



Chapter 8 : Electromagnetic Waves (Solved Examples & Exercises)

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Notes

Step-by-step solutions · NCERT Examples 8.1–8.2 and Exercises 8.1–8.10 · Class 12 Physics

Part A — NCERT Solved Examples

Example 8.1

Question: A plane electromagnetic wave of frequency 25 MHz travels in free space along the x-direction. At a particular point in space and time, $\vec{E} = 6.3 \hat{j}$ V/m. What is $\vec{B}$ at this point?

Solution: The magnitude of B is

$$B = \frac{E}{c} = \frac{6.3}{3 \times 10^8} = 2.1 \times 10^{-8} \text{ T}$$

Direction: $\vec{E}$ is along y and the wave propagates along x, so $\vec{B}$ must be perpendicular to both x and y. Using vector algebra, $\vec{E} \times \vec{B}$ should be along x. Since $(+\hat{j}) \times (+\hat{k}) = \hat{i}$, $\vec{B}$ is along the z-direction.

Answer: $\vec{B} = 2.1 \times 10^{-8} \hat{k}$ T

Example 8.2

Question: The magnetic field in a plane electromagnetic wave is given by $B_y = (2 \times 10^{-7}) \text{ T} \sin (0.5 \times 10^3 x + 1.5 \times 10^{11} t)$. (a) What is the wavelength and frequency of the wave? (b) Write an expression for the electric field.

Solution:

(a) Comparing with $B_y = B_0 \sin \left[2\pi \left(\frac{x}{\lambda} + \frac{t}{T} \right) \right]$:

$$\lambda = \frac{2\pi}{0.5 \times 10^3} \text{ m} = 1.26 \text{ cm}, \quad \nu = \frac{1}{T} = \frac{1.5 \times 10^{11}}{2\pi} = 23.9 \text{ GHz}$$

(b)

$$E_0 = B_0 c = 2 \times 10^{-7} \times 3 \times 10^8 = 6 \times 10^1 \text{ V/m}$$

The electric field is perpendicular to both the direction of propagation (x) and the magnetic field (y), so it is along z:

$$E_z = 60 \sin (0.5 \times 10^3 x + 1.5 \times 10^{11} t) \text{ V/m}$$

Part B — NCERT Exercises

Exercise 8.1

Question: Figure 8.5 shows a capacitor made of two circular plates each of radius 12 cm, and separated by 5.0 cm. The capacitor is being charged by an external source (not shown in the figure). The charging current is constant and equal to 0.15 A. (a) Calculate the capacitance and the rate of change of potential difference between the plates. (b) Obtain the displacement current across the plates. (c) Is Kirchhoff's first rule (junction rule) valid at each plate of the capacitor? Explain.

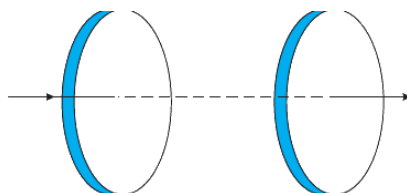


Fig. 8.5

Solution:

(a)

$$C = \frac{\epsilon_0 A}{d} = \frac{8.85 \times 10^{-12} \times \pi \times (0.12)^2}{0.05} = 8.0 \times 10^{-12} \text{ F} = 8.0 \text{ pF}$$

From $Q = CV$, $i = C \frac{dV}{dt}$:

$$\frac{dV}{dt} = \frac{i}{C} = \frac{0.15}{8.0 \times 10^{-12}} = 1.87 \times 10^{10} \text{ V s}^{-1}$$

(b) The displacement current equals the conduction current: $i_d = \epsilon_0 \frac{d\Phi_E}{dt} = 0.15 \text{ A}$.

(c) **Yes**, Kirchhoff's junction rule is valid at each plate, provided we take the current to be the **sum of the conduction and displacement currents**. The conduction current flowing into a plate equals the displacement current leaving it (across the gap).

Answer: (a) 8.0 pF, $1.87 \times 10^{10} \text{ V/s}$ (b) 0.15 A (c) Yes

Exercise 8.2

Question: A parallel plate capacitor (Fig. 8.6) made of circular plates each of radius $R = 6.0 \text{ cm}$ has a capacitance $C = 100 \text{ pF}$. The capacitor is connected to a 230 V ac supply with an (angular) frequency of 300 rad s^{-1} . (a) What is the rms value of the conduction current? (b) Is the conduction current equal to the displacement current? (c) Determine the amplitude of B at a point 3.0 cm from the axis between the plates.

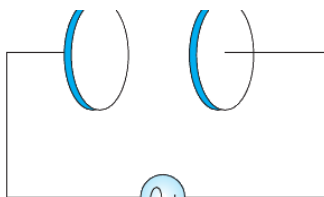


Fig. 8.6

Solution:

(a)

$$I_{rms} = \frac{V_{rms}}{X_C} = V_{rms} \omega C = 230 \times 300 \times 100 \times 10^{-12} = 6.9 \times 10^{-6} \text{ A} = 6.9 \mu\text{A}$$

(b) **Yes.** The displacement current between the plates equals the conduction current in the wires (whether the current is steady or alternating).

(c) Inside the plates, the displacement current through a circle of radius r is $i_d \frac{r^2}{R^2}$. By the Ampere–Maxwell law:

$$B (2\pi r) = \mu_0 i_d \frac{r^2}{R^2} \Rightarrow B = \frac{\mu_0 r}{2\pi R^2} i_d$$

Using the peak current $i_0 = \sqrt{2} \times 6.9 \times 10^{-6} = 9.76 \times 10^{-6}$ A:

$$B_0 = \frac{4\pi \times 10^{-7} \times 0.03 \times 9.76 \times 10^{-6}}{2\pi \times (0.06)^2} = 1.63 \times 10^{-11} \text{ T}$$

Answer: (a) $6.9 \mu\text{A}$ (b) Yes (c) 1.63×10^{-11} T

Exercise 8.3

Question: What physical quantity is the same for X-rays of wavelength 10^{-10} m, red light of wavelength $6800 \text{ \AA}$ and radiowaves of wavelength 500 m?

Answer: The **speed in vacuum**, $c = 3 \times 10^8$ m/s, is the same for all of them. It is independent of wavelength.

Exercise 8.4

Question: A plane electromagnetic wave travels in vacuum along z-direction. What can you say about the directions of its electric and magnetic field vectors? If the frequency of the wave is 30 MHz , what is its wavelength?

Solution: The electric and magnetic field vectors lie in the **x-y plane**, perpendicular to each other and to the z-direction (so that $\vec{E} \times \vec{B}$ is along +z).

$$\lambda = \frac{c}{\nu} = \frac{3 \times 10^8}{30 \times 10^6} = 10 \text{ m}$$

Answer: $\vec{E}$ and $\vec{B}$ in the x-y plane, mutually perpendicular; $\lambda = 10$ m

Exercise 8.5

Question: A radio can tune in to any station in the 7.5 MHz to 12 MHz band. What is the corresponding wavelength band?

Solution:

$$\lambda_1 = \frac{3 \times 10^8}{7.5 \times 10^6} = 40 \text{ m}, \quad \lambda_2 = \frac{3 \times 10^8}{12 \times 10^6} = 25 \text{ m}$$

Answer: 40 m to 25 m

Exercise 8.6

Question: A charged particle oscillates about its mean equilibrium position with a frequency of 10^9 Hz . What is the frequency of the electromagnetic waves produced by the oscillator?

Answer: The frequency of the EM waves equals the frequency of oscillation of the charge: **10^9 Hz** .

Exercise 8.7

Question: The amplitude of the magnetic field part of a harmonic electromagnetic wave in vacuum is $B_0 = 510$ nT. What is the amplitude of the electric field part of the wave?

Solution:

$$E_0 = cB_0 = 3 \times 10^8 \times 510 \times 10^{-9} = 153 \text{ N/C}$$

Answer: 153 N/C (V/m)

Exercise 8.8

Question: Suppose that the electric field amplitude of an electromagnetic wave is $E_0 = 120 \text{ N/C}$ and that its frequency is $\nu = 50.0 \text{ MHz}$. (a) Determine B_0 , ω , k , and λ . (b) Find expressions for E and B .

Solution:

(a)

$$B_0 = \frac{E_0}{c} = \frac{120}{3 \times 10^8} = 4 \times 10^{-7} \text{ T} = 400 \text{ nT}$$

$$\omega = 2\pi\nu = 2\pi \times 50 \times 10^6 = 3.14 \times 10^8 \text{ rad/s}$$

$$k = \frac{\omega}{c} = \frac{3.14 \times 10^8}{3 \times 10^8} = 1.05 \text{ rad/m}, \quad \lambda = \frac{c}{\nu} = 6.0 \text{ m}$$

(b) Taking the wave along +x, with $\vec{E}$ along y and $\vec{B}$ along z:

$$\vec{E} = 120 \sin (1.05x - 3.14 \times 10^8 t) \hat{j} \text{ N/C}$$

$$\vec{B} = 4 \times 10^{-7} \sin (1.05x - 3.14 \times 10^8 t) \hat{k} \text{ T}$$

Exercise 8.9

Question: The terminology of different parts of the electromagnetic spectrum is given in the text. Use the formula $E = h\nu$ (for energy of a quantum of radiation: photon) and obtain the photon energy in units of eV for different parts of the electromagnetic spectrum. In what way are the different scales of photon energies that you obtain related to the sources of electromagnetic radiation?

Solution: $E = \frac{hc}{\lambda}$; in convenient units, $E (\text{eV}) = \frac{1240}{\lambda (\text{nm})} = \frac{1.24 \times 10^{-6}}{\lambda (\text{m})}$.

Region	Typical λ	Photon energy
γ -rays	10^{-12} m	$\approx 10^6 \text{ eV (MeV)}$
X-rays	10^{-10} m	$\approx 10^4 \text{ eV (keV)}$
Ultraviolet	10^{-8} m	$\approx 10^2 \text{ eV}$
Visible	$5 \times 10^{-7} \text{ m}$	$\approx 2.5 \text{ eV}$
Infrared	10^{-5} m	$\approx 10^{-1} \text{ eV}$
Microwaves	10^{-2} m	$\approx 10^{-4} \text{ eV}$
Radio waves	1 m	$\approx 10^{-6} \text{ eV}$

Relation to sources: The photon energy indicates the spacing of energy levels in the source. γ -ray photons (MeV) come from **nuclear** energy levels; X-rays (keV) from **inner-shell** electrons of heavy atoms; visible and UV (eV) from **outer** (valence) electrons in atoms; infrared from **molecular vibrations**; microwaves and radio waves (very small energies) from the **oscillation of charges** in circuits and antennas.

Exercise 8.10

Question: In a plane electromagnetic wave, the electric field oscillates sinusoidally at a frequency of $2.0 \times 10^{10} \text{ Hz}$ and amplitude 48 V m^{-1} . (a) What is the wavelength of the wave? (b) What is the amplitude of the oscillating

magnetic field? (c) Show that the average energy density of the E field equals the average energy density of the B field. [$c = 3 \times 10^8 \text{ m s}^{-1}$.]

Solution:

(a)

$$\lambda = \frac{c}{\nu} = \frac{3 \times 10^8}{2 \times 10^{10}} = 1.5 \times 10^{-2} \text{ m}$$

(b)

$$B_0 = \frac{E_0}{c} = \frac{48}{3 \times 10^8} = 1.6 \times 10^{-7} \text{ T}$$

(c) Average energy densities:

$$\bar{u}_E = \frac{1}{4} \epsilon_0 E_0^2, \quad \bar{u}_B = \frac{B_0^2}{4\mu_0}$$

Using $B_0 = E_0/c$ and $c^2 = \frac{1}{\mu_0 \epsilon_0}$:

$$\bar{u}_B = \frac{E_0^2}{4\mu_0 c^2} = \frac{E_0^2 \mu_0 \epsilon_0}{4\mu_0} = \frac{1}{4} \epsilon_0 E_0^2 = \bar{u}_E$$

Numerically, $\bar{u}_E = \bar{u}_B = \frac{1}{4} \times 8.85 \times 10^{-12} \times (48)^2 = 5.1 \times 10^{-9} \text{ J/m}^3$.

