (current lags).
- 7. Bandwidth** $\Delta\omega = R/L$. Increasing R lowers Q and broadens the resonance curve.
- 8. Wattless current** = $I_{rms} \sin \phi$; power component = $I_{rms} \cos \phi$.

9. **dc vs ac:** an inductor passes dc freely ($X_L = 0$) but opposes high frequencies; a capacitor blocks dc ($X_C = \infty$) but passes high frequencies.
10. **LC oscillations:** energy is exchanged between C and L; the energies are equal when $q = q_0/\sqrt{2}$ (after $T/8$).
Maximum current $I_0 = q_0\omega = V_0\sqrt{C/L}$.
11. **Only resistors dissipate power.** Average power in ideal L or C is zero.
12. **Transformer:** works only on ac; frequency is unchanged. Real transformer: $P_{out} < P_{in}$. Laminated core reduces eddy currents; soft iron reduces hysteresis loss.
13. **Choke coil** is preferred to a resistor for controlling ac current because it consumes almost no power.
14. **Lamp at 50% of maximum power** $\rightarrow \cos \varphi = 1/2 \rightarrow \varphi = 60^\circ$.

